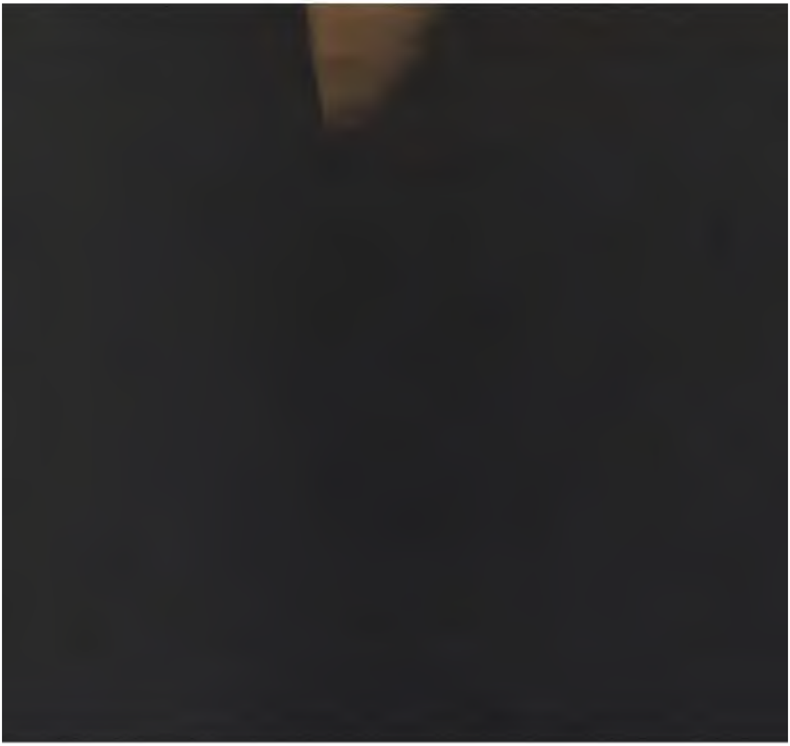

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THE AMERICAN STUDENT'S GUIDE,

CONTAINING

A COMPENDIOUS SYSTEM OF THEORETICAL AND PRACTICAL

ARITHMETIC,

COMPILED FOR THE

USE OF SCHOOLS AND PRIVATE STUDENTS

IN THE

UNITED STATES.

BY GEORGE ALFRED,
A SCHOOLMASTER IN VIRGINIA.

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Mrs. Howard G. Funkhouser
By exchange

THE

AMERICAN STUDENT'S GUIDE

Is thankfully and humbly inscribed to those gentlemen who have so liberally patronized the publication of the work by their subscriptions.

Gentlemen, now permit me to subscribe myself

Your and your children's well wisher,

GEORGE ALFRED.

THE PREFACE EXPLANATORY.

In the compilation of the "AMERICAN STUDENT'S GUIDE," brevity, correctness, and perspicuity have been carefully and punctually attended to, throughout the whole work. The *Rules* are differently arranged from any author I have heretofore noticed, but I have followed that order which an experience of nearly forty years in the practice of tuition has taught me to be the most alleviating to preceptors, and beneficial to the students. The different *Rules* are divided into various *Cases*, and the most improving and useful questions are selected for each *Case*, one of which is explicitly wrought, and explained at the beginning of the *Case*, and the answers given to the rest. Several new modes of contracted operations and explanatory notes will appear in a plain and familiar manner. The *Rules* of Proportion, Practice, and Simple Interest, Vulgar and Decimal Fractions, Involution or the raising of *Powers*, Evolution or the extraction of *Roots*, Annuities, Pensions, &c. in *Arrears*, Single and Double *Position*, Alligation, Progression, Permutation, and Combination of Numbers, a sketch of Mensuration, Artificers' *work*, and Gauging, are explained and exemplified so conspicuously, that it is believed they are intelligible to the most common capacity, and that a youth of tolerable genius and assiduous application may acquire a competent knowledge of *Arithmetic*, without the assistance of a teacher, after having learned the elementary *Rules*, namely, Numeration, Addition, Subtraction, Multiplication, and Division.

Now, I most sincerely and candidly inform the young students of Arithmetic (and hope it will not be taken amiss, as it is for their own benefit,) that a correct knowledge of it is indispensably necessary in their studies and progress through all the calculating sciences. My own experience in the tuition of youth has convinced me, that no proficiency of any consequence can be made by a student in any of the calculating sciences, who is not well versed in the science of Arithmetic. It is also requisite in the common occurrences of business, and nothing in commercial transactions can be done accurately without it. With much pleasure I now embrace the opportu-

nity of expressing my gratitude to those gentlemen who have so liberally patronized the publication of the "AMERICAN STUDENT'S GUIDE," by subscribing for the same, and also request them to excuse the errors of the press, (should any occur, as there is no perfection in this world,) and my own defects. The intimation of an error, given by a friend, will be thankfully received by the compiler, and well wisher of the present and rising generation.

GEORGE ALFRED.

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AN EXPLANATION

Of the marks or characters used in the following system of Arithmetic.

$=$ Two parallel lines signify equality, as 20s.$=$1£, that is, 20 shillings are equal to one pound sterling; so, whatever precedes the sign of equality is equal in value to that which follows it.

$+$ Plus or more, signifies addition, as $8+4=12$, that is, 8 plus 4 is equal to 12, or 8 and 4 make 12.

$-$ Minus or less, signifies subtraction, as $16-4=12$, that is, 16 minus 4 is equal to 12.

$\times$ An ex signifies multiplication, as $6\times5=30$, that is, 6 multiplied by 5 is equal to 30.

$\div$ A line between two dots signifies division, as $30\div6=5$, that is, 30 divided by 6 is equal to 5.

) (A parenthesis reversed also signifies division, as 6)24(4, that is, 24 divided by 6 gives 4 for the quotient.

$\frac{96}{12}$ Numbers placed in the form of a fraction likewise signify division, the upper one is the dividend, and the lower one the divisor, thus $\frac{96}{12}$ expresses the same as $96\div12$, or 12)96(8, that is, 96 contains 12 eight times.

∴ Three dots in the form of A, are sometimes used for *As* or *If*, at the beginning of a stating in proportion.

Two dots like a colon signify *is to*, A double colon signifies *so is*, A diæresis is used for *to*: The foregoing dots are used as the signs of proportionality, as 2 : 4 :: 6 .. 12, or ∴ 2 : 4 :: 6 .. 12 to be read by the scholar, As 2 is to 4 so is 6 to 12.

—— A vinculum denotes that the several numbers or quantities over which it is drawn are to be considered as one simple number or quantity, as $\overline{12-4}+10=18$, that is, the difference between 12 and 4 added to 10 makes 18, and $19-\overline{2+12}$ shows that the sum of 2 and 12 taken from $19=5$, also $\overline{15-10}\times6=30$, shows that the difference between 15 and 10 multiplied by 6 is equal to 30, likewise, $9-2\times\overline{8+4}=84$, shows that the difference between 9 and 2 multiplied by the sum of 8 and 4 is equal to 84.

5^2 Signifies the 2d power or square of 5, which is 25.

5^3 Signifies the 3d power or cube of 5, which is 125.

6^4 Signifies the 4th power of 6, which is 1296.

$\sqrt{}$ or $\sqrt[2]{}$ Prefixed to any number or quantity, signifies that the square root of that number is extracted or required.

$\sqrt[3]{}$ Prefixed to any number or quantity, signifies that the cube root of that number is extracted or required.

$\sqrt[4]{}$ Prefixed to any number or quantity, signifies that the biquadrate root of that number is extracted or required.

ARITHMETIC

Is the art or science of computing by numbers. The rules on which all its operations depend, are, Numeration, Addition, Subtraction, Multiplication, and Division.

NUMERATION.

Numeration teaches us how to express numbers by figures, set down, or named, and consists of two parts, namely:

First. The right method of setting them down;

Second. The true way of valuing each figure in its proper place; both of which are exhibited in the following tables:

TABLE 1.

Hundreds of Millions	Tens of Millions	Millions	Hundreds of Thousands	Tens of Thousands	Thousands	Hundreds	Tens	Units	
								1	One.
							2	1	Twenty-one.
						3	2	1	Three hundred and twenty-one.
					4,	3	2	1	Four thousands 321.
				5	4,	3	2	1	54 thousands 321.
			6	5	4,	3	2	1	654 thousands 321.
		7,	6	5	4,	3	2	1	7 millions 654 thousands 321.
	8	7,	6	5	4,	3	2	1	87 millions 654 thousands 321.
9	8	7,	6	5	4,	3	2	1	987 millions 654 thousand 321.

TABLE 2.

1	2	3,	4	5	6,	7	8	9	123 millions 456 thousands 789.
	2	3,	4	5	6,	7	8	9	23 millions 458 thousands 729.
		3,	4	5	6,	7	8	9	3 millions 456 thousands 789.
			4	5	6,	7	8	9	456 thousands 789.
				5	6,	7	8	9	56 thousands 789.
					6,	7	8	9	6 thousands 789.
						7	8	9	7 hundred and 89.
							8	9	Eighty-nine.
								9	Nine.

TABLE 3.

C. of thousands of millions.	X. of thousands of millions.	Thousands of millions.	C. of millions.	X. of millions.	Millions.	C. of thousands.	X. of thousands.	Thousands.	Hundreds.	Tens.	Units.
2	1	0,	9	8	7,	6	5	4,	3	2	1
	1	0,	9	8	7,	6	5	4,	3	2	1
		1,	9	8	7,	6	5	4,	3	2	1
			9	8	7,	6	5	4,	3	2	1
				8	7,	6	5	4,	3	2	1
					7,	6	5	4,	3	2	1
						6	5	4,	3	2	1
							5	4,	3	2	1
								4,	3	2	1
									3	2	1
										2	1
											1

Thousands of Millions.	Millions.	Thousands.	Units.
123,	456,	789,	012
34,	567,	890,	123
1,	234,	567,	890
	654,	321,	200
	56,	789,	100
	4,	321,	987
		890,	300
		21,	400
		9,	501
			109
			80
			9

TABLE 4.

0	Nothing, nought, or a cipher.
10	Ten.
100	One hundred.
1,000	One thousand.
10,000	Ten thousand.
100,000	One hundred thousand.
1,000,000	One million.
10,000,000	Ten millions.
100,000,000	One hundred millions.
1,000,000,000	One thousand millions.
10,000,000,000	Ten thousand millions.
100,000,000,000	One hundred thousand millions.

Although the preceding enumeration tables are fully sufficient to instruct us how to express any number in common use, yet the following one may be thought necessary by the curious and inquisitive student of arithmetic.

TABLE 5.

Duodecillions.	Undecillions.	Decillions.	Nonillions.	Octillions.
123,456;	789,012;	345,678;	901,234;	567,890

Septillions.	Sextillions.	Quintillions.	Quatrillions.	Trillions.
123,456;	789,012;	345,678;	901,234;	567,890

Billions.	Millions.	Units.
123,456;	789,012;	345,678.

The above table must be read in the following manner, beginning at 1, under duodecillions, and terminating with 8, under units. Thus—

One hundred and twenty-three thousand 456 duodecillions, 789 thousand no hundred and 12 undecillions, 345 thousand 678 decillions, nine hundred and one thousand 234 nonillions, 567 thousand 890 octillions, 123 thousand 456 septillions, 789 thousand 012 sextillions, 345 thousand 678 quintillions, 901 thousand 234 quatrillions, 567 thousand 890 trillions, 123 thousand 456 billions, 789 thousand 012 millions, 345 thousand, 678 units.

NOTE.—The student will do well by transcribing the fifth table on a slip of paper that will contain all the figures in a straight line, and write the name of each period over it as it is in the table. Then he may begin the rotation at 8, under units, and assign to each figure its proper place and name, from the right hand to the left, by saying—units, tens, hundreds, thousands, tens of thousands, hundreds of thousands, millions, tens of millions, hundreds of millions, thousands of millions, tens of thousands of millions, hundreds of thousands of millions, billions, tens of billions, hundreds of billions, thousands of billions, tens of thousands of billions, hundreds of thousands of billions, trillions, tens of trillions, hundreds of trillions, &c., till he arrives at 1, under duodecillions; and then read from the left hand to the right, in the order of numeration, as above directed.

NOTATION BY LETTERS.

A less literal number placed after a greater increases the value of the greater; a less literal number placed before a greater, lessens the value of the greater: Thus, vi. is 6 and iv. is 4—xi. is 11 and ix. is 9. A line drawn over any literal number, less than a thousand, signifies so many thousands: Thus, $\overline{\text{X}}$. is 10,000, $\overline{\text{C}}$. is 100,000, $\overline{\text{CC}}$. is 200,000, $\overline{\text{M}}$. is one million. Hence, by a proper combination of the numeral letters, any number may be formed.

A TABLE OF LITERAL NUMBERS.

i.	=	1	xxxiii.	=	33	lxv.	=	65	xcvii.	=	97
ii.	=	2	xxxiv.	=	34	lxvi.	=	66	xcviii.	=	98
iii.	=	3	xxxv.	=	35	lxvii.	=	67	xcix.	=	99
iv.	=	4	xxxvi.	=	36	lxviii.	=	68	c.	=	100
v.	=	5	xxxvii.	=	37	lxix.	=	69	ci.	=	101
vi.	=	6	xxxviii.	=	38	lxx.	=	70	cii.	=	102
vii.	=	7	xxxix.	=	39	lxxi.	=	71	ciii.	=	103
viii.	=	8	xl.	=	40	lxxii.	=	72	civ.	=	104
ix.	=	9	xli.	=	41	lxxiii.	=	73	cv.	=	105
x.	=	10	xlii.	=	42	lxxiv.	=	74	cvi.	=	106
xi.	=	11	xliii.	=	43	lxxv.	=	75	cvii.	=	107
xii.	=	12	xliv.	=	44	lxxvi.	=	76	cviii.	=	108
xiii.	=	13	xlv.	=	45	lxxvii.	=	77	cix.	=	109
xiv.	=	14	xlvi.	=	46	lxxviii.	=	78	cx.	=	110
xv.	=	15	xlvii.	=	47	lxxix.	=	79	cxx.	=	120
xvi.	=	16	xlviii.	=	48	lxxx.	=	80	cxxx.	=	130
xvii.	=	17	xlix.	=	49	lxxxi.	=	81	cxl.	=	140
xviii.	=	18	l.	=	50	lxxxii.	=	82	cl.	=	150
xix.	=	19	li.	=	51	lxxxiii.	=	83	clx.	=	160
xx.	=	20	lii.	=	52	lxxxiv.	=	84	clxx.	=	170
xxi.	=	21	liii.	=	53	lxxxv.	=	85	clxxx.	=	180
xxii.	=	22	liv.	=	54	lxxxvi.	=	86	cxc.	=	190
xxiii.	=	23	lv.	=	55	lxxxvii.	=	87	cc.	=	200
xxiv.	=	24	lvi.	=	56	lxxxviii	=	88	ccc.	=	300
xxv.	=	25	lvii.	=	57	lxxxix.	=	89	cccc.	=	400
xxvi.	=	26	lviii.	=	58	xc.	=	90	d.	=	500
xxvii.	=	27	lix.	=	59	xci.	=	91	dc.	=	600
xxviii.	=	28	lx.	=	60	xcii.	=	92	dcc.	=	700
xxix.	=	29	lxi.	=	61	xciii.	=	93	dccc.	=	800
xxx.	=	30	lxii.	=	62	xciv.	=	94	dcccc.	=	900
xxxi.	=	31	lxiii.	=	63	xcv.	=	95	m.	=	1000
xxxii.	=	32	lxiv.	=	64	xcvi.	=	96	md.	=	1500

SIMPLE ADDITION.

Simple Addition teaches us how to add several numbers of like name, into one total sum.

RULE.

Place units under units, tens under tens, &c. then, begin at the right hand and add up the first column; if the sum be less than 10, set it down; if it be more than 10, set down the excess, and carry 1 for every 10, to the next column, and so on to the last, at which set down the whole sum.

Proof.

Cut off the top line of figures, and find the amount of the rest; then, if the said amount and top line, when added together, be equal to the total sum, the operation is right.

Examples.

10	16	22	28	34	40	46	52	58
11	17	23	29	35	41	47	53	59
12	18	24	30	36	42	48	54	60
13	19	25	31	37	43	49	55	61
14	20	26	32	38	44	50	56	62
15	21	27	33	39	45	51	57	63
75	111	147	153	219	255	291	327	363
65	95	125	125	185	215	245	275	305
75	111	147	153	219	255	291	327	363

64	69	74	79	84	89	94	12	41
65	70	75	80	85	90	95	21	14
66	71	76	81	86	91	96	13	51
67	72	77	82	87	92	97	31	15
68	73	78	83	88	93	98	99	61

17	91	66	100	200	3009	11110	78945
71	19	77	101	301	9003	12100	12345
37	93	88	102	219	1001	90101	67890
72	39	99	901	912	2018	10090	98765
28	44	56	909	429	8120	81049	43213
82	55	65	110	924	9000	99999	14301

999999	800180	1111111	98765432	14354028
111111	108201	1000001	2345678	1342091
110120	666200	9000909	987654	900752
300909	200666	8009009	56789	42130
510601	900109	1090028	876	32

123456789	987654321	101101101	2109876543	12345678901
100001001	98000129	699999999	1111111111	98765432
909909109	966660	600106	9999999999	45678
500100025	5005	96	1011011010	90

9	123456789
89	87654321
789	2345678
6789	901234
56789	56789
456789	1234
3456789	567
23456789	89
123456789	9

Application.

1. Add 1000, one hundred and one, 90 and 9, together. Ans. 1200.

2. Add 10,000, 2,002, 999, twenty and 9, into one sum. Ans. 13,030.

3. Add 9, 19, 102, one thousand and one, and twelve thousand one hundred, into one total sum. Ans. 13,231.

4. Add ninety thousand one hundred and one, ten thousand and ninety, three hundred and one, and ten together. Ans. 100,502.

5. A owes me 60 dollars, B 425, C 265, D 250 and E 1000. What sum is owing to me in all? Ans. 2000 dollars.

6. I hold B's bond for 1000 dollars, on which there are 125 dollars of interest now due. What sum will discharge the debt? Ans. 1125 dollars.

7. A merchant bought 50 barrels of flour for 300 dollars; 75 barrels for 525 dollars, and 125 barrels for 1000. How many barrels did he buy, and what sum did he pay for the whole quantity? Ans. 250 barrels, and paid 1825 dollars.

8. A vintner bought 6 pipes of wine, containing 120, 118, 125, 121, 127 and 119 gallons, respectively. How many gallons did he buy in all? Ans. 730 gallons.

9. A man went out to collect money, and received of one debtor, ninety dollars; of another, one hundred and forty dollars; of another, one hundred and one dollars; and of another, twenty-nine dollars. How much did he collect in all? Ans. 360 dollars.

10. I have in cash, nine hundred and one dollars; in bank stock, two thousand and eighteen dollars; in good bonds, eleven thousand and eleven dollars; in book debts, four thousand and seventy dollars. What is the whole amount of my estate? Ans. 18,000.

11. If Josiah give Harvey twenty oranges, and John give him 10, and Edward give him twenty-one, and James give him thirty. How many will he have? Ans. 81.

SIMPLE SUBTRACTION.

Simple subtraction teaches us how to find the difference between any two numbers of the same denomination.

Rule.

1. Set the less number under the greater, with units under units, tens under tens, &c.
2. Begin at the right hand, and take each lower figure from the one above it, and set down the remainder.
3. If any one of the lower figures be greater than the one above it, add 10 to the top figure, and take the lower one from that sum—set down the remainder, and add 1 to the next lower figure, and so on. The upper number is called the *minuend*, and the lower one the *subtrahend*.

Proof.

Add the remainder, or difference, and lower line of figures together, and their sum will be equal to the top line, if the work is right.

Examples.

From	836746	From	78367864
Take	415623	Take	41678917
Ans.	421123 the remainder.		36688947 the difference.
	836746 proof.		78367864 proof.

From	56789012	456785	678900000000	90004000
Take	27919126	161436	143612345678	18136136
Diff.				
Proof				

From	783678	9876543210	10001234	683981
Take	176789	987654322	12223457	461649
Diff.				
Proof				

From	123456789000	81000000	100000	101001010100
Take	81917191245	61234667	12345	10110101012
Rem.				
Proof				

From	100	1000	100	100000	1000
Take	6	25	34	125	1
Rem.					
Proof					

APPLICATION.

1. Subtract 156 from 320. Ans. 164 the difference.

2. Subtract fifteen thousand five hundred and nine from twenty thousand and fifty-four. Ans. 4545.

3. Subtract ten thousand and ninety from ninety thousand one hundred and one. Ans. 80,011.

4. Subtract eighty-one thousand and forty-nine from ninety thousand and twenty-one. Ans. 8972.

5. William is nine years old, and James is twenty-one. How much older is James than William? Ans. 12 years.

6. John owns thirty-two marbles, and Charles twenty-four. How many has John more than Charles? Ans. 8.

7. Alexander bought one hundred apples, of which he gave his brother thirty and his sister twenty. How many had he left? Ans. 50.

8. A certain person had nine thousand dollars in bank, and drew out one thousand one hundred and twelve. How much money had he remaining in the bank? Ans. 7888.

9. America was discovered in the year 1492, and her independence declared in the year 1776. How many years passed between those two dates? Ans. 284 years.

10. General George Washington was born in the year 1732, and died 1799. How old was he? Ans. 67 years.

11. Sir Isaac Newton was born in the year 1642, and died in 1727. How old was he at the time of his defunction? Ans. 85 years.

12. The mariner's compass was invented in the year 1302 and the art of printing in 1440. How many years were there between those discoveries? Ans. 138

13. Gunpowder was invented in 1344. How many years have passed since that invention to the present date? Ans.

14. The sun is 95,173,000 miles distant from this earth, and the moon 240,000. Which of them is the farthest off, and how many miles? Ans.

15. How many years have passed from the discovery of America to the present date? Ans.

16. How long has it been from the declaration of independence, by the Congress of the United States, to the present year? Ans.

SIMPLE MULTIPLICATION.

Simple Multiplication is a short way of perfoming many additions, without respect to the denomination.

There are three principal members in multiplication, namely:

1. The *multiplicand*, or number given to be multiplied.
2. The *multiplier*, or number given to multiply by.
3. The *product*, which contains the multiplicand as many times as there are units contained in the multiplier.

The multiplicand and multiplier are sometimes called *factors*, and the product a *fact* or *rectangle*.

THE MULTIPLICATION TABLE.*

2 times	3 times	4 times	5 times	6 times	7 times
1 make 2	1 make 3	1 make 4	1 make 5	1 make 6	1 make 7
2 make 4	2 make 6	2 make 8	2 . . 10	2 . . 12	2 . . 14
3 make 6	3 make 9	3 . . 12	3 . . 15	3 . . 18	3 . . 21
4 . . 8	4 . . 12	4 . . 16	4 . . 20	4 . . 24	4 . . 28
5 . . 10	5 . . 15	5 . . 20	5 . . 25	5 . . 30	5 . . 35
6 . . 12	6 . . 18	6 . . 24	6 . . 30	6 . . 36	6 . . 42
7 . . 14	7 . . 21	7 . . 28	7 . . 35	7 . . 42	7 . . 49
8 . . 16	8 . . 24	8 . . 32	8 . . 40	8 . . 48	8 . . 56
9 . . 18	9 . . 27	9 . . 36	9 . . 45	9 . . 54	9 . . 63
10 . . 20	10 . . 30	10 . . 40	10 . . 50	10 . . 60	10 . . 70
11 . . 22	11 . . 33	11 . . 44	11 . . 55	11 . . 66	11 . . 77
12 . . 24	12 . . 36	12 . . 48	12 . . 60	12 . . 72	12 . . 84

8 times	9 times	10 times	11 times	12 times
1 make 8	1 make 9	1 make 10	1 make 11	1 make 12
2 . . 16	2 . . 18	2 . . 20	2 . . 22	2 . . 24
3 . . 24	3 . . 27	3 . . 30	3 . . 33	3 . . 36
4 . . 32	4 . . 36	4 . . 40	4 . . 44	4 . . 48
5 . . 40	5 . . 45	5 . . 50	5 . . 55	5 . . 60
6 . . 48	6 . . 54	6 . . 60	6 . . 66	6 . . 72
7 . . 56	7 . . 63	7 . . 70	7 . . 77	7 . . 84
8 . . 64	8 . . 72	8 . . 80	8 . . 88	8 . . 96
9 . . 72	9 . . 81	9 . . 90	9 . . 99	9 . 108
10 . . 80	10 . . 90	10 . . 100	10 . . 110	10 . 120
11 . . 88	11 . . 99	11 . . 110	11 . . 121	11 . 132
12 . . 96	12 . . 108	12 . . 120	12 . . 132	12 . 144

*The student must memorize the multiplication table thoroughly.

TO PROVE MULTIPLICATION.

1. Cast the nines out of the multiplicand, and set the remainder on the left hand of a cross, as 2 in the margin.

2. Do the same by the multiplier, and set the remainder on the right hand of the said cross, as 3 in the margin.

3. Multiply these two remainders together, and if the product be less than 9, set it down in the top of the cross, as six; but, if it be more than 9, reject the nines and set down the excess instead of the 6.

6
2 3
6

4. Cast the nines out of the product, and set the remainder in the bottom of the said cross; if the top and bottom figures are equal, the work is right.

Case 1.

When the multiplier does not exceed 12.

Rule.

1. Set the multiplier under the right hand figure in the multiplicand, then multiply the several figures successively from the right hand to the left, as the table directs.

2. Set down the units figure of each particular product, and add the rest to the next product, and so on to the last, at which set down the whole amount.

Examples.

Multiplicand	76792263	3	6989864	6
Multiplier	2	6 2	3	5 3
Product	153584526	3	20969592	6
Multiplicand	96812612		9689638	
Multiplier	4		5	
Product				
Multiplicand	45678923		7894356	
Multiplier	6		7	
Product				
Multiplicand	98765432		2345678	
Multiplier	8		9	
Product				

Multiplicand	81927364	9184753
Multiplier	10	11
Product		

Multiplicand	13946875	8096087
Multiplier	12	9
Product		

CASE 2.

When the multiplier exceeds 12.

RULE.

1. Set the multiplier under the multiplicand, with units under units, tens under tens, &c.
2. Make as many separate products as there are figures in the multiplier, omitting the cipher.
3. Set the first figure of each particular product directly under that figure of the multiplier by which it is produced.
4. Add these several products together, and their sum will be the total product required.

EXAMPLES.

Multiplicand	437856946		237896584	
Multiplier	23		45	
1st Product	1313570838		1189482920	
2d Product	875713892		951586336	
Total Product	10070709758		10705346280	

Multiplicand	678945	568943
Multiplier	43009	60709
1st Product	61110505	5120487
2d Product	2036835	3982601
3d Product	2715780	3413658
Total Product	29200745505	34539960587

Multiplicand	875643296	683547928
Multiplier	67	89
Total Product	58668100832	60835765414

Multiplicand	765432896	678954327
Multiplier	234	667
Answer	179111297664	384967103409

Multiplicand	4329685	8754326
Multiplier	8923	4567
Answer	38633779255	39981006842

Multiplicand	6784359	9436785
Multiplier	6708	50083
Answer	45509480172	472622503155

Multiplicand	9208057	784536
Multiplier	90508	70306
Answer	833402822956	55157588016

Multiplicand	329187	987654
Multiplier	6007	80009
Answer	1977426309	79021208886

Multiplicand	98765	41325
Multiplier	56789	52314

Answer and proof required.

Case 3.

When there are ciphers at the right hand of either the multiplicand or multiplier, or at both of them.

Rule.

Place the significant figures under one another, then multiply by them only; add the several products together and annex as many ciphers to the total product as there are in both the said factors.

Examples.

Multiplicand	679100	930137000
Multiplier	5600	9500
Total Product	3802960000	8836301500000

Multiplicand	35926000	8196000	579	63975
Multiplier	7364	59180	100	40000
Ans.	264559064000	485039280000	57900	2559000000

CASE 4.

When the multiplier is any number between 12 and 20.

RULE.

Multiply by the unit figure only; and, as you multiply, add to the product of each single figure that of the multiplicand, which stands next on the right hand of the one you multiplied, and to the last figure in the multiplicand add what you have to carry.

EXAMPLES.

Multiplicand	65497	84916	19345	67895
Multiplier	13	14	15	16
Ans.	851461	1188824	290175	1086320
Multiplicand	46789	12345	67891	23456
Multiplier	17	18	19	15
Ans.				

CASE 5.

When the multiplier is the product of any two numbers in the multiplication table.

RULE.

Multiply the given sum by one of those numbers, and the product thence arising by the other; the last product will be the answer required.

EXAMPLES.

Multiply	83676 by 14	Multiply	67836 by 15
	2		3
First product	167352	First product	203508
	7		5
Answer	1171464	Answer	1017540

Multiply	36745 by 16	89436 by 18	456789 by 21	987654 by 24
Ans.	587920	1609848		

Multiply	56787 by 96	89123 by 132	23456 by 108	45678 by 144
Ans.				

Application.

1. A dollar is equal to 100 cents; therefore, how many cents are contained in 50 dollars? Ans. 5000 cents.

2. Four quarters of a cent make a whole one; therefore, how many quarters are there in 50 cents? Ans. 200 quarters.

3. A pound sterling is equal to 20 shillings; therefore, how many shillings are equal to 20 pounds sterling? Ans. 400 shillings.

4. Twelve pence make one shilling; therefore, how many pence are contained in 400 shillings? Ans. 4800 pence.

5. Four farthings make one penny; therefore, how many farthings are contained in 72 pence? Ans. 288 farthings.

6. Suppose 40 men were concerned in the payment of a debt, and each man paid 50 dollars. How much was the debt?

Ans. 2000 dollars.

7. How many square feet are in a floor that is 46 feet long and 35 feet wide? Ans. 1610 square feet.

8. If a man travel 36 miles in a day, how far will he go in 12 days? Ans. 432 miles.

9. A merchant bought 342 bales of linen; each bale contained 56 pieces, and each piece 25 yards. How many pieces and yards were there in all? Ans. 19,152 pieces and 478,800 yards.

10. How many corn hills are in a field which contains 125 rows, with 80 hills in each row? Ans. 10,000.

11. If an orchard contain 126 rows, with 109 trees in each row, and 1007 apples on each tree, how many trees and apples are in the said orchard? Ans. 13784 trees and 13830138 apples.

12. A certain island contains 52 counties, each county 42 townships, each township 56 houses, and each house 12 persons. How many townships, houses, and persons are there on the said island?

Ans. 2184 townships, 122,304 houses, and 1,467,648 persons.

SIMPLE DIVISION.

Simple Division is a short way of performing many subtractions, without respect to the denomination. It shows how often one number is contained in another, and what remains.

There are four principal members in division; namely:

1. The *dividend*, or number given to be divided.
2. The *divisor*, or number given to divide by.
3. The *quotient*, or answer to the question, which shows how many times the divisor is contained in the dividend.
4. The *remainder*, (if any) which must be less than the divisor, and of the same name with the dividend. The divisor, dividend, and quotient, are certain members; but the remainder is uncertain, because many questions in division have no remainders.

PROOF.

RULE 1.—Multiply the quotient and divisor together, and add the remainder to the product, which will make a sum equal to the dividend, if the operation is right.

RULE 2.—Add all the *subtrahends* and remainder together, according to the order in which they stand in the work; that sum will be equal to the dividend, if the operation is right.

RULE 3.—1. Cast the nines out of the divisor, and set the excess on the left hand of a cross, as in multiplication.

2. Do the same by the *quotient*, and set the excess on the right hand of the said cross.

3. Multiply these two figures together, add the product to the remainder, cast the nines out of that sum, and set the excess in the top of the said cross.

4. Cast the nines out of the *dividend*, and set the excess at the bottom of the said cross; if the operation be right, the top and bottom figures will agree.

SHORT DIVISION.

Short Division is when the divisor does not exceed 12.

RULE.

Seek how often the divisor is contained in the first left hand figure or figures of the dividend, under which set the result; and for each 1 that remains add 10 to the next right hand figure; try how often the divisor is contained in that sum, set down the result, and proceed on till all the dividend figures are used.

EXAMPLES.

	Dividend.		Dividend.
Divisor	2)676367684	Divisor	3)867676895
Quotient	338183842	Quotient	289225631+2 Rem.
			3
Proof	676367684	Proof	867676895

	Dividend.		Dividend.
Divisor	4)676768654	Divisor	5)968123456
Quotient	169192163+2 Rem.	Quotient	193624691+1 Rem.
	4		5
Proof	676768654	Proof	968123456

Dividend.	Dividend.
Divisor 6)7890123456	Divisor 7)678901234
Quotient 131502057+3 Rem.	Quotient 96985890+4 Rem.
6	7
Proof 7890123456	Proof 678901234

Dividend.	Dividend.	Dividend.
Divisor 8)567890120	Divisor 9)36126018	Divisor 10)123456789
Quotient		
Proof		

11)98765431	12)984100836	3)100000000	4)561236412
6)725442012	5)345401045	7)119455217	8)9000124808

LONG DIVISION.

Long Division is when the divisor exceeds 12.

Rule.

1. *For the first dividual*, take as many of the left hand figures of the dividend as are just necessary to contain the divisor.

2. Try how often the divisor is contained in the said dividual, and set the result of this trial on the right hand of the dividend, for the first quotient figure.

3. Multiply the divisor by the quotient figure, now found—set the product under the aforesaid dividual and subtract it therefrom.

4. Bring down the next figure of the dividend, and join it on the right hand of the remainder to form a second dividual.

5. Try how often the divisor will go in the second dividual, and set the result in the quotient for the second figure thereof; then, multiply and subtract, as before directed.

6. But, if the second dividual be not sufficient to contain the divisor, set a cipher in the quotient; then, affix the next figure of the dividend (not yet brought down) to the said dividual, and so on, till a competent number of the dividend figures has been annexed to the remainder to form a complete dividual.

7. Try how often the divisor will go in this complete dividual, and proceed on, as before directed, till all the dividend figures are used.

8. The *dividual* is a partial dividend or, so many of the dividend

figures as are taken to be divided at one time, and which produce one quotient figure.

EXAMPLES.

Divisor. Dividend. Quotient.
13)768685(59129
65 13
118 768677
117 rem. 8 added.
16 768685 proof.
13
38
26
125
117
8 remainder.

Divisor. Dividend. Quotient.
15)1350002679(90000178
135 15
. . . 00026 . 450000890
15 90000178
117 . 1350002670
105 . 9
129 1350002679 proof
120
9 remainder.

Divisor. Dividend. Quotient.
14)1132613608(80900972
112 14
126 323603888
126 80900972
. . . 136 1132613608 proof.
126
100
98
28
28
0

Divisor. Dividend. Quotient.
16)19201592015(1200099500
16 16
32 19201592000
32 . . . 15
0159 19201592015
144 . Proof.
152 .
144
80
80
. . 15 remainder.

Dividend. Quotient.
Divisor 21)89365605(4255505
0 remainder.

Dividend. Quotient.
Divisor 22)3678268(167194
0 remainder.

Dividend. Quotient.
Divisor 23)678682000(29507913
1 remainder

Dividend Quotient.
Divisor 24)906016000(37750666
16 remainder.

Divisor. Dividend. Quotient.
17)1678686719(98746277
153
———
148
136 proved by casting
———
126
119 out 8 5 5 5 the
———
78
68 nines.
———
106
102
———
47
34
———
131
119
———
129
119
———
10 remainder.

Divisor. Dividend. Quotient.
19)38685004(2036052
38
———
..68
57
———
115
114
———
100
95
———
54
38
———
16 remainder.
38 is the first subtrahend.
..57=the 2nd subtrahend.
..114=the 3d.
.....95=the 4th.
......38=the 5th.
.......16=the remainder.
———
38685004 proved by adding the subtrahends and remainder into one sum which is equal to the div'd.

Divisor 28)263832687(9422595 (Dividend. Quotient.)
———
27 remainder.

Div'r. 36)8836736823(245464911 (Dividend. Quotient.)
———
27 remainder.

---

42)13682822832(325781496
———
0 remainder.

47)4698765432(99973732
———
28 remainder.

---

52)6419753028(123456789
———
0 remainder.

54)36764394285(680822116
———
21 remainder.

---

63)62875438900(998022839
———
43 remainder.

84)73519670196(875234169
———
0 remainder.

---

98)49594939282(506070809
———
0 remainder.

112)943210000032(8421517857
———
48 remainder.

Divi'r. Dividend. Quotient.	Divi'r. Dividend. Quotient.
144)836746612345(5810736966	286)1234567890000(4316670944
121 remainder.	116 remainder.
365)24580044985(67342589	479)40006736900(83521371
0 remainder.	191 remainder.
598)367467234589(614493703	632)600000000000(949367088
195 remainder.	384 remainder.
754)376345689126(499132213	863)862987345604(999985336
524 remainder.	636 remainder.
987)7400689842(7498166	2345)836465631890(356701762
0 remainder.	0 remainder.
6432)223100987654(34686098	41659)756360804(18156
5318 remainder.	0 remainder.
7589)683056789000(90006165	87648)98713695542(112625
2815 remainder.	13542 remainder.
8736)398569420704(45623789	175296)19742712000(112625
0 remainder.	0 remainder.
9876)98750001234560(9998987856	210472)352107027680(1672940
9200 remainder.	0 remainder.
12896)11678368362254(905580677	976294)79697670002(8163
4222 remainder.	279080 rem.
25382)836764203682(32966834	9876543)456789012345(46249
23094 remainder.	8775138 rem.

47821682)2011527857382426(42063093

0 remainder.

Case 2.

1. When there is one cipher, or more, at the right hand of the divisor, it or they may be cut off; also, cut off the same number of figures at the right hand of the dividend, and proceed as in the last case.

2. The figures which were cut off from the dividend must be placed at the right hand of the remainder; but, if ciphers only were cut off from the dividend, they must not be annexed to the remainder.

Examples.

```
Divisor.  Dividend.   Quotient.
45|000)76453|674(1698
       45          45000
       ---        ------
       314         8490
       270        6792
       ---       --------
        445      76410000
        405         43674
        ---      --------
        403      76453674 proof.
        360
        ---
The true 43674 remainder.
```

```
Divisor.   Dividend.   Quotient.
96|0000)43776|0000(456
        384          960000
        ---          ------
        537          2736
        480         4104
        ---        ---------
         576       437760000
         576         proof.
         ---
         000
```

```
Divisor.  Dividend.   Quotient.
345|000)8092320|000( . . . .
                0 remainder.
```

```
Divisor.  Dividend.  Quotient.
3746|000)8448679268( . . . .
               1449268 rem.
```

```
Divisor.   Dividend.   Quotient.
6073|0000)94367426802( . . . .
          53736802 remainder.
```

```
Divisor. Dividend. Quotient.
365|00)9840000( . . .
            215 rem.
```

Case 3.

When the divisor is the product of any two or more numbers in the multiplication table.

Rule.

Divide the given sum by one of those numbers, the quotient thence arising by the other, and so on; the last quotient will be the answer required.

Then to find the true remainder.

Rule.

1. Multiply the quotient by the whole divisor, and subtract the product from the dividend, the result will be the true remainder; or,

2. Multiply the last remainder by the preceding divisor, or the last but one; and to the product add the preceding remainder; multiply the sum by the next preceding divisor, and to the product add the next preceding remainder, and so on, till you have gone through all the divisors and remainders, to the first; the result will be the true remainder required.

EXAMPLES.

Divide 4678 by 16.

4)4678

4)1169+2 the first remainder.

292+1 the second remainder.

Now, $1\times4+2=6$ the true remainder; consequently, the true answer is 292, and 6 rem.

Divide 367845 by 1728.

12)367845

12)30653+9

12)2554+5

212+10=last remainder.
12=preced'g divisor.

120
5=preceding rem.

125
12=next pre. divis'r.

1500
9=first remainder.

1509 true remainder.

Consequently the true quotient is 212 and 1509 remainder.

Dividend=367845
$1728\times212=366336$

1509 true remainder found by the first rule.

Divide 87946 by 864.

12)87946

9)7328+10

8)814+2

101+6 last remainder.
9 preceding divisor.

54
2 preceding rem.

56
12 the first divisor.

672
10 first remainder.

682 true remainder.

The best method of proving examples in this case, is—

1st. Multiply the last quotient by the last divisor, and take in the last remainder.

2d. Multiply this sum by the second divisor, and take in the second remainder.

3d. Multiply this last sum by the first divisor, and take in the first remainder; this last result will be equal to the dividend if the work is right.

1. Divide 89463 by 24. Answer, 3727+15 remainder.
2. Divide 673682 by 36. Answer, 18713+14 remainder.
3. Divide 376836 by 63. Answer, 5981+33 remainder.
4. Divide 836782 by 84. Answer.
5. Divide 987654 by 132. Answer.
6. Divide 467826 by 144. Answer.

CASE 4.

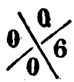
To perform division without setting down the subtrahends.

RULE.

Multiply the divisor by the quotient figures, as before, and subtract the product of each figure in the divisor, from the dividend, as you produce it; set down the remainder, and carry as many to the product of the next figure, as there were tens borrowed before; and so on, till all the dividend figures have been employed.

EXAMPLES.

Div'r. Dividend. Quotient.
756)7012656(9276
2086
5745
4536
0000



Div'r. Dividend. Quotient.
459)137901246(300438
2012
1764
3876
204 remainder.

In the first example, I find the first quotient figure to be 9, then, I say 9 times 6 are 54, which I take from 62, and 8 remains; next, I say 9 times 5 are 45 and 6 that I carry (for 6 tens which I borrowed) make 51, which I take from 51, and nothing remains; again, I say 9 times 7 are 63 and 5 that I carry (for 5 tens which I borrowed) make 68, which I take from 70, and 2 remains; so, the whole remainder is 208, to which I annex 6, with a dot, over it, and the sum is 2086. The second quotient figure is 2, consequently I say twice 6 are 12, which I take from 16, and 4 remains; next, I say twice 5 are 10 and 1 that I carry (for 10 which I borrowed) make 11, which I take from 18, and 7 remains; again, I say twice 7 are 14 and 1 I carry (for 10 I borrowed) make 15, which I take from 20, and 5 remains; wherefore, the second remainder is 574, to which I annex 5, with a dot over it, and the sum is 5745, in which the divisor goes 7 times; of course, the third quotient figure is 7, consequently, I say 7 times 6 are 42, which I take from 45, and 3 remains, which I set down under the 5, and proceed on in the same manner, through the whole dividend.

APPLICATION.

1. One hundred cents make a dollar; therefore, how many dollars are in 5000 cents? Ans. 50 dollars.

2. How many dollars are in 8400 cents? Ans. 84 dollars.

3. Twenty shillings make one pound sterling; therefore, how many pounds sterling are in 1500 shillings? Ans. 75.

4. Twelve pence make one shilling; therefore, how many shillings do 1152 pence make? Ans. 96.

5. Four farthings made a penny; therefore, how many pence do 48 farthings make? Ans. 12 pence.

6. If a man go a journey of 432 miles in 12 days, how many miles did he travel each day? Ans. 36 miles.

7. If a man travel 36 miles in a day, in how many days will he go a journey of 432 miles? Ans. 12.

8. The expense of building a certain court-house is 5022 dollars, which is to be defrayed equally by 186 men. How much must each man pay? Ans. 27 dollars.

9. I am desirous to plant out 2812 apple trees in 19 rows. How many trees must I put in each row? Ans. 148.

10. A man planted 2812 peach trees, and put 148 trees in each row. How many rows were there in his orchard? Ans. 19.

11. How many pieces and bales are contained in 478800 yards of linen, allowing 25 yards to be in each piece, and 56 pieces in each bale? Ans. 19152 pieces and 342 bales.

12. Several boys went out to gather chestnuts and collected 9900, which were so divided among them that each boy had 825. How many boys were there in company? Ans. 12.

13. How many times will a wheel, which is 208 inches in circumference, turn round between Richmond and Staunton, which is 7603232 inches? Ans. 36554 times.

ARITHMETICAL PROBLEMS.

PROBLEM I.—Having the least of two numbers, and the difference between them, given, to find the greater number.

RULE.—Add them together, and their sum will be the greatest number.

1. The least of two numbers is 127 and their difference is 198. What is the greatest number? Ans. 325.

2. The least of two numbers is 9709, and the difference between them is 1192. What is the greatest number? Ans. 10901.

PROBLEM II.—Having the sum of two numbers, and one of them given, to find the other one.

RULE.—Subtract the given number from the given sum, and the remainder will be the other number.

1. If 325 be the sum of two numbers, and one of them 198, what is the other number? Ans. 127.

2. If 10901 be the sum of two numbers, and one of them 9709, what is the other number? Ans. 1192.

Problem III.—Having the greater of two numbers, and the difference between them given, to find the less.

Rule.—Subtract the difference from the greater number, and the remainder will be the less number.

1. If the greater of two numbers be 1001, and the difference between them 825, what is the less? Ans. 176.

2. If the difference between two numbers be 334, and the greater one 2000, what is the less? Ans. 1666.

Problem IV.—Having the divisor and quotient given, to find the dividend.

Rule.—Multiply them together, and the product will be the dividend.

1. The divisor of an operation in division is 12, and the quotient 144. What is the dividend? Ans. 1728.

2. What dividend will produce 9276 for the quotient, when the divisor is 756? Ans. 7012656.

Problem V.—Having the product of two numbers, and one of them given, to find the other one.

Rule.—Divide the product by the given number, and the quotient will be the number required.

1. If the product of two numbers, be 1728, and one of them 144, what is the other number? Ans. 12.

2. The product arising from the multiplication of two numbers is 7012656, and one of them is 9276. Required the other number. Ans. 756.

Problem VI.—Having the dividend and quotient given, to find the divisor.

Rule.—Divide the dividend by the quotient, and the quotient thence arising will be the divisor.

1. A certain dividend is 288 and the quotient 32. What is the divisor? Ans. 9.

2. The quotient of an operation in division is 365 and the dividend 18980. What is the divisor? Ans. 52.

Problem VII.—Having the sum and difference of two numbers given, to find those numbers.

Rule.—1. To half the sum add half the difference, and that sum will be the greatest number.

2. From half the sum take half the difference, and the remainder will be the least number.

1. What are those two numbers whose sum is 144 and difference 32.

The sum $144 \div 2 = 72$ the half sum.

The difference $32 \div 2 = 16$ the half difference.

Therefore, $72 + 16 = 88$ the greatest number, and $72 - 16 = 56$ the least number.

Proof. $88 + 56 = 144$ the given sum.

And $88 - 56 = 32$ the given difference.

2. What are those two numbers whose sum is 2000 and difference 1200? Ans. 1600 and 400.

PROBLEM VIII.—Having the sum of two numbers, and the difference of their squares given, to find those numbers.

RULE.—Divide the difference of their squares by the sum of the two numbers, and the quotient will be their difference. You will then have their sum and difference to find the numbers by problem the seventh.

1. What are those two numbers whose sum is 32, and the difference of their squares 256?

32)256(8 the difference of the two given numbers. Now, by the seventh problem,

To 16 half of the given sum,
Add 4 the half difference.

20 the greatest number.
And from 16 the half sum,
Subtract 4 the half difference.

12 the least number.

2. What are those two numbers whose sum is 2000, and the difference of their squares 240000? Ans. 1600 and 400.

PROBLEM IX.—Having the difference of two numbers, and the difference of their squares given, to find those numbers.

RULE.—Divide the difference of their squares by the difference of the numbers, and the quotient will be their sum. You will then have their sum and difference to find the numbers by the seventh problem.

1. What are those two numbers whose difference is 20, and the difference of whose squares is 2000?

20)2000(100=the sum of the two given numbers. Now, by the seventh problem, To 50 the half sum,
Add 10 the half difference.

60 the greatest number.
$50 - 10 = 40$ the least number.

2. What are those two numbers whose difference is 50, and the difference of whose squares is 5000? Ans. 75 and 25.

APPLICATION OF THE PROBLEMS.

1. I once borrowed a number of dollars from my friend, and have since paid him 275; I still owe him 125. How many did I borrow at first. Ans. 400 dollars.

2. A boy put 750 hazlenuts in two bags, one of which held 380. How many were in the other bag? Ans. 370.

3. If I sell goods to the amount of 1000 dollars, and receive 825 in payment, what sum remains due to me? Ans. 175.

4. A gentleman by his will left his whole estate to be equally divided among his 12 children, each one of whom received 1275 dollars. What was the whole amount of the estate? Ans. 15300 dollars.

5. The sum of 4600 dollars is to be distributed among a regiment of soldiers, so that each one may have 25 dollars. Please to inform me how many soldiers there were in the said regiment? Ans. 184.

6. A gentleman who happened to die intestate, left a tract of land containing 520 acres, which was so divided among his children that each one inherited 65 acres. How many heirs were there in his family? Ans. 8.

7. Two men, namely, A and B, together deposited 1000 dollars in the Bank of Virginia, but A put in 200 dollars more than B. Please to inform me the amount of each man's deposite?

Ans. A deposited 600 dollars and B 400.—By problem 7.

8. Two boys, namely, Alexander and Benjamin, had 14 marbles apiece when they commenced playing, but, after several games, Benjamin refused playing any longer, because he had lost some of his marbles, at which time it was found that the difference of the squares of the numbers, which each of them then had, was 336. How many marbles had Benjamin left, and what number did he lose?

Ans. He had 8 left, and consequently he lost 6.—By problem 8.

9. Said Henry to his friend Charles, my father gave me 12 apples more than he gave to my brother James, and the difference of the squares of our separate parcels was 288. Now, if you are arithmetician enough to tell how many he gave to each of us, you shall have half of mine.

Ans. He gave Henry 18 and James 6.—By problem 9.

DECIMAL FRACTIONS.

A *decimal fraction* is a part, or parts of a unit, denoted by a *dot* placed before it—thus, .5 .25 and .125 are decimal fractions. The first figure after the dot is so many tenths of a unit, the second is so many hundredths, the third, so many thousandths, &c. They are commonly read .5 tenths, .25 hundredths, .125 thousandths, &c. But, more convenient and equally accurate—thus, .5 decimals, .25

decimals, .125 decimals, .1234 decimals, &c., particularly when there are many figures to be expressed decimally.

A *mixt number* is composed of a whole number and a decimal—thus, 12.5, 4.25, 6.375, are mixt numbers; the figures on the left hand of the *dot* are whole numbers.

Ciphers, joined on the right hand of decimals do not alter their value, for .5 .50 .500 .5000, &c. are decimals of the same value. But, when they are placed on the left hand, they decrease their value in a ten-fold proportion—thus, .5 .05 .005 .0005 are all different in value, because the significant figure is removed further from the *decimal point* in each successive expression.

NOTATION OF DECIMALS.

We may consider *unity* as a fixt point, from whence whole numbers proceed, infinitely increasing towards the left hand, and *decimals* infinitely decreasing towards the right hand to 0, as in the following

TABLE.

Whole numbers.									Decimal fractions.							
C. of millions.	X. of millions.	Millions.	C. of thousands.	X. of thousands.	Thousands.	Hundreds.	Tens.	Units.	Tenth parts.	Hundredth parts.	Thousandth parts.	X. thousandth parts.	C. thousandth parts.	Millionth parts.	X. millionth parts.	C. millionth parts.
9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9

From the above table it is self-evident, that, in *decimals* as well as in *whole numbers*, each figure takes its value according to its distance from the place of *unity*. The *decimal point* must not be omitted, because a *decimal fraction* cannot be distinguished from a *whole number* without it.

ADDITION OF DECIMALS.

RULE.

1. Place the numbers, whether mixt or pure decimals, under each other, according to their value—that is, units under units, tens under tens, &c.
2. Place tenths under tenths, hundredths under hundredths, &c.
3. Add them together as in Simple Addition.

4. Point off so many figures at the right hand of the total sum, as are equal to the greatest number of decimal places, in any one of the given numbers.

EXAMPLES.

46846.6	4.8625	.8368	.67836854
7368.57	64.4	364.65	.3210234
683.885	67.68	6.5	.567890
68.7685	0.9	.365	.12345
5.836525	9.0	18.785	.6789
.2654	0.5	.8	.012
.56	86.25	.25	.34
.8	4.375	8.5	.5
54975.285425	237.9675	400.6868	3.22163194

.1275	.9	.426789	987.
.0005	.87	.000009	654.
.005	.654	.900000	32.
.05	.321	.9876543	1.
.5	.0987	.9876642198	.1
12345.	.65432	.8765	.98
4.75	.123456	.987	.987
93.125	.7890123	.89	.9876
6.1625	.45678912	.9	.98765

APPLICATION.

1. What is the total sum of 19.073+2.3597+223.+.0197581+3478.1+12.358? Ans. 3734.9104581.
2. What is the total sum of 429.+21.37+355.003+1.07+1.7? Ans. 808.143.
3. What is the total sum of 5.3+11.973+49.+.9+1.7314+34.3? Ans. 103.2044.
4. What is the total sum of 973.+19.+1.75+93.7164+.9501? Ans. 1088.4165.
5. Add 450.+31.47+376.004+1.08+456.+.76+.05 together. Ans. 1315.364.
6. Add 2476.8471+94.9+9.8941+867.05+84.9+271.007+5.1008 together. Ans. 3809.699.
7. Add 4.+.5+.01+.001+.0009+4.388+1.0001 into one sum. Ans. 9.9.
8. Add 1000.+.00009+5.25+6.5+100.125+50.06+838.0649 together. Ans. 1999.99999.

SUBTRACTION OF DECIMALS.

Rule.

Place the numbers according to their value; then subtract as in whole numbers, and point off the decimals, as in addition.

Examples.

From	1793.13.....	1.	46.	5.	1111.1111
Take	817.0569375	0.9999	9.875	0.9555	222.3334
Rem.	976.0730625	.0001	36.125	4.0445	888.7777

Application.

1. What is the difference between 171.195 and 125.9176? Ans. 45.2774.
2. What is the difference between 219.1384 and 195.91? Ans. 23.2284.
3. What is the difference between 480. and 245.0075? Ans. 234.9925.
4. What is the difference between 94.1 and 5.2112? Ans. 88.8888.
5. What is the difference between 10. and .00001? Ans. 9.99999.
6. What is the difference between 1000. and 333.3334? Ans. 666.6666

MULTIPLICATION OF DECIMALS.

Rule.

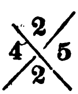
1. Set the right hand figure of the multiplier under the right hand figure of the multiplicand; then, multiply precisely as in multiplication of whole numbers.

2. Count how many decimal figures there are in both the factors together; then, point off that many figures at the right hand of the product for decimals.

3. If there are not figures enough in the product, supply the defect by prefixing ciphers to it, before you place on the decimal point; which must not be omitted.

Note.—When any number, either whole or mixed, is multiplied by a decimal-fraction, the product is always less than the multiplicand, in the same proportion as the multiplying decimal is less than a unit, or 1.

Examples.

Multiplicand	.000041		.00000715	
Multiplier	.000017		23.	
	287		2145	
	41		1430	
Product	.000000000697		.00016445 answer.	

Multiplicand	48.	4.6	437.	686.78	.3 759
Multiplier	.48	5.8	.75	.25	.945
Product	23.04	26.6 8	327.75	171.69 50	.3552 255

Multiply	.84179	3 2.1	.6 3478	.3 85746
By	.00385	9 8.7	.8204	.00464
	.00324 08915	3168.2 7	.52077 3512	.001789 86144

Multiply	·09	345678	.172839
	.09	.102507	205014
	.00 81	35434.414746	35434.414746

Case 2.

To contract the operation so as to retain as many decimal places in the product as may be thought necessary.

Rule.

1. Reverse the multiplier, and set it under the multiplicand; so that its unit's place may stand under the lowest decimal figure, that is to be retained in the product.

2. In multiplying, reject all the figures on the right hand of the multiplying digit, observing to increase the first figure of each line by adding 1 for every 8 in the product of the omitted figure by the multiplier.

3. The first figures of every respective product must stand directly under each other.

Examples.

1. Multiply 27.14986 by 92.41035 and retain only four decimal figures in the product.

Contracted.	Common way.	
27.149 86 multiplicand.	27.14986	
Multiplier=53 014.29 reversed.	92.41035	
244 348 74	13	5 74930
5 429 97 .	81	4 4958
1 085 99 . .	2714	9 86
27 15 . . .	10 8599	4 4
81	54 2997	2
14	2443 4874	.
2508.92 80	2508.9280	6 50510

2. Multiply .7854 by 2.348 and retain 3 decimals in the product.

```
 .78 54
 843.2
 157 1
  23 5
   3 1
     7
 1.84 4 answer.
```

3. Multiply 3.4567 by 1234 and retain 4 decimals in the product.

```
 3.456 7
   432.1
 3 456 7
   691 3
   103 7
    13 8
 4.265 5 answer.
```

4. Multiply .248264 by .725234 and retain four, five and six figures in the several products respectively.

```
    .248 264      .2482 64       .24826 4
     527.          2527.        4 32527.
The .180 0        .1800 5       .1 80049 products
```

DIVISION OF DECIMALS.

Rule.

1. Annex so many ciphers to the dividend as will make the decimal places thereof equal to those in the divisor at least.

2. Divide precisely as in whole numbers, and, from the right hand of the quotient, point off so many figures for decimals, as the decimal places of the dividend exceed those in the divisor.

3. If the quotient do not contain so many figures as the rule requires to be pointed off, supply the deficit by prefixing ciphers to the left hand of it, before you place on the decimal point.

4. Ciphers may be annexed to the remainder, and the quotient carried on to any degree of exactness.

Note 1. When the dividend is greater than the divisor, the quotient will be greater than the dividend; but, when the dividend is less than the divisor, then the quotient will be less than the dividend, and in the same proportion as a unit is greater or less than the dividing decimal fraction.

2. Division and multiplication prove each other reciprocally.

Examples.

1. Divisor. Dividend. Quotient.

```
2.4)2039.64(849.85
    192. .
     119
      96
      236
      216
       204
       192
        12.0
        12 0
```

In this example there is one cipher annexed to the remainder 12, which make the decimal places of the dividend three; therefore, two figures are pointed off at the right hand of the quotient as decimals, because there is one decimal figure in the divisor, for the decimal places of the divisor and quotient counted together must be equal to those in the dividend.

```
2. Divisor.   Dividend.        Quotient.
      219).1178410749(.0005380871
           1095......
           ----
            834
            657
            ----
            1771
            1752
            ----
             1907
             1752
             ----
              1554
              1533
              ----
               219
               219
               ---
               ...
```

In the second example, the divisor having no decimals, the quotient must have so many as there are in the dividend; wherefore, I have prefixed three ciphers to the left hand of the first quotient figure, before I put on the decimal point.

```
3. Divisor. Dividend.   Quotient.
   .3719)38.0000(102.178+18
          37 19..
          -----
           8100
           7438
           -----
            662.0
            371 9
            -----
            290 10
            260 33
            ------
             29 770
             29 752
             ------
Remainder        18
```

In the third example, the dividend, being a whole number, must have at least so many ciphers annexed to it as there are decimals in the divisor, and so far the quotient will be whole numbers; then, by annexing more ciphers and continuing the operation, we obtain decimals in the quotient according to the rule.

```
4. Divisor. Dividend.  Quotient.
   .125).0000710(.000568
          625
          ----
           85.0
           75 0
           ----
           10 00
           10 00
           -----
           .....
```

In the fourth example, there are nine decimals in the dividend, including the two ciphers annexed. The divisor has three decimals, and there are three figures in the quotient, which make six, and the three ciphers prefixed to the first quotient figure make up nine decimals, which are equal in number to the decimals in the dividend.

5. Divide 1. by 25. Ans. .04

6. Divide 6. by .375 Ans. 16.

7. Divide .05 by .625	Ans. .08
8. Divide .6 by .375	Ans. 1.6
9. Divide .863972 by 92.	Ans. .009391
10. Divide 89. by .853	Ans. 104.3376+272 rem.
11. Divide 80.3 by 2.2	Ans. 36.5
12. Divide 80.3 by 22.	Ans. 3.65
13. Divide the number one by .04	Ans. 25.
14. Divide .71 by .000568	Ans. 1250.
15. Divide 5. by .825	Ans. 6.060606+50 rem.
16. Divide 71. by .000568	Ans. 125000
17. Divide 14 by .7854	Ans. 17.825+2450 rem.
18. Divide 234.70525 by 64.25	Ans. 3.653
19. Divide 9. by .9	Ans. 10.
20. Divide .8 by 8.	Ans. .1
21. Divide 7. by .875	Ans. 8.
22. Divide .7 by .875	Ans. .8
23. Divide .07 by .875	Ans. .08
24. Divide 222. by .365	Ans. 608.219178+30 rem.
25. Divide .48624097 by 179.	Ans. .00271643

DIVISION OF DECIMALS CONTRACTED.

Rule.

1. Find the first quotient figure, as in common division.

2. Let each succeeding remainder be a new dividual, and for every such dividual reject the right hand figure of the preceding divisor.

3. Increase the next subtrahend by adding 1 for every 8 in the product of the omitted figure.

4. When there are not so many figures in the divisor as are required to be in the quotient, continue the operation, as usual, till the number of figures in the divisor and those wanting in the quotient are equal; then use the contracted method to find the rest.

1. Divisor. Dividend. Quotient.

```
.3719)38.0000(102.178
      37 19
      -----
        8100
        7438
        ----
   371)662
       372 increased by 1.
       ---
    37)290
       259
       ---
     3)31
       31 increased by 7.
```

2. Divisor. Dividend. Quotient.

```
945)3552255(3759
    2835
    ----
     7172
     6615
     ----
   94)557
      472 increased by 2.
      ---
     9)85
       85 increased by 4.
       --
        0
```

3. Divisor. Dividend. Quotient.

```
92.41035)2508.9280650510(27.14986
          1848 2070
          ---------
 92.4103)660 7210
         646 8725 increased 4
         --------
  92.410)13 8485
          9 2410
         -------
   92.41)4 6075
         3 6964
         ------
    92.4)9111
         8317 increased by 1
         ----
     92.)794
         740 increased by 4
         ---
       9)54
         54
         --
          0
```

ROUND DIVISION.

This method of performing division has not appeared in any Arithmetic that I have seen. The operation is the same as in the fourth case of Simple Division, except bringing down the figures from the dividend, which is not done in this case. It is the most concise method I have ever learned, and becomes quite easy with a little practice.

EXAMPLES.

1. Divisor. Dividend. Quotient.

```
946)746394(789
     84110
      850
        0
```

In this example, the first dividual is 7463, which contains the divisor 7 times; now I set a dot over the last figure 3, and say 7 times 6 are 42, which I take from 43, and 1 remains; this sum I set down under 3, and say 7 times 4 are 28, and 4 that I carry (for 4 tens which I borrowed) make 32, which I take from 36, and 4 remains; this sum I set down under 6, and then say 7 times 9 are 63, and 3 that I carry (for 3 tens which I borrowed) make 66; this sum I take from 74, and 8 remains, which I set down under 4, and the whole remainder is 841; next I set a dot over 9, and the second dividual is 8419: the second quotient figure is 8, consequently I say 8 times 6 are 48; this sum I take from 49, and 1 remains, which I set down under 9, and say 8

times 4 are 32, and 4 that I carry (for 4 tens which I borrowed) make 36; this sum I take from 41 under 3, and 5 remains, which I set down under 1, and say 8 times 9 are 72, and 4 that I carry (for 4 tens which I borrowed) make 76; this sum I take from 84, and 8 remains, which I set down under 4, and the whole remainder is 851; then I set a dot over the last figure, and the third dividual is 8514: the last quotient figure is 9, of course I say 9 times 6 are 54; this sum I take from 54, and nought remains, which I set down under 4, and proceed on in the same way, and nothing remains all round.

2. Divisor. Dividend. Quotient.

```
92.41035)2508.9280650510(27.14986
          660 7210100010
           13 848686920
            4 60756260
               9111740
                 79440
                   550
                     0
```

The curious and inquisitive student will please to work this example at large by long division, and he will be convinced of the propriety of using this method in preference to any other, notwithstanding it is too difficult for young beginners, unless they work their questions at large by the common rule, and set down the several remainders as in the above examples. I learned the above method of performing division of a travelling gentleman, who never authorized me to publish his name.

FEDERAL MONEY.

Federal money being purely decimal, is, of course, added, subtracted, multiplied, and divided in the same manner as decimals.

NOTATION OF FEDERAL MONEY.

RULE.

1. In setting down sums of federal money, the cents are placed on the right hand of the dollars, and separated from them by a dot, in the same manner that decimals are separated from whole numbers.

2. If the number of cents be less than 10, a cipher must be put in the ten's place; and if there are no cents in the given sum, two ciphers are placed on the right hand of the dollars.

3. If the dot, which separates the dollars from the cents be removed, the whole sum may be called cents, or decimals of a dollar.

4. If the sum be cents only, separate two figures from the right hand for cents, and all, on the left hand of the dot, will be dollars, which is fully exemplified in the following

TABLE.

Dollars.		Dimes, or tenth parts of a dollar.	Cents, or 100th parts of a dol'r.	Mills, or 1000th parts of a doll.	Decimal parts of a mill.	
6 5 4 3 2 1	.	1	2	3	4 5 6	&c. as in decimals.

N. B. In calculations of federal money, we commonly say, so many dollars, cents, mills and decimal parts of a mill; instead of saying, so many eagles, dollars, dimes, cents, &c. Wherefore the above table may be read 654 thousand 321 dollars, 12 cents, 3 mills and 456 decimals of a mill. Consequently, any number of dollars, dimes, cents, mills, &c. may be simply expressed in dollars and decimal parts of a dollar. Thus, 21 dollars and one dime are expressed 21 dollars and .1 tenth, or 21 dollars 10 cents. 12 dollars, 4 dimes, 7 cents, 9 mills, are equal to 12 dollars 479 decimals, or 12 dollars, 47 cents, 9 mills. 99 dols. 9 dimes, 9 cents, 9 mills, 9 tenths, are expressed, 99 dols. 9999 decimal parts of a dollar.

ADDITION OF FEDERAL MONEY.

The denominations of federal money, are

10 mills	(*m.*)	make	1 cent.	*c.*
10 cents		make	1 dime.	*d.*
10 dimes			1 dollar.	\$ or *D.*
10 dollars			1 eagle.	*E.*

RULE.

Place the numbers so that those of like name may stand directly under each other. Then, add them together as in decimals.

Examples.

Written down according to the table.					Written down in dollars, cents and mills.			Written down decimally.	
Eagles.	Dolls.	Dimes.	Cents.	Mills.	Dollars.	c.	m.	$	. Decimals.
99	9	9	9	9	999	99	9	999	.999
38	8	8	8	8	388	88	8	388	.888
57	6	5	4	3	576	54	3	576	.543
21	0	1	2	3	210	12	3	210	.123
45	6	7	8	9	456	78	9	456	.789
09	0	0	2	5	90	02	5	90	.025
04	0	0	0	7	40	00	7	40	.007
00	8	9	0	1	8	90	1	8	.901
00	0	2	0	3		20	3		.203
00	0	0	0	5		00	5		.005
277	1	4	8	3	2771	48	3	2771	.48,3

1. I have written down the above example three ways, in order to convince the student of the propriety of using the decimal process.

2. When a question is written down in dollars, cents and mills, it is necessary to separate the mills from the cents by a comma.

3. When a question is written down decimally, the two first figures on the right hand of the decimal point are cents, the next one mills, and the rest decimal parts of a mill.

4. Sometimes it is very convenient, in business, to use the fractions of a cent, instead of the mills, &c. Therefore:

$\frac{1}{4}$ of a cent is equal to .25 decimals of a cent, or 2 mills and 5 tenths.

$\frac{1}{2}$ of a cent is equal to .5 decimals of a cent, or 5 mills=.005 of a dollar.

$\frac{3}{4}$ of a cent is equal to .75 decimals of a cent, or 7 mills and 5 tenths.

N. B. In adding the fractions, we always add the top figures together, and divide the sum by the lower one; but as $\frac{1}{2}$ is used instead of $\frac{2}{4}$, we add the lower figure 2, in the place of the top 1. Therefore, the student will be careful to add those figures only, which are marked with +, in the first example, and proceed in the same manner with the remaining examples. If the remainder is 1, set down $\frac{1}{4}$, if it is 2, set down $\frac{1}{2}$, and if it is 3, set down $\frac{3}{4}$, under the column of fractions, and add the quotient to the first column of the cents; then, proceed as before directed. Observe this note in sterling money, &c.

EXAMPLES BOTH WAYS.

Cents.	c. m. &c.	Cents.	c. m. &c.	$ c.	$ c. m.
.06¼+	.0 6 25	.56¼	.56 25	5.12½	5.12 5
.12½+	.1 2 50	.62½	.62 5	6.18¾	6.18 75
.18¾+	.1 8 75	.68¾	.68 75	7.31¼	7.31 25
.31¼+	.3.1 25	.81¼	.81 25	8.37½	8.37 5
.37½+	.3.7 50	.87½	.87 5	9.43¾	9.43 75
.43¾+	.4.3 75	.93¾	.93 75	10.87½	10.87 5
$ 1.50.	$1.50,00	$1.50.	4.50,00	47.31¼	47.31,25
1.43¾	1.43,75	3.93¾	3.93,75	42.18¾	42.18,75
1.50.	1.50	4.50	4.50	47.31¼	47.31,25

c.	$ c.	$ c.	$ c.
.08⅓	0.66⅔	5.66⅔	9.91⅔
.16⅔	0.83⅓	4.33⅓	8.08⅓
.33⅓	0.91⅔	3.16⅓	7.16⅔
.41⅔	1.08⅓	2.58⅓	6.33⅓
.53⅓	2.16⅔	1.83⅓	5.00
1.58⅓	5.66⅔	17.58⅓	36.50

APPLICATION.

1. Add 99 dols. 6¼ cents, 78 dols. 56¼ cents, 37 dols. 62½ cents, 73 dols. 63¾ cents, 18 dols. 81¼ cents, and 81 dols. 93¾ into one total sum. Ans. 389 dols. 68¾.

2. Add 187 dols. 12½ cts., 781 dols. 18¾ cts., 694 dols. 6¼ cts., 495 dols. 31¼ cts., 271 dollars 62½ cts. and 172 dols. 87½ cts. together. Ans. 2603 dols. 18 cts. 75 decimals.

3. I have bought a bible for 4 dols. 87½ cents, a dictionary for 2 dols. 62½ cents, an arithmetic for 93¾ cents, an introduction to the English Reader for 37½ cents, an English Reader for 62½ cents, a sequel to the English Reader for 81¼, what did the whole bill of articles amount to? Ans. 10 dols. 25 cts.

4. I have bought cloth to make a coat for 16 dols., a hat for 7½ dols., shirting linen for 7 dols, a pair of shoes for 2 dols. 18¾ cents, a pocket handkerchief for 87½ cents, and paid the tailor 6 dols. 43¾ cents. How much money did I lay out in all? Ans. 40 dollars.

SUBTRACTION OF FEDERAL MONEY.

RULE.

Place the less number under the greater, so that dollars may stand under dollars, cents under cents, &c. Then subtract as in decimals.

EXAMPLES.

	E. D. d. c. m.	$ c. m.	$ c. m.
From	99 . 1 . 5 . 4 . 6	854 . 54 . 5	764 . 87 , 525
Take	79 . 2 . 7 . 5 . 8	498 . 76 . 6	278 . 93 , 125
Rem.	19 . 8 . 7 . 8 . 8	355 . 77 . 9	485 . 94 , 400

	$ c. m.	Dol's. c. m.	Dols. c. m
From	987 . 00 , 0	1000 . 00 , 0	495621 . 00 , 1
Take	889 . 76 , 5	999 . 00 , 2	486738 . 02 , 2

APPLICATION.

1. Subtract 25 cents from 100 dollars. Ans. 99 dols. 75 cents.
2. Subtract 37 cents 5 mills from 1 eagle. Ans. 9 dols. 62 cents 5 mills.
3. Take 1 mill from 1 dollar. Ans. 99 cts. 9 mills.
4. Take .25 decimals of a cent from 25 dollars. Ans. 24 dols. 99 cts. 75 decimals.
5. Take 9 dols. 12½ cents from 50 dollars. Ans. 40 dols. 87½ cts.
6. Subtract 62 dols. 62 cents 5 mills from 10 eagles. Ans. 37 dols. 37½ cents.

MULTIPLICATION OF FEDERAL MONEY.

RULE.

1. Multiplication of Federal Money is performed, in all respects, like multiplication of decimals.

2. When you have pointed off the decimals in the product according to the rule given in multiplication of decimals, all the figures on the right hand of the mills, or third place of decimals, are so many decimal parts of a mill.

3. But, if there should be a fraction of a cent in the given sum, instead of mills, &c. as $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, &c. you must multiply the top figure of the fraction by the multiplying figure and divide that product by the lower one—set down the remainder (if any) under the given fraction, and add the quotient to the product of cents, then proceed as in multiplication of decimals.

CASE 1.

When the multiplier does not exceed 12—set it under the right hand figure of the sum given to be multiplied, and multiply as above directed.

Examples.

	$ d. c. m.	$ d. c. m.	$ d. c. m.	$ c. m.	$ c. m.
Multiply	144.9.8.9	98.7.8.5	340.5.6.3	79.87.5	69.75.8
By	2	3	4	5	6
Ans.	289.9.7.8	296.3.5.5	1362.2.5.2	399.37.5	418.54.8

	$ c. m.	$ c. m.	$. c. m.	$ c. m.	$. c. m.
Multiply	461.36.4	97.93.75	125.62.7	674.12.5	999.99.9
By	7	8	9	10	11
Ans.	3229.54.8	783.50.00	1130.64.3	6741.25.0	10999.98.9

Examples

With cents, and the fractions of a cent.

	c.	c.	c.	c.	c.	c.	c.	c.
Multiply	.06¼	.08⅓	.12½	.16⅔	.18¾	.31¼	37½	43¾
By	4	5	6	7	8	9	10	12
Ans.	.25	.41⅔	.75	$1.16⅔	$1.50	$2.81¼	$3.75	$5.25

Case 2.

When the multiplier exceeds 12, and is the exact product of any two numbers in the multiplication table.

Rule.

Multiply the given sum by one of those numbers, and that product by the other one; the last product will be the answer required.

N. B. If there should be no fraction in the given sum, proceed by common multiplication of decimals.

Examples.

1. Multiply 56 cents by 14.

By parts.		Common way.
.56 cents.		.56 cts.
2		14
1.12		224
7		56
$7.84	Answer.	$7.84

2. Multiply 56¼ cents by 15.

By parts.		Common way.
.56¼ cents	=	.5625 decimals.
3		15
1.68¾		28125
5		5625
$8.43¾	Ans.	$8.43,75

3. Multiply 62½ cents by 16.

62½ cents	=	.625 decimals.
4		16
2.50		3750
4		625
$10.00	Ans.	$10.000

4. Multiply 68¾ cents by 18.

68¾ cents	=	6875 dec. of $1.
6		18
4.12½		55000
3		6875
$12.37½	Ans.	$12.375 m. = 12.37½

5.	Multiply .75 cents by 20.	Answer, 15 dollars.
6.	Multiply .66⅔ cents by 24.	Answer, 16 dollars.
7.	Multiply .81¼ cents by 25.	Answer, $20.31¼ cents.
8.	Multiply .83⅓ cents by 27.	Answer, $22.50 cents.
9.	Multiply .87½ cents by 28.	Answer, $24.50 cents.
10.	Multiply .91⅔ cents by 30.	Answer, $27.50 cents.
11.	Multiply .93¾ cents by 32.	Answer, $30.
12.	Multiply .06¼ cents by 36.	Answer, $2.25 cents.
13.	Multiply .08⅓ cents by 42.	Answer, $3.50 cents.
14.	Multiply .12½ cents by 48.	Answer, $6.00.
15.	Multiply .16⅔ cents by 54.	Answer, $9.00.
16.	Multiply .18¾ cents by 56.	Answer, $10.50 cents.
17.	Multiply .25 cents by 60.	Answer, $15.00.
18.	Multiply .31¼ cents by 64.	Answer, $20.00.
19.	Multiply .36 cents by 66.	Answer, $23.76 cents.
20.	Multiply .37½ cents by 70.	Answer, $26.25 cents.
21.	Multiply .56¼ cents by 72.	Answer, $40.50 cents.
22.	Multiply .58⅓ cents by 84.	Answer, $49.00.
23.	Multiply .62½ cents by 96.	Answer, $60.00.
24.	Multiply .75 cents by 108.	Answer, $81.00.
25.	Multiply .81¼ cents by 120.	Answer, $97.50.
26.	Multiply .83⅓ cents by 121.	Answer, $100.83⅓ cents.
27.	Multiply .87½ cents by 132.	Answer, $115.50 cents.
28.	Multiply .93¾ cents by 144.	Answer, $135.

EXAMPLES

With Dollars, Cents, &c.

		$ c.		$ c.
1.	Multiply	2.18¾ by 21.	Answer,	45.93¾.
2.	Multiply	3.25 by 22.	Answer,	71.50.
3.	Multiply	4.37½ by 35.	Answer,	153.12½.
4.	Multiply	5.41⅔ by 44.	Answer,	238.33⅓.
5.	Multiply	6.56¼ by 63.	Answer,	413.43¾.
6.	Multiply	7.62½ by 77.	Answer,	587.12½.
7.	Multiply	8.66⅔ by 80.	Answer,	693.33⅓.
8.	Multiply	9.87½ by 88.	Answer,	869.00.
9.	Multiply	33.33⅓ by 33.	Answer,	1100.00.
10.	Multiply	56.62½ by 64.	Answer,	3624.00.
11.	Multiply	333.33⅓ by 99.	Answer,	33000.00.
12.	Multiply	234.37½ by 144.	Answer,	33750.00.

CASE 3.

When the multiplier exceeds 12, and is not the exact product of some two numbers in the multiplication table.

*4

RULE.

1. Choose two numbers that will produce the nearest product to the given multiplier, and proceed with them, as in case the second; then add or subtract for the deficiency or excess.

2. If the sum given to be multiplied be dollars and cents only, or dollars, cents, mills, &c., proceed by multiplication of decimals.

EXAMPLES.

1. Multiply 12½ cents by 13.

```
         12
        ----
Product 1.50 by 12
   Add   12½ for deficiency.
        -----
Answer, $1.62½ cents.
```

The same by decimals.

```
12½ cents =.125 decimals.
             13
          ------
Answer,   1.62,5 } as before.
          $ c. m.}
```

2. Multiply 18¾ cents by 17.

```
          4
        ----
         .75
          4
        ----
        3.00
         18¾ added.
        -----
Answer, $3.18¾ cents.
```

The same by decimals.

```
.1875 decimals of a dollar.
   17
 -----
$3.18,75 answer, as before.
```

3. Multiply 31¼ c. by 19 decimally.

```
.3125 decimals.
   19
-------
$5.93,75
```

4. Multiply 2 dols. 37½ cts. by 31.

```
         $2.37½
              4
          -----
           9.50
              8
          -----
Product  76.00 by 32
Subtract  2.37½ for the excess of 31
         ------
        $73.62½ cents answer.
```

The same by decimals.

```
$2.375
    31
 -----
  2 375
 71 25
 ------
$73.625 answer as before.
```

Another way.

```
         $2.37½
             10
          -----
          23.75
              3
          -----
Product   71.25 by 30
Add        2.37½ for the deficiency
          ------
         $73.62½ cents, as before.
```

N. B. By comparing the preceding examples, the student must be convinced in favor of the decimal method of calculation, both for conciseness and perspicuity.

5. Multiply .25 cents by 23. Answer, $5.75 cents.
6. Multiply .375 mills by 26. Answer, $9.75 cents.
7. Multiply .45 cents by 29. Answer, $13.05 cents.
8. Multiply .50 cents by 34. Answer, $17.00.
9. Multiply .56$\frac{1}{4}$ cents by 37. Answer, 20.81\frac{1}{4}$ cents.
10. Multiply .62$\frac{1}{2}$ cents by 39. Answer, 24.37\frac{1}{2}$ cents.
11. Multiply .81$\frac{1}{4}$ cents by 41. Answer, 33.31\frac{1}{4}$ cents.
12. Multiply .87$\frac{1}{2}$ cents by 43. Answer, 37.62\frac{1}{2}$ cents.
13. Multiply 1.93\frac{3}{4}$ cents by 46. Answer, 89.12\frac{1}{2}$ cents.
14. Multiply 2.12\frac{1}{2}$ cents by 47. Answer, 99.87\frac{1}{2}$ cents.
15. Multiply $3.25 cents by 51. Answer, $165.75 cents.
16. Multiply $4.35 cents by 52. Answer, $226.20 cents.
17. Multiply 5.62\frac{1}{2}$ cents by 53. Answer, 298.12\frac{1}{2}$ cents.

CASE 4.

When there is a fraction, as $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, &c. in the multiplier.

RULE.

1. Multiply the given sum by the whole number only; then add $\frac{1}{2}$, $\frac{2}{3}$, or $\frac{3}{4}$, of the given sum to that product, and the sum thence arising will be the answer. Or,

2. Multiply the given sum by the top figure of the fraction and divide the product by the lower one—add the quotient to the product by the whole number, and the sum thence arising will be the answer.

EXAMPLES.

1. Multiply 24 dols. 60 cts. by 5$\frac{1}{2}$.

```
   $  c.
 2)24.60
       5
 ------
  123.00
Add 12.30=½ the given sum.
 ------
$135.30 cents, answer.
```

2. Mult. $44.45 c. 6 mills by 6$\frac{1}{4}$.

```
   $  c. m.
 4)44.45,6
         6
 ---------
  266.73,6
Add 11.11,4=¼ the given sum.
 ---------
$277.85,0 m. answer.
```

---

3. Mult. $12.36 cts. 9 mills by 4$\frac{1}{3}$.

```
   $. c. m.
 3)12.36,9
         4
 ---------
   49.47,6
Add 4.12,3=⅓ given sum.
 ---------
$53.59,9 m. answer.
```

4. Mult. 7 dols. 25 cts. by 4$\frac{3}{4}$.

```
   $ c.
 2)7.25
      4
 ------
  29.00
2) 3.62½=½ the given sum }
   1.81¼=¼ the given sum } =¾ ad.
 ------
$34.43¾ cents, answer.
```

---

5. Mult. 2 dols. 54 cents by 15⅔.

$ c.
⅓)2.54
15

38.10
84⅔ = ⅓ the giv'n sum
84⅔ = the same } = ⅔ ad.

39.79⅓ cents, answer.

6. Mult. 8 dols. 25 cts. by 11⅗

$ c.
8.25
11

90.75
4.95 = ⅗ added.

$95.70 answer.

8.25
3

5)24.75

4.95

---

7. Multiply 7 dols. and ⅝ by 12.

$
7
12

84
7.50 = ⅝ added.

$91.50 cents, answer.

$ c.
12×5÷8=7.50

8. Multiply 10 dols. by 6⅞.

$ $
$ 10×6=60 c.
10× 7÷8= 8.75

$68.75 cents, ans.

Case 5.

When the multiplier exceeds the product of any two numbers in the multiplication table:

Rule.

Multiply decimally, or change the factors and proceed by one of the foregoing cases.

Examples.

1. Multiply .06¼ by 378.

.06¼ cts. = .0625 decimals of a dollar.
378

5000
4 375
18 75

$ c.
$23.6250 = 23.62½ answer.

The factors changed.

4)378
06¼

22 68
94.5

Answer 23.62,5
$ c. m.

	$ c.		$ c.
2. Multiply	0.12½ by 176.	Answer,	22.00
3. Multiply	0.18¾ by 244.	Answer,	45.75
4. Multiply	1.25 by 325.	Answer,	406.25
5. Multiply	2.37½ by 416.	Answer,	988.00
6. Multiply	3.50 by 532.	Answer,	1862.00
7. Multiply	4.62½ by 678.	Answer,	3135.75
8. Multiply	5.33⅓ by 333.	Answer,	1776.00

APPLICATION.

$ c.

1. What will 9 oranges cost, at 12½ cents a piece? Ans. 1.12½
2. What will 10 lbs. of sugar cost, at 16⅔ cts. per lb.? Ans. 1.66⅔
3. What will 11 gallons of molasses cost, at 37½ cts. per gallon? Ans. 4.12½
4. What will 12 bushels of rye cost, at 62½ cts. per bushel? Ans. 7.50
5. What will 13 yards of velvet cost, at 75 cts. per yard? Ans. 9.75
6. What will 14 lbs. of coffee cost, at 18¾ cts. per pound? Ans. 2.62½
7. What will 15 lbs. of sugar cost, at 25 cts. per pound? Ans. 3.75
8. What will 16 yards of satin cost, at 93¾ cts. per yard? Ans. 15.00
9. What will 17 yards of muslin cost, at $1.25 cts. per yard? Ans. 21.25
10. What will 18 bushels of corn cost, at 56 cts. per bushel? Ans. 10.12½
11. What will 19 yards of cloth cost, at $2.37½ cts. per yard? Ans. 45.12½
12. What will 20 yards of fine linen cost, at $1.12½ cents per yard? Ans. 22.50
13. What will 21 yds. of calico cost, at 66⅔ cts. per yard? Ans. 14.00
14. What will 22 skeins of sewing silk cost, at .06¼ cents per skein? Ans. 1.37½
15. What will 23 hanks of thread cost, at .08⅓ cts. per hank? Ans. 1.91⅔
16. What will 24 gallons of wine cost, at $1.62½ cents per gallon? Ans. 39.00
17. What will 25 gallons of cider cost, at 18¾ cts. per gallon? Ans. 4.68¾
18. What will 26 lbs. of iron cost, at .07½ cts. per lb.? Ans. 1.95
19. What will 27 lbs. of steel cost, at 27½ cts. per lb.? Ans. 7.42½
20. What will 28 lbs. of loaf sugar cost, at 22½ cts. per pound? Ans. 6.30
21. What will 642 pipes cost, at 5 mills each? Ans. 3.21
22. What will 48 yards of ferret cost, at .06¼ cents per yard? Ans. 3.00
23. What will 74 yards of tape cost, at .07 cts. per yard? Ans. 5.18
24. What will 64 yards of cambric cost, at $1.45 cts. per yard. Ans. 92.80
25. What will 87 barrels of corn cost, at 2.56½ per barrel? Ans. 223.15½

		$ c.
26.	What will 49 lbs. of coffee cost, at 16 cents per pound?	Ans. 7.84
27.	What will 341 pair of shoes cost, at $1.45 cts. per pair?	Ans. 494.45
28.	What will 214 pair of stockings cost, at $2.04 cents per pair?	Ans. 436.56
29.	What will 36 yards of coating cost, at $3.00 cts. 7 mills per yard?	Ans. 108.00,2 m.
30.	What will 317 yards of velveret cost, at $2.12 cts. 5 mills per yard?	Ans. 673.62,5 m.

DIVISION OF FEDERAL MONEY.

Rule.

1. Division of Federal Money is performed, in all respects, like division of decimals.

2. When you have pointed off the decimals in the quotient according to the rule given in division of decimals, the two first figures on the right hand of the decimal point will be cents, the next one mills, and the rest decimal parts of a mill.

3. If there should be a fraction of a cent in the given sum, instead of the mills, &c., as $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, &c., continue the operation, as above directed, till two decimal figures are obtained in the quotient for the cents; then multiply the remainder (if any) by the lower figure of the given fraction, and add the upper one to the product; divide that sum by the same divisor, and the quotient will be so many parts of a cent, according to the figure you multiplied by, which must be annexed to the cents in order to complete the answer.

Examples.

1. Divide 21 dols. $12\frac{1}{2}$ cts. by 13.

13)21.12½(1.62½ answer.
13
—
81
78
—
32
26
—
6
2
—
13)13(½ cent.
13
—
.

2. Divide 3 dols. $56\frac{1}{4}$ cts. by 19.

19)3.56¼(0.18¾ answer.
19
—
166
152
—
14
4
—
19)57(¾ of a cent.
57
—
...

3. Divide 70 dollars by 21.

21)70.00($3.33\frac{1}{3}$ answer.
63
—
70
63
—
70
63
—
7
3
—
21)21($\frac{1}{3}$ of a cent.
21
—
00

A short method.
7)70.00
———
3)10.00
———
$\$3.33\frac{1}{3}$ answer.

4. Divide 603 dollars by 48.

48)603.00($12.56\frac{1}{4}$ answer.
48
—
123
96
—
270
240
—
300
288
—
12
4
—
48)48($\frac{1}{4}$ of a cent.
48
—
0

Another way.
6)603.00
———
8)100.50
———
$\$12.56\frac{1}{4}$ answer.

---

5. 2)4321.18,75 ($ c. m.)	6. 8)1754.76,0 ($ c. m.)	7. 3)293.81,25 ($ c. m.)
Ans.	......	......

8. 5)315.62,5 ($ c. m.)	9. 9)3888.56,25 ($ c. m.)	10. 4)2092.60 ($ c.)	11. 7)506.62,5 ($ c. m.)
Ans.	....	.....	.....

		$ c.				$ c.
12.	Divide	402.50	by	6.	Answer,	$67.08\frac{1}{3}$
13.	Divide	$198.62\frac{1}{2}$	by	7.	Answer,	$28.37\frac{1}{2}$
14.	Divide	$266.66\frac{2}{3}$	by	8.	Answer,	$33.33\frac{1}{3}$
15.	Divide	300.00	by	9.	Answer,	$33.33\frac{1}{3}$
16.	Divide	$000.91\frac{2}{3}$	by	11.	Answer,	$00.08\frac{1}{3}$
17.	Divide	0.75	by	12.	Answer,	$0.06\frac{1}{4}$
18.	Divide	$8.43\frac{3}{4}$	by	15.	Answer,	$0.56\frac{1}{4}$
19.	Divide	10.00	by	16.	Answer,	$0.62\frac{1}{2}$
20.	Divide	$12.37\frac{1}{2}$	by	18.	Answer,	$0.68\frac{3}{4}$
21.	Divide	16.00	by	24.	Answer,	$0.66\frac{2}{3}$
22.	Divide	30.00	by	32.	Answer,	$0.93\frac{3}{4}$

Application.

		$ c.
1.	If 9 cocoa-nuts cost $1.12½ cts. what will 1 cost?	Ans. 0.12½
2.	If 10 lbs. of sugar cost $1.66⅝ cts. what will 1 lb. come to?	Ans. 0.16⅝
3.	If 11 yards of gingham cost $4.58¼ cts. what will 1 yard come to?	Ans. 0.41⅝
4.	If 12 bushels of rye cost $7.50 cents, what will 1 bushel cost?	Ans. 0.62½
5.	If 15 lbs. of sugar cost $3.75 c. what will 1 lb. cost?	Ans. 0.25
6.	If 17 yards of muslin cost $21.25 cents, what will 1 yard cost?	Ans. 1.25
7.	If 24 gallons of wine cost $39, what will 1 cost?	Ans. 1.62½
8.	If 25 gallons of cider cost $4.68¾ cents, what will 1 gallon cost?	Ans. 0.18¾
9.	If 64 yards of cambric cost $92.80 cents, what will 1 yard cost?	Ans. 1.45
10.	If 87 barrels of corn cost $223.15½ cents, what will 1 barrel cost?	Ans. 2.56½
11.	If 317 yards of velveret cost $673.62½ cents, what will 1 yard cost?	Ans. 2.12½
12.	If 532 sacks of salt cost $1862, what will 1 cost?	Ans. 3.50

COMPOUND ADDITION.

Compound Addition is the adding of several sums or quantities of different denominations into one total sum or quantity.

General Rule.

1. Place the sums or quantities so that those of the same denomination may stand directly under each other.

2. Begin at the right hand and add up the first column—divide the amount by as many of the same denomination as will make 1 of the next greater.

3. Set down the remainder (if any) under the said column, and add the quotient to the next column.

4. Continue the process to the last column, and set down the whole amount.

I. OF STERLING MONEY.

The denominations of Sterling Money, are—

4 farthings	(*qrs.*)	make 1 penny.	*d.*
12 pence		make 1 shilling.	*s.*
20 shillings		make 1 pound.	£.

EXAMPLES.

10	20	12 4		20	12 4		20	12 4	10	20	12 4
£	s.	d. qrs.	£	s.	d. qrs.	£	s.	d. qrs.	£	s.	d. q.
7	13	7¼	3	19	11¾	9	16	7¼	371	10	11½
6	14	8½	2	18	10½	8	15	6½	60	19	1¼
8	16	9¾	3	6	1¾	7	14	5¾	9	00	6.
9	18	4¼	1	10	7½	6	13	4.	0	15	9.
1	7	2¼	9	10	9½	5	12	0½	0	18	7½
1	3	1¾	1	17	1¾	4	11	9¼	0	00	10½
6	0	2.	4	16	2¼	3	10	8¾	9	18	00
2	1	10½	3	17	8.	2	9	1.	0	16	8¼
43	15	10¼	31	17	5.	49	3	7.	454	00	6¾
36	2	3.	27	17	5¼	39	6	11¾	82	09	7¼
43	15	10¼	31	17	5.	49	3	7.	454	00	6¾

£	s.	d. qrs.	£	s.	d. qrs.	£	s.	d. qrs.	£	s.	d. q.
99	19	11¾	123	07	04½	678	18	11¾	9	19	11¾
88	18	8½	321	09	07½	954	19	11¼	19	10	7½
77	17	7¼	9	00	00¼	321	17	10½	267	15	9½
66	16	6¼	0	13	06.	100	00	00.	374	13	3¾
55	15	4.	0	16	10½	000	00	7½	67	12	6.
44	00	..	9	10	00.	000	00	4½	8	11	3.
3	00	4½	6	00	00.	009	01	6.	0	19	6.
0	19	6.	0	00	10½	300	00	1½	0	00	10½

II. OF TROY WEIGHT.

The denominations of Troy Weight are—

24 grains (grs.)	make	1 pennyweight.	dwt. or pwt.
20 pennyweights	make	1 ounce.	oz.
12 ounces	make	1 pound.	lb.

Troy Weight is used for weighing jewels, gold, silver, pearls, diamonds, electuaries, liquors, and all other things which are not subject to gross weight.

EXAMPLES.

10	12	20	24								
lbs.	oz.	dwt.	grs.	lbs.	oz.	dwt.	grs.	lbs.	oz.	dwt.	grs.
19	11	19	23	14	10	11	18	19	11	19	23
68	10	18	22	11	11	10	20	18	10	18	22
74	9	17	21	6	6	4	22	00	00	18	16
32	8	16	20	1	8	18	23	16	00	00	20
67	7	15	19	4	7	19	18	00	10	00	12
18	6	14	18	9	8	12	20	08	00	00	00

DIRECTION.—1. Add up the column of grains as in Simple Addition and divide the sum by 24, (because 24 grains make 1 pennyweight)—set down the remainder (if any) under the column of grains and add the quotient to the column of pennyweights.

2. Add up the pennyweights in like manner and divide the sum by 20, (because 20 dwts. make 1 ounce)—set down the remainder (if any) under the column of pwts. and add the quotient to the column of ounces.

3. Add up the ounces in the same manner and divide the sum by 12, (because 12 oz. make 1 pound, Troy Weight)—set down the remainder (if any) under the column of oz. and add the quotient to the column of pounds.

4. Add up the pounds as before directed, and set down the whole amount.

5. Cut off the top line, as in Simple Addition, and find the amount of the rest; then, if this last amount and the top line when added together produce a sum equal to the whole amount of the several quantities, the operation is right.

III. OF AVOIRDUPOIS WEIGHT.

The denominations of Avoirdupois Weight are—

16 drams	(dr.)	make	1 ounce.	oz.
16 ounces		make	1 pound.	lb.
28 pounds		make	1 quarter.	qr.
4 quarters,	(or 112 lbs.)	make	1 hundred weight.	cwt.
20 hundred weight		make	1 ton.	T.

Avoirdupois Weight is used for weighing cotton, hemp, flax, leather, iron, steel, tobacco, sugar, coffee, and all other articles of merchandize, which are of a coarse and drossy nature, and therefore subject to gross weight.

EXAMPLES.

	20	4	28	16	16
T.	*cwt.*	*qrs.*	*lb.*	*ozs.*	*drs.*
7	19	3	27	15	15
6	10	2	20	12	14
5	7	1	19	13	13
4	6	3	26	14	12
3	18	2	25	11	11
1	17	1	24	10	10
1	16	3	23	9	9
9	12	2	18	8	8

	20	4	28	16	16
T.	*cwt.*	*qrs.*	*lb.*	*ozs.*	*drs.*
86	19	1	27	6	2
95	18	3	26	15	15
00	17	3	25	14	14
00	00	2	24	13	13
		1	27	14	15
			20	12	12
9	18	3	4	00	00
2	16	1	0	00	15

DIRECTION.—1. Add up the column of drams, as in Simple Ad-*tion, and divide* the sum by 16, (because 16 drs. make 1 oz.)—set

the remainder under the column of drams, and add the quotient to the ounces. 2. Add up the ounces and divide the sum by 16, (because 16 ozs. make 1 lb. Avoirdupois Weight)—set the remainder under the ounces and add the quotient to the pounds. 3. Add up the pounds and divide the sum by 28, (because 28 lbs. make 1 qr. of a cwt.)—set the remainder under the pounds and add the quotient to the quarters. 4. Add up the quarters and divide the sum by 4, (because 4 qrs. make 1 cwt.)—set the remainder under the quarters, and add the quotient to the hundreds weight. 5. Add up the hundreds weight and divide the sum by 20, (because 20 cwt. make 1 ton)—set the remainder under the hundreds weight and add the quotient to the tons. 6. Add up the tons and set down the total sum. 7. Prove the operation by the rule given under Simple Addition, and also under Troy Weight.

IV. OF APOTHECARIES WEIGHT.

The denominations of Apothecaries Weight are—

20 grains	(gr.)	make	1 scruple.	℈	or sc.
3 scruples		make	1 drachm.	ʒ	or dr.
8 drachms		make	1 ounce.	℥	or oz.
12 ounces		make	1 pound.	℔	or lb.

Apothecaries use this weight in compounding their medicine, but buy and sell their drugs by Avoirdupois Weight.

EXAMPLES.

10	12	8	3	20
℔	℥	ʒ	℈	*gr.*
19	11	7	2	19
18	10	6	1	18
41	9	5	2	17
10	8	4	2	16
9	7	3	1	14
8	10	2	1	15
6	9	7	2	13
56	6	6	2	12
12	9	5	1	11

℔	℥	ʒ	℈	*gr.*
56	7	3	1	18
71	11	7	2	19
36	10	6	2	17
63	6	7	1	16
74	10	4	2	19
19	4	3	2	18
67	7	2	1	14
41	6	6	1	12
10	7	6	2	16

DIRECTION.—1. Add up the column of grs. and divide the sum by 20, (because 20 grains make 1 scruple)—set the remainder under the grains and add the quotient to the scruples. 2. Add up the scruples and divide the sum by 3, (because 3 scruples make 1 dr.)—set the remainder under the scruples, and add the quotient to the drachms. 3. Add up the drs. and divide the sum by 8, (because 8 drachms make 1 ounce, set the remainder under the drs. and add the quotient to the ounces. 4. Add the ounces and divide the sum by 12, (because 12 ounces make 1 *lb. Apothecaries* Weight)—set the remainder under the oun-

ces, and add the quotient to the pounds. 5. Add the pounds and set down the whole amount.

V. OF WOOL WEIGHT.

The denominations of Wool Weight are—

7	pounds (lb.)	make	1 clove.	clo.
2	cloves, or 14 lb.	make	1 stone.	st.
2	stones	make	1 tod.	to.
6½	tods, or 182 lbs.	make	1 wey.	w.
2	weys, or 364 lbs.	make	1 sack.	sa.
12	sacks	make	1 last.	la.

Wool Weight is supplemental to Avoirdupois Weight, and is used for weighing wool and cotton in the large factories of Europe, and probably may be introduced into our American factories.

EXAMPLES.

	12	2	6½	2	2	7
La.	*sac.*	*w.*	*tod.*	*sto.*	*clo.*	*lb.*
44	11	1	6	1	1	6
37	10	1	4½	1	1	4
67	9	1	2½	1	1	3
28	6	1	6	1	1	6
16	5	1	3½	1	1	4
68	10	1	3½	1	1	6
42	9	1	4½	1	1	4
24	8	1	2	1	1	5
332	3	0	½	0	1	3
287	3	0	0	0	1	4
332	3	0	½	0	1	3

	2	2	7
Tods.	*sto.*	*clo.*	*lb.*
20	1	1	6
19	1	1	5
49	1	1	4
68	1	1	3
37	1	1	6
41	1	0	0
00	1	1	6
10	1	0	4

DIRECTION.—When you have to divide by 6½, which may sometimes occur—you must multiply the total sum of the tods by 2, and divide the result by 13, (because 13 half tods make 1 whole tod,) then set down half the remainder under the tods, and add the quotient to the weys.

VI. OF CLOTH MEASURE.

The denominations of Cloth Measure are—

2¼	inches (in.)	make	1 nail.	na.
4	nails, or 9 inches	make	1 quarter of a yard.	qr.
4	quarters	make	1 yard.	yd.
5	quarters	make	1 English ell.	E. E.
3	quarters	make	1 Flemish ell.	F. E.
6	quarters	make	1 French ell.	Fr. E.

NOTE 1.—The yard is used for measuring all sorts of woollen and cotton goods, wrought silks, linen, tape gartering, &c.

NOTE 2.—The English ell was formerly used for measuring a particular kind of linen called Holland.

Note 3.—The Flemish ell is used in measuring tapestry.
Note 4.—The yard is now commonly used in all cases.

EXAMPLES.

	4	4		4	9		5	4		5	9
yds.	*qrs.*	*na.*	*yds.*	*qrs.*	*in.*	*E. E.*	*qrs.*	*na.*	*E. E.*	*qrs.*	*in.*
67	3	3	61	3	8	77	4	3	70	4	8
68	3	2	40	2	7	66	3	2	12	3	7
71	3	3	30	3	6	37	2	3	39	4	8
67	2	2	60	2	4	68	3	2	36	4	7

	3	9		3	4		6	9		6	4
F.E.	*qrs.*	*in.*	*F. E.*	*qrs.*	*na.*	*Fr. E.*	*qrs.*	*in.*	*Fr. E.*	*qrs.*	*na.*
18	2	8	31	2	3	67	5	7	39	5	3
39	2	7	16	1	3	38	4	8	93	2	2
67	0	8	61	2	3	69	5	7	14	5	1
28	2	6	69	1	3	26	4	6	42	4	1

VII. OF DRY MEASURE.

The denominations of Dry Measure are—

2 pints (pts.)	make	1 quart.	qt.
4 quarts	make	1 gallon.	ga.
2 gallons	make	1 peck.	pk.
4 pecks	make	1 bushel.	bu.
5 bushels	make	1 barrel.	bl.
4 bushels	make	1 coom.	coo.
2 cooms, or 8 bushels,	make	1 quarter.	qr.
4 quarters	make	1 chaldron.	ch.
5 quarters	make	1 wey.	w.
2 weys	make	1 last.	la.

Note.—A chaldron of coals is 36 bushels.

Dry Measure is used to measure corn, wheat, rye, oats, potatoes, beans, peas, timothy seeds, clover seeds, salt, sand, coals, and all other dry articles.

The standard bushel is $18\frac{1}{2}$ inches wide, and 8 inches deep.

A gallon of dry measure contains $268\frac{4}{5}$ solid inches, and a bushel $2150\frac{2}{5}$ solid inches.

EXAMPLES.

	4	2	4	4	2	4	2
ch.	*qrs.*	*coo.*	*bu.*	*pks.*	*ga.*	*qts.*	*pts.*
9	3	1	3	3	1	3	1
8	2	1	2	2	1	2	1
7	1	1	1	1	1	1	1
6	1	1	1	1	1	1	1
9	3	1	3	3	1	3	1

	5	4	2	4	2
bls.	*bu.*	*pks.*	*g.*	*qts.*	*pts.*
46	4	3	1	3	1
46	2	2	1	2	1
27	4	3	1	3	1
58	3	2	1	3	1
67	1	3	1	2	1

VIII. OF WINCHESTER MEASURE.

The denominations of Winchester Measure are—

2 pints (pts.)	make	1	quart.	qt.
2 quarts	make	1	pottle.	pot.
2 pottles	make	1	gallon.	ga.
8 gallons	make	1	firkin of ale.	fir.
9 gallons	make	1	firkin of beer.	fir.
2 firkins	make	1	kilderkin.	kil.
2 kilderkins	make	1	barrel.	bl.
1½ bls., or 54 gals.	make	1	hogshead.	hhd.

Winchester Measure is used in measuring ale, beer, cider, perry metheglin, vinegar, &c.

EXAMPLES.

bls.	kil.	fir.	g.	pot.	qt.	pt.
28	1	1	7	1	1	1
37	1	1	6	1	1	1
91	1	1	5	1	1	1
12	1	1	4	1	1	1
10	1	1	3	1	1	1
13	1	1	2	1	1	1

bls.	kil.	fir.	gal.	pot.	qts.	pts.
9	1	1	8	1	1	1
9	1	1	7	1	1	1
9	1	1	6	1	1	1
8	1	1	8	1	1	1
9	1	1	6	1	1	1
6	1	1	5	1	1	1

IX. OF WINE MEASURE.

The denominations of Wine Measure are—

4 gills (gi.)	make	1	pint.	pt.
2 pints	make	1	quart.	qt.
4 quarts	make	1	gallon.	gal.
5 gallons	make	1	half anchor.	hf. an.
10 gallons	make	1	anchor.	an.
42 gallons	make	1	tierce.	tier.
2 tierces, or 84 gallons,	make	1	puncheon.	pun.
63 gallons	make	1	hogshead.	hhd.
2 hogsheads	make	1	pipe or butt.	pi. or but.
2 pipes, or butts,	make	1	tun.	tun.

Wine Measure is used in measuring all sorts of strong liquors, as brandy, whiskey, rum, and wine.

EXAMPLES.

	2	2	63	4	2	4
T.	pi.	hhd.	gal.	qt.	pt.	gi.
46	1	1	62	3	1	3
76	1	1	61	2	1	3
14	1	1	42	2	0	2
12	1	1	39	2	0	3
16	1	1	40	2	1	2
14	1	1	24	2	0	3

	2	42	4	2	4
pun.	tier.	gal.	qts.	pt.	gi.
36	1	42	3	1	3
70	0	40	3	0	3
40	1	26	2	1	2
37	0	36	3	0	3
68	1	43	2	1	2
22	0	36	3	0	3

X. OF LONG MEASURE.

The denominations of Long Measure are—

3	barley corns (b.c.)	make	1	inch.	in.
4	inches	make	1	hand.	hd.
12	inches	make	1	foot.	ft.
3	feet	make	1	yard.	yd.
6	feet	make	1	fathom.	fa.
5½	yards, or 16½ feet,	make	1	rod, perch, or pole.	per. or po.
40	poles	make	1	furlong.	fur.
8	furlongs	make	1	mile.	mi.
3	miles	make	1	league.	le.
20	leagues	make	1	degree.	deg.
60 geographic, or 69½ English miles		make	1	degree.	deg. or °

Long Measure is used to measure the distance from one place to another, and also to ascertain the true length of any particular thing, without regard to its breadth. The hand is used to measure horses. The fathom is used to measure depths.

EXAMPLES.

	20	3	8	40
deg.	*le.*	*mi.*	*fur.*	*po.*
120	19	2	7	39
132	18	2	6	38
194	17	1	5	37
95	16	2	4	36
42	15	1	3	35
30	14	2	2	34

	3	12	3
yds.	*ft.*	*in.*	*b.c.*
781	2	11	2
289	2	10	2
789	2	9	2
799	1	9	2
876	2	7	2
988	2	6	2

		8	40	5½	3	12	3
	mi.	*fur.*	*po.*	*yds.*	*ft.*	*in.*	*bc.*
	376	7	10	5	2	9	1
	267	6	8	4½	1	8	2
	182	5	39	3½	2	10	2
	289	7	24	5	2	11	2
	126	6	10	2½	1	10	1
Answer	1244	1	15	2½	0	2	2
	867	2	4	2	0	5	1
Proof	1244	1	15	2½	0	2	2

XI. OF LAND OR SQUARE MEASURE.

The denominations of Land Measure are—

144	square inches (sq. in.)	make	1	square foot.	ft.
9	square feet	make	1	square yard.	yd.
30¼	square yards	make	1	square pole.	po.
40	square poles	make	1	square rood.	ro.
4	square roods	make	1	acre.	A.

Land Measure is used to measure land and gardeners' work. The first part is used to measure plank, glass, pavements, plastering, wainscotting, tiling, flooring, roofing, and every other dimension *of length and breadth* only.

EXAMPLES.

A.	ro.	po.
	4	40
73	3	39
49	3	14
91	2	29
19	3	36
99	3	38

A.	ro.	po.
	4	40
276	3	39
498	2	28
578	3	32
671	2	28
684	3	38

Sq. yds.	ft.	in.
	9	144
2794	8	143
1962	7	109
1243	6	108
999	5	140
168	4	142

Add up the column of square inches, as in Simple Addition, and divide the sum by 144, (because 144 square inches make 1 square foot,) set the remainder under the square inches, and add the quotient to the square feet; then proceed with the rest according to former directions.

XII. OF SOLID MEASURE.

The denominations of Solid Measure are—

1728	solid inches	make	1 solid foot.
27	solid feet	make	1 solid yard.
40 / 50	solid feet of round timber, or of hewn timber	make	1 ton, or load.
128	solid feet, that is, 8 in length, 4 in width, and 4 in height	make	1 cord of wood.

Solid Measure is used in measuring all solid bodies, or things that have length, breadth, and depth.

NOTE.—The wine gallon contains 231 solid inches, and the beer gallon 282. A gallon of dry measure contains $268\frac{4}{5}$ solid inches, and a bushel $2150\frac{2}{5}$ ditto.

EXAMPLES.

Tons of round timber.	Solid ft.	Solid in.
	40	1728
4	39	1727
3	36	68
2	27	578
3	32	1683
6	38	1572

Tons of hewn timber.	Solid ft.	Solid in.
	50	1728
6	49	1727
8	32	1648
7	48	1268
3	37	1472
6	34	1674

Cords.	Solid ft.	Solid in.
	128	1728
427	127	1727
376	100	1689
637	78	987
236	99	29
120	126	896

XIII. OF TIME.

The denominations of Time are—

60 seconds	(sec.)	make	1 minute.	min.
60 minutes		make	1 hour.	hr.
24 hours		make	1 natural day.	da.
7 days		make	1 week.	wk.
4 weeks		make	1 month.	mo.
13 months, 1 day, and 6 hours		make	1 Julian year.	yr.
12 calendar months		make	1 common year.	

The number of days in each calendar month may be designated in the following manner:

Thirty days are in September,
In April, June, and November;
In all the rest just thirty-one,
Except February alone,
Which has but twenty-eight, in fine,
Till leap year gives it twenty-nine.
Now add them up, and let me hear
How many days are in the year.
As sure as you and I'm alive,
There's three hundred and sixty-five.

N. B. When the year of our Lord can be divided by 4 without a remainder, it is leap-year or bissextile; centurial years excepted.

Examples.

	13	4	7	24	60	60
yrs.	*mo.*	*w*	*d.*	*hrs.*	*m.*	*sec.*
33	12	3	6	23	59	59
82	10	3	5	20	49	56
63	11	2	4	19	54	52
47	9	2	3	18	50	46
26	8	1	2	16	30	48
37	0	0	4	00	00	50
00	0	1	0	20	00	10
44	0	3	0	00	11	00

	13	4	7	24	60	60
yrs.	*mo.*	*w.*	*d.*	*hrs.*	*m.*	*sec.*
9	6	1	2	10	30	30
8	12	3	6	10	16	47
9	11	2	5	6	24	50
4	10	3	4	20	29	59
3	9	1	1	14	40	46
9	0	0	0	20	00	28
6	0	0	0	00	40	50
1	10	3	6	15	00	45

Add up the column of seconds as in Simple Addition and divide the total sum by 60, (because 60 seconds make 1 minute)—set the remainder under the seconds and add the quotient to the column of minutes—proceed on adding up the several columns and dividing their respective sums by the numbers placed over each column, till *you arrive at the last;* then set down the whole sum as before directed.

XIV. OF MOTION OR CIRCLE MEASURE.

The denominations of Motion (in the heavenly bodies) are—

60 seconds	(″)	make	1 prime, or minute.	′
60 minutes	(′)	make	1 degree.	°
30 degrees	(°)	make	1 sign.	si.
12 signs	(si)	make	1 complete circle.	

The zodiac is a great circle of the sphere, or globe, containing the 12 following signs, through which the sun is supposed to pass once in every solar year.

THE NAMES OF THE 12 SIGNS OF THE ZODIAC.

The Sun enters

Aries,	♈	March	20th.	Libra,	♎	September	22nd.
Taurus,	♉	April	19th.	Scorpio,	♏	October	22nd.
Gemini,	♊	May	20th.	Sagitarius,	♐	November	21st.
Cancer,	♋	June	21st	Capricorn,	♑	December	21st.
Leo,	♌	July	22nd.	Aquarius,	♒	January	19th.
Virgo,	♍	August	22nd.	Pisces,	♓	February	18th.

Motion, or circle measure, is principally used in astronomical computations, as eclipses, solar and lunar transits, &c.

EXAMPLES.

si.	°	′	″	°	′	″	°	′	″
1	29	59	59	79	29	59	89	59	59
1	24	40	44	31	41	46	38	40	48
1	21	54	52	81	52	37	79	01	10
1	10	46	37	98	59	28	124	6	00
1	12	12	18	87	40	45	12	8	12
0	18	44	56	99	00	00	8	0	58

XV. OF DUODECIMALS.

The denominations of Duodecimals are—

12 fourths	(⁗)	make	1 third.	‴
12 thirds	(‴)	make	1 second.	″
12 seconds	(″)	make	1 inch.	′ or in.
12 inches	(′)	make	1 foot.	ft.

Duodecimals are supplemental to Square Measure, and are used in measuring all sorts of painting and joinery, glazing and mason-

ty, plastering, wainscotting, ceiling of rooms, &c. to a greater degree of exactness than that measure will admit of alone.

EXAMPLES.

F.	in.	''	'''	''''
112	11	11	11	11
110	10	10	10	10
89	9	9	1	1
9	6	4	2	3
8	2	6	4	6
7	7	9	8	9
6	8	8	9	4
3	10	11	11	9

F.	in.	''	'''	''''
986	9	8	5	5
674	6	1	2	2
1272	10	10	10	10
5786	11	11	11	11
28644	00	00	00	1
900	6	4	2	4
65	4	2	0	0
9	2	1	0	1

APPLICATION.

1. Bought a horse for 18£, a saddle for 3£ 18s. 10½d., a bridle for 19s. 6d., a cow for 4£ 10s. 3d., a quantity of hay for 6£, and a quantity of corn for 12£ 13s. 7½d. How much did the whole cost? Ans. 46£ 2s. 3d.

2. A store keeper opened his store on Monday and received 12£, on Tuesday he took in 20£ 17s. 3d. ¾qrs., on Wednesday 10s. 8d. ¼, on Thursday 7£ and 6d., on Friday, only 7½d., and on Saturday he sold goods to the amount of 30£ and 4½d. How much money did he receive in the whole week? Ans. 70£ 9s. 6d.

3. A johannes weighs 18 pennyweights, a half johannes 9 pwts., a doubloon 16 pwts. 21 grs., a moidore 6 pwts. 18 grs., an English guinea 5 pwts. 6 grs., a French guinea 5 pwts. 5 grs. a Spanish pistole 4 pwts. 6 grs., and a French pistole 4 pwts. 4 grs. How much do they all weigh together? Ans. 3 oz. 9 pwts. 12 grs.

4. A grocer sold 5 cwt. 3 qrs. 25 lbs. of tobacco, 2 cwt. 2 qrs. 18 lbs. of brown sugar, 1 cwt. 2 qrs. 19 lbs. of white sugar, a quantity of raisins weighing 3 qrs. 13 lbs. 12 oz., and a parcel of tea weighing 1 qr. 18 lbs. 7 ozs. How much did the whole weigh together? Ans. 11 cwt. 2 qrs. 10 lbs. 3 oz.

5. An apothecary compounded several parcels of medicines, the first weighed 2 lbs. 10 oz. 6 drs. 2 sc. 15 grs.; the 2d, 11 oz. and 18 grs.; the 3d. 5 oz. 1sc. and the 4th. 8 oz. 7 drs. and 3 grs. How much did the whole composition weigh?

Ans. 4 lb. 7 oz. 3 drs. 1 sc. 16 grs.

6. A farmer sold 4 tods of wool to one man, to a second man he sold 1 stone and 6 lbs., to a third, 1 wey and 2 tods, and to a fourth, 1 wey and 3 lbs.? How much did he sell in all?

Ans. 1 sack, 0 weys, 6 tods, 1 stone, 1 clo. 2 lbs.

7. A linen draper bought 4 pieces of linen, the 1st. contained 28

yds. 3 qrs. 8 in.; the 2d, 25 yds. and 4 in.; the 3d, 26 yds. 2 qrs. and the 4th, 27 yds. 6 in. How much did he buy in all? Ans. 107 yds. 3 qrs.

8. A distiller bought 50 barrels and 3 quarts of corn from one man, of another he bought 20 bls. and 4 bushels, of a third he bought 68 bls. and 3 pks., and of a fourth 10 bls. 1 pk. 3 quarts. How much did he buy in all? Ans. 149 bls. 0 bs. 0 p. 1 gal. 2 qts.

9. A miller had 428 bushels, 3 pecks, 1 gallon, 3 quarts of wheat in one garner; 576 bu. 2 p. 1 g. 2 qts. in another one; 294 bu. 3 p. 1 g. 2 qts. in a third; and 99 bu. 3 p. 3 qts. in a fourth. How much was there in the said mill?

Ans. 1400 bu. 1 pk. 1 g. and 1 qt.

10. A man sold 4 hogsheads of cider—the 1st held 98 gallons, 3 quarts, 1 pint; the 2d, 68 gal. 2 qts.; the 3d. 120 gallons; and the 4th, 76 g. 2 qts. 1 pt. How much did he sell in all?

Ans. 364 gallons.

11. A man started a journey, on Monday morning, and travelled 30 miles; on Tuesday he travelled 32 m. 7 fur. 35 po.; on Wednesday, 36 m. 5 fur. 26 po.; on Thursday, 37 m. 2 fur. 39 po.; on Friday, 28 m. 4 fur., and on Saturday, 25 m. 0 fur. 25 po. How far did he travel in all? Ans. 190 miles 5 furlongs.

12. A man bought several tracts of land—the 1st contained 100 acres; the 2d, 75 a. 3 r. 20 p.; the 3d, 50 a. 1 r. 38 p.; the 4th, 5 a. 2 r. 13 p.; and the 5th, 15 a. 3 r. 25 p. How much did he buy in all? Ans. 247 a. 3 r. 16 per.

13. A tanner bought several loads of bark—the 1st contained 127 solid feet; the 2d, 100 s. f.; the 3d, 1 cord; the 4th, 1 c. 64 s. f.; and the 5th, 1 c. 93 s. f. How much did he buy in all? Ans. 6 cords.

14. A joiner having finished several pieces of work, wishes to know the whole content: now, the 1st piece measured, 17 ft. 10 in. 2 seconds and 1 thd.; the 2d piece, 20 ft. 4 in. and 7 thirds; the 3d, 49 ft. 6 in. 8 seconds; the 4th, four-score feet, 10 in. and 3 thirds; the 5th, 17 ft. and 4 thirds; the 6th three-score ft. 10 in. and the 7th, 37 ft. 1 in. and 9 thirds. What is the content in square measure? Ans. 283 feet, 6 inches.

COMPOUND SUBTRACTION.

Compound Subtraction teaches us how to find the difference between any two sums, or quantities, consisting of several denominations.

Rule.

1. Place the sums, or quantities, as in Compound Addition, with the less under the greater; then begin at the right hand, and take each separate part in the lower line from the one directly over it, and set down the remainder.

2. But, if any part in the lower line be greater than the upper one, take it from as many of the same denomination as will make 1 of the next greater.

3. Add the difference (if any) to the upper number, and set down that sum; then carry 1 to the next part in the lower line, and proceed as above directed, till the operation is finished.

4. Add the difference to the lower line, and that sum will be equal to the upper line if the work is right.

I. OF STERLING MONEY.

	10	20	12 4	10	20	12 4	10	20	12 4
	£	s.	d. qrs.	£	s.	d. qrs.	£	s.	d. q.
From	100	00	0.	399	18	$10\frac{3}{4}$	463	1	$7\frac{1}{2}$
Take	000	00	$0\frac{1}{4}$	243	10	$4\frac{1}{4}$	276	6	$1\frac{3}{4}$
Dif.	99	19	$11\frac{3}{4}$	156	8	$6\frac{1}{2}$	186	15	$5\frac{3}{4}$
Proof	100	00	0.	399	18	$10\frac{3}{4}$	463	1	$7\frac{1}{2}$

	£	s.	d.		£	s.	d.		£	s.	d.
Borrow'd	1000	00	0	Lent	1012	6	$4\frac{1}{2}$	Lent	3	10	0
Paid at several times	100	10	$2\frac{1}{2}$	Received at sundry times	461	2	$7\frac{1}{2}$	Receiv'd at sundry times		19	6
	96	7	$3\frac{1}{4}$		99	19	$10\frac{1}{4}$			15	$4\frac{1}{2}$
	203	19	$9\frac{3}{4}$		296	17	$9\frac{1}{2}$			12	9
	109	18	0		84	6	$4\frac{1}{4}$			19	$4\frac{1}{4}$
	279	19	$1\frac{1}{2}$		00	18	6			00	6
Paid in all				Received				Received			
Due				Due				Due			

II. OF TROY WEIGHT.

	lb.	oz.	pwts.	gr.	lb.	oz.	pwts.	gr.	lb.	oz.	pwts.	gr.
From	41	7	12	18	100	0	0	0	31	11	0	6
Take	36	8	6	20	19	9	9	9	11	11	10	10
Difference												
Proof												

III. OF AVOIRDUPOIS WEIGHT.

	T.	cwt.	qr.	lbs.	oz.	dr.	T.	cwt.	qr.	lb.	oz.	dr.
From	4	13	1	10	3	12	6	0	0	0	5	1
Take	2	17	2	27	1	13	5	0	0	0	12	11
Difference												
Proof												

IV. OF APOTHECARIES WEIGHT.

	℔	℥	ʒ	℈	gr.	℔	℥	ʒ	℈	gr.	℔	℥	ʒ	℈	gr.
From	16	7	6	1	6	80	0	0	1	10	12	1	2	0	0
Take	13	8	7	1	12	49	1	1	1	15	9	2	5	0	16
Difference															
Proof															

V. OF WOOL WEIGHT.

	la.	sac.	w.	Tods	sto.	clo.	lbs.	sac.	w.	t.	st.	cl.	lb.
From	46	4	0	11	0	0	3	6	0	6	0	0	0
Take	29	8	1	8	1	1	4	1	0	3½	1	0	5
Difference													
Proof													

VI. OF CLOTH MEASURE.

		4	4		4	9		5	4		5	9
	yds.	qrs.	na.	yds.	qrs.	in.	E. E.	qrs.	na.	E. E.	qrs.	in.
From	27	0	0	174	0	2	61	0	0	100	0	0
Take	19	1	2	99	1	8	51	4	2	39	3	4
Difference												
Proof												

	Fr. E.	qr.	na.	Fr. E.	qr.	in.	Fl. E.	qr.	na.	Fl. E.	qr.	in.
From	100	0	0	110	0	0	200	0	0	1000	0	0
Take	89	1	3	106	5	6	119	2	2	899	0	7
Difference												
Proof												

VII. OF DRY MEASURE.

	bl.	bu.	pk.	g.	qt.	pt.	ch.	qr.	coo.	bu.	pk.	g.	qt.	pt.
From	74	2	2	0	2	0	10	2	0	2	1	0	2	0
Take	67	3	3	0	2	1	8	3	1	2	1	1	3	1
Difference														
Proof														

VIII. OF WINCHESTER MEASURE.

	A. bls.	*bu.*	*pk.*	*qt.*	*pt.*	*gi.*	*pt.*
From	61	0	0	4	0	0	0
Take	59	0	1	3	1	1	1
Difference							
Proof							

	B. bls.	*kil.*	*fir.*	*gal.*	*pot.*	*qts.*	*pts.*
From	100	0	0	0	0	0	0
Take	97	1	1	8	1	0	1
Difference							
Proof							

IX. OF WINE MEASURE.

	T.	*pi.*	*hhd.*	*gal.*	*qt.*	*pt.*	*gi.*
From	6	0	0	50	0	0	0
Take	4	1	1	60	1	1	1
Difference							
Proof							

	pun.	*tier.*	*gal.*	*qts.*	*pt.*	*gi.*
From	40	0	40	0	0	0
Take	38	1	41	0	0	2
Difference						
Proof						

X. OF LONG MEASURE.

	deg.	*le.*	*mi.*	*fur.*	*po.*
From	100	10	1	6	30
Take	49	16	1	7	38
Difference					
Proof					

	yds.	*ft.*	*in.*	*b.c.*
From	1760	0	0	0
Take	879	1	5	1
Difference				
Proof				

	Deg.	*le.*	*mi.*	*fur.*	*po.*	*yds.*	*ft.*	*in.*	*b.c.*
From	251	12	0	6	32	$2\frac{1}{2}$	1	6	1
Take	199	18	1	7	35	$3\frac{1}{2}$	1	8	2
Difference									
Proof									

XI. OF LAND OR SQUARE MEASURE.

	A.	*ro.*	*po.*
From	1000	1	20
Take	799	1	32
Difference			
Proof			

	A.	*ro.*	*po.*
From	6721	2	0
Take	4890	3	30
Difference			
Proof			

	Sq. yds.	*ft.*	*in.*
From	6000	0	127
Take	4000	6	143
Difference			
Proof			

XII. OF SOLID MEASURE.

	Tons of round timber.	*Solid ft.*	*Solid in.*
From	120	30	1200
Take	115	38	1600

	Tons of hewn timber.	*Solid ft.*	*Solid in.*
From	112	40	1628
Take	107	49	1700

	Cords.	*Solid ft.*	*Solid in.*
From	100	100	700
Take	91	108	1428

XIII. OF TIME.

yrs.	mo.	w.	d.	hrs.	m.	sec.		yrs.	mo.	w.	d.	hrs.	m.	sec.
100	0	0	0	0	0	0		1828	10	2	2	2	20	30
87	6	1	0	11	53	54		1728	4	3	6	5	29	40
Difference														
Proof														

N. B.—The distance of time between any two given dates (according to the calendar) may be easily found by the following

Rule.

1. Set the first date under the last, and subtract it therefrom.

2. When there are more days in the lower number than the upper one, subtract them from as many days as make the lower month, (according to the calandar,) and add the difference to the upper number of days; then carry one to the months in the lower date, and subtract that sum from the months in the upper one, increased by twelve when necessary.

3. When twelve are added to the months in the upper date, one must be added to the years in the lower date before the subtraction is made.

4. Begin with January, and number the months 1, 2, 3, 4, &c. to 12, inclusive.

5. When one of the dates is in the old style and the other in the new style, eleven days must be taken from the difference.

Examples.

1. A bond was given on the 20th day of December, 1825, and taken up on the 10th day of August, 1830. For what space of time must interest be calculated thereon?

Years.	mo.	days.	
1830	8	10	
1825	12	20	
4	7	21	Answer.

2. General George Washington was born on the 11th day of February, 1732, old style, and died on the 14th day of December, 1799, new-style. How old was he on the day of his defunction?

	Years.	mo.	days.
	1799	12	14
	1732	2	11
	67	10	3
Subtract			11
Answer	67	9	20

Calculated by new style.

Years.	mo.	days.	
1799	12	14	
1732	2	22	
67	9	20	Answer.

3. The massacre at Boston by the British troops happened on the 5th day of March, 1770, and the battle at Lexington April 19th, 1775. What space of time passed between those events?

Ans. 5 yrs. 1 mo. 14 ds.

4. General Burgoyne and his army were captured October 17th, 1777, and Earl Cornwallis and his army on the 19th of October, 1781. What length of time passed between those events?

Ans. 4 yrs. and 2 ds.

5. The Revolutionary War, between England and America, commenced April 19th, 1775, and a general peace took place on the 20th day of January, 1783. How long did the war continue?

Ans. 7 yrs. 9 mo. 1 day.

6. Land was first discovered in the Western hemisphere by Christopher Columbus on the 12th day of October, 1492, old style, and the independence of the United States declared at Philadalphia on the 4th day of July, 1776. What duration of time passed between those memorable events? Ans. 283 yrs. 8 mo. 12 ds.

XIV. OF MOTION OR CIRCLE MEASURE.

	si.	°	′	″	°	′	″	°	′	″
From	12	00	00	00	180	40	52	91	00	00
Take	6	6	6	6	90	50	54	45	29	46
Answer										
Proof										

XV. OF DUODECIMALS.

	F.	in.	″	‴	⁗	F.	in.	″	‴	⁗	Ft.	in.	″	‴	⁗
From	172	8	7	10	6	1210	4	6	4	0	2000	10	0	4	7
Take	91	9	8	11	8	918	5	1	4	6	999	11	1	5	9
Answer															
Proof															

COMPOUND MULTIPLICATION.

Compound Multiplication teaches us how to multiply any sum or quantity consisting of several denominations, but is principally used to find the amount of any given quantity of goods, when the price of one article is known.

General Rule.

Set the multiplier under the lowest denomination of the given sum or quantity, then multiply, as in whole numbers, and divide

the product by as many of the same denomination as will make one of the next greater; set down the remainder, (if any,) and add the quotient to the product of the next denomination: proceed on in the same manner till the work is completed.

EXAMPLES IN STERLING MONEY.

	£	s.	d.	£	s.	d.	£	s.	d.
Multiply	19	19	$11\frac{3}{4}$	15	18	$9\frac{1}{2}$	13	17	$8\frac{1}{4}$
By			2			3			4
Answers	39	19	$11\frac{1}{2}$	47	16	$4\frac{1}{2}$	55	10	9.

	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.
Multiply	2	13	$4\frac{1}{2}$	1	12	$3\frac{1}{4}$	10	10	$10\frac{3}{4}$	11	12	$10\frac{3}{4}$
By			8			9			10			12

EXAMPLES IN WEIGHTS AND MEASURES.

	lbs.	oz.	pwts.	grs.
Multiply	9	11	19	23
By				2

T.	cwt.	qrs.	lb.	ozs.	drs.
2	17	3	27	15	15
					3

lb.	oz.	dr.	sc.	gr.
3	11	7	2	19
				4

	yds.	qrs.	na.	yds.	qrs.	in.	E. E.	qrs.	na.	E. E.	qrs.	in.
Multiply	27	3	3	19	3	8	16	4	3	16	4	8
By			5			6			7			8

bls.	bu.	pks.	ga.	qts.	pts.
9	4	3	1	3	1
					9

qrs.	bu.	pks.	ga.	qts.	pts.
25	7	3	1	3	1
					10

B.A.	kil.	fir.	gal.	po.	qt.	pt.
2	1	1	7	1	1	1
						11

B.B.	kil.	fir.	gal.	po.	qt.	pt.
2	1	1	8	1	1	1
						12

T.	pi.	hhd.	gal.	qt.	pts.	gil.
3	1	1	62	3	1	3
						5

Pi.	hhd.	gal.	qts.	pts.	gil.
2	1	60	3	1	3
					8

Deg.	lea.	mi.	fur.	po.
3	19	3	7	39
				6

yds.	ft.	in.	b.c.
89	2	11	2
			8

A.	roo.	po.
46	3	28
		9

A.	roo.	po.
22	2	36
		11

Sq. yds.	sq. ft.	sq. in.
576	7	72
		12

yrs.	mo.	w.	da.	hr.	min.	sec.
36	12	3	6	23	54	56
						2

signs	°	′	″
1	15	32	36
			7

To find the amount of any quantity of goods, or other mercantile articles, at any given price by the integer.

Case 1.

When the number of integers in the given quantity does not exceed 12, multiply the price of an integer by the given number, and the product will be the answer required.

Examples.

1. What will 5 gallons of molasses cost, at 7s. 6d. per gallon? Ans. 1£ 17s. 6d.
2. What will 6 lbs. of cinnamon cost, at 10s. 9d. per lb.? Ans. 3£ 4s. 6d.
3. What will 9 lbs. of iron cost, at $7\frac{1}{2}$d. per lb.? Ans. 0£ 5s. $7\frac{1}{2}$d.
4. What will 12 yards of drab cost, at 18s. $6\frac{1}{2}$d. per yd.? Ans. 11£ 2s. 6d.

Case 2.

When the number of integers in the given quantity exceeds 12, and is the exact product of any two factors in the multiplication table.

Rule.

Multiply the price of an integer by one of the said factors, and the product thence arising by the other—the last product will be the answer required.

Examples.

1. What will 14 yards of cloth cost at

	s.	d.	
	17	6	per yard.
		2	
£1	15	0	price of 2 yds.
		7	
Ans. 12	5	0	price of 14 yds.

2. What will 16 yards of cassinet cost, at 7s. 10d. per yard? Ans. 6£ 5s. 4d.
3. What will 18 lbs. of sugar come to at $10\frac{1}{2}$d. per lb.? Ans. 15s. 9d.
4. What will 20 lbs. of tobacco come to at 1s. $7\frac{1}{2}$d. per lb.? Ans. 1£ 12s. 6d.
5. What will 21 lbs. of pepper come to at 4s. 6d. per lb.? Ans. 4£ 14s. 6d.
6. What will 22 lbs. of butter come to at $7\frac{1}{2}$d. per lb.? Ans. 13s. 9d.
7. What will 24 yards of scarlet come to at 23s. 6d. per yard? Ans. 28£ 4s.

Case 3.

When the number of integers in the given quantity exceeds 12, and is not the exact product of some two factors in the multiplication table.

Rule.

1. Choose two such factors as will produce the nearest product to the given number of integers and proceed with them as in the last case.

2. Multiply the given price of one integer by any number that will supply the deficit, and add the result to the amount produced by the last factor, and that sum will be the answer required.

Examples.

1. What will 118 lbs. of bacon come to at 10½d. per lb.?

The price of *1lb.*=10½*d.*×8

	£	s.	d.	
			10	
		8	9.	value of 10 lbs.
			11	
	4	16	3.	value of 110 lbs.
		7	0.	value of 8 lbs.
Answer	5	3	3.	value of 118 lbs.

2. What will 13 yards of linen come to at 5s. 6d. per yard?
Ans. 3£ 11s. 6d.

3. What will 17 bushels of rye come to at 3s. 9d. per bushel?
Ans. 3£ 3s. 9d.

4. What will 19 bushels of wheat come to at 13s. 6d. per bushel?
Ans. 12£ 16s. 6d.

5. What will 23 lbs. of sugar come to at 10½d. per lb.?
Ans. 1£ 0s. 1½d.

6. What will 26 bushels of corn come to at 2s. 6d. per bushel?
Ans. 3£ 5s.

Case 4.

When the number of integers in the given quantity is greater than the product of any two numbers in the Multiplication Table.

Rule.

Multiply continually by as many tens, less one, as there are figures in the given quantity; then multiply the last product by the left hand figure of the given quantity, (if it be more than one.) Again: multiply the units figure into the given price of one integer, *and that in the ten's* place into the price of 10, and that in the hun-

dred's place into the price of 100, &c. Place the several products as in Compound Addition, and their sum will be the answer required.

EXAMPLES.

1. What will 7859 lbs. of tobacco come to, at 13½ per lb.?

		£	s.	d.		
The price of 1 lb. is 13½d. or			1 s.	1½ d. ×9=	0 £ 10 s. 1½ d.	
				10		
The price of 10 lbs. is			11	3.×5=	2 16 3	
				10		
The price of 100 lbs. is		5	12	6.×8=	45 0 0	
				10		
The price of 1000 lbs. is		56	5	0.		
				7		
		£393	15	0.	=the price of 7000 lbs.	
		45	0	0.	=the price of 800 lbs.	
		2	16	3.	=the price of 50 lbs.	
			10	1½	=the price of 9 lbs.	
Answer		£442	1	4½	=the price of 7859 lbs.	

2. What will 352 lbs. of beef come to, at 3¼d. per lb.?
Ans. 4£ 15s. 4d.

3. What will 195 lbs. of loaf sugar come to, at 14d. per lb.?
Ans. 11£ 7s. 6d.

4. What will 390 lbs. of bacon come to, at 7d. per lb.?
Ans. 11£ 7s. 6d.

5. What will 407 lbs. of Crawley steel cost, at 1s. 7½d. per lb.?
Ans. 33£ 1s. 4½d.

MULTIPLICATION OF DUODECIMALS,

COMMONLY CALLED

CROSS MULTIPLICATION.

Multiplication of Duodecimals is used for finding the superficial content of hewn timber, planks, boards, and scantling. It is also used for calculating the superficial content of carpenters', joiners', bricklayers', plasterers', painters', and glaziers' work; and, likewise, for ascertaining the solidity or superficial content of any thing else, when the dimensions are taken in feet, inches, and parts of an inch.

General Rule.

Feet multiplied by feet give feet.
Feet multiplied by inches give inches.
Feet multiplied by seconds give seconds.
Inches multiplied by inches give seconds.
Inches multiplied by seconds give thirds.
Seconds multiplied by seconds give fourths, &c.

Case 1.—Of feet and inches.

Examples.

	ft.	in.	
Multiply	7	3	
By	3	9	
	21	0	
	0	9	
	5	3	''
	0	2	3
Answer	27	2	3

In this example, I say 3 times 7 are 21 feet, which I set down in the first line under feet, and place a cipher in the second, under inches. Next, I say 3 feet multiplied by 3 inches give 9 inches, which I set down in the second line under inches, and place a cipher in the first, under feet. Now, I begin with 9 inches, and say 9 inches multiplied by 7 feet give 63 inches, which are equal to 5 feet 3 inches; that I set down for the third line, under feet and inches. Lastly, I say 9 inches multiplied by 3 inches give 27 seconds, which make 2 inches and 3 seconds; that I set down for the last line, and then add them together, and the answer is 27 ft. 2 inches, and 3 seconds.

Proved by whole numbers.

7 feet 3 inches are equal to 87 inches, and
3 feet 9 inches are equal to 45 inches.

```
          435
         348
       ─────
     12)3915 seconds.
       ─────
      12)326  3''
       ─────
```

Answer the same as before, 27 ft. 2 in. 3''

---

	ft.	in.		ft.	in.		ft.	in.		
Multiply	4	7		3	11		7	10		27 feet
By	5	10		9	5		8	11		9 inches
	20	0		27	0		56	0		12)243 inches
	2	11		8	3		6	8		
	3	4		1	3		6	5		20 ft. 3 in.
	0	*5*	10''	0	4	7''	0	9	2''	Answer.
Answers	26	8	10	36	10	7	69	10	2	

	ft. in.	ft. in.	ft. in.	ft. in.	ft. in.
Multiply	4 6	9 7	8 3	3 8	9 7
By	5 8	9 7	6 4	7 6	3 6
Answers	25 6	91 10 1″	52 3	27 6	33 6 6″

Case 2.—Of feet, inches, and seconds.

Examples.

	F.	in.	″		
Multiply	8	5	9		
The register ‖ By	7	8	6		
7ft. × 8ft. =	56	0	0		
7ft. × 5in. =	2	11	0		
7ft. × 9″ =	0	5	3		
8in. × 8ft. =	5	4	0		
8in. × 5in. =	0	3	4		
8in. × 9″ =	0	0	6		
6″ × 8ft. =	0	4	0	‴	
6″ × 5in. =	0	0	2	6	⁗
6″ × 9″ =	0	0	0	4	6
Answer	65	4	3	10	6

	F.	in.	″		
Multiply	27	9	6		
The register ‖ By	0	11	6		
11in. × 27ft. =	24	9	0		
11in. × 9in. =	00	8	3	‴	
11in. × 6″ =	00	0	5	6	
6″ × 27ft. =	01	1	6	0	
6″ × 9in. =	0	0	4	6	
6″ × 6″ =	0	0	0	3	
Answer	26	7	7	3	

	F. in. ″	F. in. ″	F. in. ″	F. in. ″
Multiply	27 9 6	22 11 4	7 3 2	3 8 4
By	00 11 0	2 9 6	1 7 3	3 9 2
	‴	‴	‴ ⁗	‴ ⁗
Answer	25 5 8 6	64 0 7 8	11 7 9 11 6	13 10 10 4 8

Application of Duodecimals.

1. If a plank be 18 feet long and 9 inches wide, how many square feet does it contain? Ans. 13 feet 6 inches.

2. If a plank be 22 feet long, and 11½ inches wide, how many square feet does it contain? Ans. 21 feet 1 inch.

3. There is a piece of scantling that is 6 inches wide, 4 inches thick, and 24 feet long. How many square feet does it contain?

$$\left.\begin{array}{l}\text{6 inches wide}\\ \text{4 inches thick}\end{array}\right\}\text{added}$$

10 inches = side and edge
24 feet long

12)240 inches

Ans. 20 feet.

4. *There are 12 planks*, each 17ft. 6 inches long and 8 inches

seconds wide. How many square feet do they contain? Ans. 1 plank contains 12ft. 9 in. 1 sec. 6‴, and the whole number 153ft. 1in. 6″.

5. A sawyer having sawed 24 rafters 18 feet long, 4 inches wide, and 3 inches thick, demands how many square feet of sawing he must be paid for? Ans. 252 square feet

6. A glazier having glazed 24 sixteen light windows, each light being 8 by 10 inches, would know how many feet of glazing he must be paid for.

8 inches.
10 inches.

12)80 seconds.

6′ 8″=the content of one light.
16 =no of lights in one window.

8 ft. 10 8 =content of one window.
24 =number of windows.

213 4 0 answer.

7. A joiner having wainscotted a room with cedar that measures 112 about and 9 feet 6 inches high, desires to know how many square yards of wainscotting it contains, allowing 9 square feet to each square yard. Ans. 118 yards 2 feet.

8. A carpenter having laid a floor that is 36 feet long and 24 wide, wishes to know how many squares, of 100 feet each, are contained on the said floor. Ans. 8 squares and 64 feet.

9. A plasterer having plastered the interior walls of a house that measures 120 feet round and 10 feet 6 inches high, would be glad to know how many square yards of plastering are contained thereon. Ans. 140.

COMPOUND DIVISION.

Compound Division teaches us how to divide any given quantity, consisting of several denominations, by one common divisor; also, to find the price of an integer, when the whole number and value thereof are both given.

General Rule.

1. Divide the first denomination on the left hand by Simple Division.

2. Multiply the remainder by as many of the second, or next lower *denomination*, as make one of the first, and add the number of *the second denomination* to the product.

3. Divide that sum by the same divisor, and proceed in like manner through all the denominations of the given quantity to the last.

4. If the first denomination be less than the divisor, reduce it to the next lower one by the second rule; then divide, and continue the process as above directed.

5. Prove the work by Compound Multiplication.

Case 1.

When the divisor does not exceed 12.

Rule.

Divide the first denomination on the left hand by Short Division, and set the quotient directly under the said denomination; then continue the operation according to the general rule.

Examples in Sterling Money.

	£ s. d.	£ s. d.	£ s. d.
	2)39 19 11½	3)47 16 4½	4)55 10 9¾
Answer	19 19 11¾	15 18 9½	13 17 8¼+3 rem.
	2	3	4
Proof	39 19 11½	47 16 4½	55 10 9¾

	£ s. d.	£ s. d.	£ s. d.
	5)29 18 2½	6)71 0 5¾	7)89 0 0
	r.	r.	r.
Answer	5 19 7½+4	11 16 8¾+5	12 14 3¼+5
	5	6	7
Proof	29 18 2½	71 0 5¾	89 0 0.

	£	s.	d.			£	s.	d. r.
1. Divide	4	3	4¼	by 3.	Answer,	1	7	9¼+2
2. Divide	0	11	9¾	by 4.	Answer,	0	2	11¼+3
3. Divide	6	0	0	by 7.	Answer,	0	17	1½+6
4. Divide	90	5	0	by 8.	Answer,	11	5	7½
5. Divide	10	18	7½	by 11.	Answer,	0	19	10½

Examples in Weights and Measures.

1. Divide 41 lb. 11 oz. 19 pwts. 23 grs. by 5.
Ans. 8 lb. 4 oz. 15 pwts. 23 grs.+4 r.

2. Divide 91 tons, 16 cwt. 3 qrs. 27 lbs. by 6.
Ans. 15 tons, 6 cwt. 0 qrs. 18 lbs. 8 oz.

3. Divide 9 lb. 11 oz. 6 dr. 1 sc. 18 grs. by 7.
Ans. 1 lb. 5 oz. 0 dr. 2 sc. 16 grs.+6 r.

4. Divide 17 tods, 1 stone, 1 clo. 6 lbs. 8 oz. by 8.
Ans. 2 tods, 0 sto. 0 clo. 6 lbs. 15 oz.

5. Divide 170 yds. 1 qr. 3 na. by 9. Ans. 18 yds. 3 qrs. 3 na.
6. Divide 257 yds. 0 qrs. 8 in. by 10. Ans. 25 yds. 2 qr. 8 in.
7. Divide 372 F.E. 2 qrs. 3 in. by 11. Ans. 33 F.E. 2 qrs. 6 in.
8. Divide 154 E.E. 4 qrs. 6 in. by 12. Ans. 12 E.E. 4 qrs. 5 in.
9. Divide 143 Fr.E. 5 qrs. by 9. Ans. 15 Fr.E. 5 qrs. 8 in.
10. Divide 75 bls. 3 bus. 2 pks. by 8. Ans. 9 bls. 2 bus. 1 p. 0 g. 2 q.
11. Divide 601 bush. 2 pks. 0 g. 2 qts. by 7. Ans. 85 bush. 3 pks. 1 gal. 2 qts.
12. Divide 1 tun of wine by 6. Ans. 42 gallons.
13. Divide 5 hhds. of beer by 9. Ans. 30 gallons.
14. Divide 41 deg. 48 miles, 4 fur. 35 poles by 5. Ans. 8 deg. 21 miles, 5 fur. 31 poles.
15. Divide 187 A. 3 roo. 32 po. by 4. Ans. 46 A. 3 roo. 38 po.
16. Divide 1829 years by 12. Ans. 152 yrs. 5 mo. 1 w. 4 da. 18 h.
17. Divide 12 days, 5 hrs. 40 min. 45 sec. by 7. Ans. 1 da. 17 h. 57 m. 15 sec.
18. Divide 11 signs, 20° 54′ by 8. Ans. 1 sign, 13° 51′ 45″.

Case 2.

When the given number of integers exceeds 12, and is the exact product of any two factors in the Multiplication Table.

Rule.

Divide the given price of the whole quantity by one of the said factors, and the quotient thence arising by the other one; the last quotient will be the answer required.

Examples.

1. Divide 21£ 15s. 3½d. by 24.

	£	s.	d.	
4)	21	15	3½	
6)	5	8	9¾	+2 r.
Answer		18	1½	+3 r.
			6	
	5	8	9¾	
			4	
Proof	21	15	3½	

	£	s.	d.		Answers £	s.	d.
2. Divide	6	6	0	by 16.	0	7	10½
3. Divide	1	1	10½	by 25.	0	0	10½
4. Divide	0	14	0	by 32.	0	0	5¼
5. Divide	1	10	0	by 48.	0	0	7½
6. Divide	1	0	0	by 64.	0	0	3¾
7. Divide	200	5	0	by 72.	2	15	7½
8. Divide	321	6	0	by 96.	3	6	11¼

Case 3.

When the divisor exceeds 12, and is not the exact product of some two factors in the Multiplication Table.

Rule.

Divide the greatest denomination of the given sum by Long Division, and proceed with the rest according to the general rule.

EXAMPLES.

Divide 379£ 19s. 7¼d. by 19.

```
     £    s.   d.   £    s.   d.
19)379   19   7¼(19   19   11¾ answer.
   19
   ---
   189
   171
   ---
   £18 remainder.
Multiply by 20 and take in 19 shillings.
   ---
   19)379(19 shillings.
      19
      ---
      189
      171
      ---
       18 shillings remainder.
Multiply by 12 and take in 7 pence.
      ---
   19)223(11 pence.
      19
      ---
       33
       19
      ---
       14 pence remainder.
Multiply by 4 and take in ¼.
      ---
   19)57(¾ farthings.
      57
      ---
      00
```

	£	s.	d.			£	s.	d.
1. Divide	3	11	6	by 13.	Answer,	0	5	6
2. Divide	3	3	9	by 17.	Answer,	0	3	9
3. Divide	1	0	1½	by 23.	Answer,	0	0	10½
4. Divide	17	2	10½	by 39.	Answer,	0	8	9½
5. Divide	140	4	9	by 52.	Answer,	2	13	11¼
6. Divide	1	3	5¼	by 75.	Answer,	0	0	3¾

APPLICATION.

	£	s.	d.		£	s.	d.
1. If 8 bushels of wheat cost	4	6	4,	what will 1 cost?	0	10	9½
2. If 12 yards of cloth cost	11	2	6,	what will 1 cost?	0	18	6½
3. If 20 lbs. of tobacco cost	1	12	6,	what will 1 cost?	0	1	7½
4. If 24 yards of scarlet cost	28	4	0,	what will 1 cost?	1	3	6
5. If 28 bushels of corn cost	5	5	0,	what will 1 cost?	0	3	9
6. If 29 yds. of toileynet cost	10	17	6,	what will 1 cost?	0	7	6
7. If 37 yds. of cassimere cost	19	17	9,	what will 1 cost?	0	10	9

DIVISION OF CROPS.

Division of Crops is supplemental to Compound Division, and is essentially necessary among planters and farmers in general.

RULE.

1. Add all the whole shares and parts of a share together, and reduce the sum to one denomination for a divisor, by which divide the whole crop; the quotient will be such a part of a single share as the divisor was reduced to—that is, if the divisor was reduced into halves, quarters, or thirds, &c. of a share, the quotient will be one-half, one-quarter, or one-third part of a single share.

2. Multiply the quotient by such a number as will produce a single share—that is, if it be one-half share, multiply by 2; if it be one-third, multiply by 3; and if it be one-fourth, multiply by 4, &c.

3. Multiply the single share by the number of whole sharers that the employer has under the overseer.

4. If any sharer is allowed more than a single share, add such parts together as will make up his quota.

5. Add the employer's part, the overseer's share, and the remainder (if any) together, and the sum will be equal to the whole crop, if the operation is right.

EXAMPLES.

1. Divide 8856 pounds of tobacco between an overseer and his employer, allowing the overseer one share and his employer five shares.—How much is each man's part.

5 shares.
1 share.

6)8856 pounds.

1476=1 single share.
5=employer's hands.

Ans. 7380 lbs. employer's part.
1476 lbs. overseer's share.

8856 lbs. proof.

2. Divide 9764 lbs. of tobacco between an overseer and his employer, allowing the overseer $1\frac{1}{2}$ shares and the employer 7 shares. What is each man's part?

7 whole shares.
$1\frac{1}{2}$ shares.

$8\frac{1}{2}$
2

17)9764(574=$\frac{1}{2}$ share.
85 2
126 1148=a single share.
119 7=employer's hands
74 8036=employer's part.
68 1722=overseer's share.
6 6=remainder.
9764 proof.

1148 is one single share.
574 is one-half share.

1722 the overseer's share, which is added to the employer's part, in order to prove the question, as may be seen above.

3. Divide 14671 lbs. of tobacco and 375 barrels of corn between an employer and his overseer, allowing the employer 8 shares and the overseer $1\frac{1}{4}$, and tell me each man's part.

8 sharers.
$1\frac{1}{4}$ overseer's share.
———
$9\frac{1}{4}$
4
———
37)14671(*lbs. oz.* 396 8=$\frac{1}{4}$ share.
111   4
———
357   1586 0=a single share.
333   8 hands.
———
241   12688 0=master's part.
222   1982 8
———   8 remainder.
19
16   14671 0=proof.
———
37)304(8 ounces.
296
———
8

lbs. oz.
1586 0=a single share of tobacco.
396 8=$\frac{1}{4}$ share of tobacco.
———
1982 8=the overseer's share of tob'o.

bls. corn.
37)375(
370
———
5
5
37)25(0*b*
4
37)100(2*p*
74
26
2
37)52(1*g*
37
15
4
37)60(1*qt*
37
23
2
37)46(1*p*
37
2)9 pints remainder.
4)4 1
1 0 1 remainder.
g. q. pt.

bls. b. p. g. q. pt.
10 0 2 1 1 1=$\frac{1}{4}$ share.
4
———
40 2 2 1 2 0=a single share.
8
———
324 1 2 0 0 0=master's part.
50 3 1 0 3 1=overseer's sh.
1 0 1=remainder.
———
375 0 0 0 0 0 proof.

bls. b. p. g. q. pt.
40 2 2 1 2 0=a single share.
10 0 2 1 1 1=$\frac{1}{4}$ of a share.
———
50 3 1 0 3 1=overseer's share of corn.

4. An overseer and 11 sharers raised 14720 lbs. of tobacco and 192 barrels of corn, of which the overseer is to have 1 share and $\frac{1}{2}$ of a share. I demand the master's part, the overseer's share, and proof. Answer. The master must have 12650 lbs. of tobacco and 165 barrels of corn, and the overseer must have 2070 lbs. of tobacco and 27 barrels of corn.

5. An overseer and 9 hands raised 1678 lbs. of tobacco, 376 barrels of corn, and 400 bushels of wheat, of which crop the overseer is allowed 1 share and $\frac{1}{3}$ of a share. What is the master's part, and how much must the overseer have? Answer. The master must have 14607 lbs. of tobacco, 326 bls. 3 b. 2 pks. of corn, and 347 bushels, 2 pks. 1 gallon of wheat. The overseer must have 2164 lbs. of tobacco, 48 bls. 2 bu. of corn, and 51 bush. 2 pks. of wheat.

6. A gentleman owned a plantation, on which he placed 20 hands at a single share apiece, an old hand at $\frac{5}{8}$ of a share, and a boy at $\frac{3}{4}$; he allowed his overseer 3 shares and $\frac{1}{8}$ of a share; they raised 30494 lbs. of tobacco, 772 bls. of corn, and 1544 bush. of wheat. What is *the employer's part, and* also the overseer's share of the said crop?

20 whole shares.
$3\frac{1}{8}$ for the overseer.
$\frac{5}{8}$ for the old hand.
$\frac{3}{8}$ for the boy.
$\overline{24\frac{1}{8}}$
8

lbs. tob. *lbs.*
193)30494(158=$\frac{1}{8}$ part of a share.
193 8
$\overline{1119}$
965 1264=a whole share.
$\overline{1544}$ 20 hands.
1544
.... 25280=the employer's part for 20 hands.
$\frac{1}{8}$ of a share=158×5 790=the old hand's part.
=158×3 474=the boy's part.
$\overline{26544}$=the employer's whole share of tobacco.

$$\left.\begin{array}{r}1264\times3=3792\\158\times1=\ \ 158\end{array}\right\}=\ 3950=\text{the overseer's share of tobacco.}$$

$\overline{30494}$ proof.

bls.corn. *bls.*
193)772(4=$\frac{1}{8}$ part of a share.
772 8
... 32=a whole share.
20 hands.
$\overline{640}$=the employer's part for 20 hands.
$\frac{1}{8}$ of a share 4×5= 20=the old hand's part.
4×3= 12=the boy's part.
$\overline{670}$=the employer's whole share of corn.

$$\left.\begin{array}{r}1\text{ share}=32\times3=96\\\frac{1}{8}\text{ share}=\ 4\times1=\ 4\end{array}\right\}=100=\text{the overseer's share of corn.}$$

$\overline{772}$ proof.

bus.wheat. *bus.*
193)1544(8=$\frac{1}{8}$ part of a share.
1544 8
.... 64=a whole share.
20 hands.
$\overline{1280}$=the employer's part for 20 hands.
$\frac{1}{8}$ of a share=8×5= 40=the old hand's part.
=8×3= 24=the boy's part.
$\overline{1344}$=the employer's whole share of wheat

$$\left.\begin{array}{r}1\text{ share}=64\times3=192\\\frac{1}{8}\text{ share}=\ 8\times1=\ \ 8\end{array}\right\}\ 200=\text{the overseer's share of wheat.}$$

$\overline{1544}$ proof.

A Collection of Tables necessary in the following Rules.

I.—A TABLE OF COINS WHICH PASS CURRENTLY IN THE UNITED STATES, WITH THEIR STERLING AND FEDERAL VALUE.

NAMES OF COINS.	Standard Weight.		Sterling Money.			New Hampshire, Vermont, Massachusetts, Maine, R. Island, Connecticut, & Va. currency, a dollar being 6s.			New York and North Carolina currency—the dollar being 8s.			New Jersey, Pennsylvania, Delaware and Maryland currency—the dollar being 7s. 6d.			South Carolina and Georgia currency—the dollar being 4s. 8d.			Federal Value.		
Gold.	*pwt.*	*gr.*	£	*s.*	*d.*	£	*s.*	*d.*	£	*s.*	*d.*	£	*s.*	*d.*	£	*s.*	*d.*	$	*c.*	*m.*
A Johannes	18	0	3	12	0	4	16	0	6	8	0	6	0	0	4	0	0	16	00	0
A Half Johannes	9	0	1	16	0	2	8	0	3	4	0	3	0	0	2	0	0	8	00	0
A Doubloon	16	21	3	6	0	4	8	0	5	16	0	5	12	6	3	10	0	14	93	3
A Moidore	6	18	1	7	0	1	16	0	2	8	0	2	5	0	1	8	0	6	00	0
An English Guinea	5	6	1	1	0	1	8	0	1	17	4	1	15	0	1	1	9	4	66	7
A French Guinea	5	5	1	0	7½	1	7	6	1	16	0	1	14	6	1	1	3	4	60	0
A Spanish Pistole	4	6	0	16	6	1	2	0	1	9	0	1	8	0	0	18	0	3	77	3
A French Pistole	4	4	0	16	0	1	2	0	1	8	0	1	7	6	0	17	6	3	66	7
Silver Coins.																				
An English or French Crown	19	0	0	5	0	0	6	8	0	8	9	0	8	3	0	5	0	1	10	0
The Dollar of Spain, Sweden, Denmark	17	6	0	4	6	0	6	0	0	8	0	0	7	6	0	4	8	1	00	0
An English Shilling	3	8	0	1	0	0	1	4	0	1	9	0	1	8	0	1	0	0	22	2
A Pistareen	3	11	0	0	10¾	0	1	3	0	1	7	0	1	6	0	0	11	0	20	0

All other gold coins, of equal fineness, are valued at 89 cents per pennyweight, and silver coins at 111 cents per ounce.

II.—A TABLE OF OTHER FOREIGN COINS, &c., WITH THEIR VALUE IN FEDERAL MONEY ANNEXED.

	$	c.	m.		$	c.	m.
A Pound Sterling	4	44	4	A Rupee of Bengal	0	55	5
A Pound of Ireland	4	10	0	A Guilder of the United Netherlands	0	40	0
A Pagoda of India	1	94	0	A Mark-banco of Hamburg	0	33⅓	0
A Tale of China	1	48	0	A Liver Turnois of France	0	18½	0
A Mill-ree of Portugal	1	25	0	A Real Plate of Spain	0	10	0
A Ruble of Russia	1	00	0				

III.—N. B.—THE FOLLOWING PIECES OF MONEY WERE FORMERLY CURRENT, BUT NOT SO NOW.

	s.	d.		d.		£	s.	d.
A Mark	13	4	or	160	A Groat	0	0	4
A Noble	6	8	or	80	An Angel	0	10	0
A Crown	5	0	or	60	A Carolus	1	3	0
A Half Crown	2	6	or	30	A Jacobus	1	5	0

REDUCTION.

Reduction teaches us how to reduce or change any given sum or quantity, of several denominations, to the lowest one required; and, also, from one denomination to another of the same value.

General Rule.

1. When the reduction is descending, multiply the first or greatest denomination given by as many of the second or next lower one as will make one of the first denomination, and add the number of the second denomination into the product. Again: multiply the result by as many of the third or next lower denomination as will make one in the second, and add the number of the third denomination into the product. Proceed in the same manner through all the denominations to the lowest one required.

2. When the reduction is ascending, divide the least denomination given by as many of its own name as will make one in the next greater, and so on, till you have divided by the greatest denomination required. The last quotient, with the several remainders properly annexed, will be the answer sought.

3. Reduction ascending and descending do mutually prove each other.

I. OF STERLING MONEY.

Reduction Descending.—Examples.

1. Reduce 7£ 14s. 6¼d. into shillings, pence, and farthings.

	£ s. d.	
	7 14 6¼	
	20	
Answer	154	shillings.
	12	
Ans.	1854	pence.
	4	
Ans.	7417	farthings.

2. Reduce 50£ 19s. 9½d. into shillings, pence, and half-pence.

	£ s. d.	
	50 19 9½	
	20	
Ans.	1019	shillings.
	12	
Ans.	12237	pence.
	2	half-pence make 1d.
Ans.	24475	half-pence.

3. Reduce 90£ 17s. 6d. into two-penny pieces and farthings.

	90 17 6	
	20	
	1817	
	6	two-pences make 1s.
Ans.	10905	two-penny pieces.
	8	*farthings* make 2d.
Ans.	*87240*	*farthings.*

4. Reduce 81£ 5. 7½d. into crowns, pence, and farthings.

	81 5 7½	
	4	crowns make 1£.
Ans.	325	crowns.
	60	pence make 1 crown.
Ans.	19507	pence.
	4	
Ans.	78030	farthings.

5. Reduce 19£ 19s. 11¾d. into farthings. Ans. 19199 qrs.
6. Reduce 12 crowns to shillings and pence. Ans. 60s. 720d.
7. Reduce 50 half-crowns to pence and farthings.
Ans. 1500d. and 6000 qrs.
8. Reduce 306 crowns to half-crowns and pence.
Ans. 612 half-crowns and 18360d.
9. Reduce 120 six-pences into three-pences, pence, and farthings.
Ans. 240 three-pences, 720d. 2880 qrs.
10. Reduce 210 crowns to shillings, groats, and pence.
Ans. 1050s. 3150gr. 12600d.

Reduction Ascending.—Examples.

1. In 7417 farthings, how many pounds, shillings, pence, &c.?

4)7417 quarters.
12)1854 ¼qr.
2,0)15,4 6d.
Ans. £7 14 6¼qr.

2. In 24475 half-pence, how many pounds, shillings, &c.?

2)24475 half-pence.
12)12237 ½d.
2,0)101,9 11d.
Ans. £50 19 11½d.

3. In 6431 six-pences, how many pounds, &c.?

2)6431 six-pences.
2,0)326,5+1 six-pence.
Ans. £160 15 6d.

4. In 11672 pence, how many groats, pounds, &c.?

4)11672 pence.
3)2918 groats, answer.
20)97,2s. and 2 gr., or 8d. over.
£48 12 8d. answer.

5. In 87240 qrs. how many two-pences, pounds, &c.?

d. grs.
2=8)87240 qrs.
6d.)10905 two-pences, ans.
2,0(181,7+3 two-pences or 6d. over.
£90 17 6d. answer.

6. In 78030 qrs., how many crowns, pounds, &c.?

4)78030 quarters.
cro. d.
1=6,0)1950,7½
cro.
£1=4) 325
Answer £81 5s. 7½

---

7. In 19199 farthings, how many pounds, &c.? Ans. £19 19s. 9¾d.
8. In 720d. how many shillings and crowns? Ans. 60s. 12 cro.
9. In 6000 qrs. how many pence and half-crowns?
Ans. 1500d. and 50 half-crowns.
10. In 18360d. how many half-crowns and crowns?
Ans. 612 half-crowns and 306 crowns

11. In 2880 qrs. how many pence, three-pences, and six-pences?
Ans. 720d. 240 three-pences, and 120 six-pences.
12. In 12600d. how many groats, shillings, and crowns?
Ans. 3150 groats, 1050s. and 210 crowns.

Reduction Ascending and Descending.—Examples.

1. In 720s. how many pence and crowns? Ans. 8640d. and 144c.
2. In 120s. how many crowns, half-crowns, and pounds?
Ans. 24 crowns, 48 half-crowns, and £6.
3. In 60 crowns, how many pounds, shillings, and dollars?
Ans. £15, 300s. and 50 dollars.
4. In 612 half-crowns, how many crowns, pence, and dollars?
Ans. 306 crowns, 18360 d. and 255 dollars.
5. In 40 guineas, Virginia currency, how many shillings and dolls.?
Ans. 1176s. and 196 dollars.
6. In 12600d., how many shillings and dollars?
Ans. 1050s. and 175 dolls.
7. In 63 crowns, how many shillings and guineas sterling?
Ans. 315s. and 15 guineas.
8. In 70 moidores, Virginia currency, how many pounds sterling?
Ans. £126 sterling.
9. In 12180 three-pences, how many shillings and guineas sterling?
Ans. 3045s. and 145 guineas.
10. How many crowns, groats, and pounds are in 82560 qrs.?
Ans. 344 crowns, 5160 groats, and £86.
11. How many groats, three-pences, and six-pences are in 121s.?
Ans. 363 groats, 484 three-pences and 242 six-pences.
12. How many shillings, crowns, and pounds are in 1120 six-pences?
Ans. 560s. 112 crowns, and £28.

II. OF FEDERAL MONEY.

Case 1.

To reduce the currency of any State, where the dollar passes for 6s. into Federal Money.

Rule.

1. When the sum is pounds only, annex three ciphers to it; then divide by 3 and the quotient will be the answer in cents.

2. When the sum is shillings only, annex two ciphers and divide by 6—the quotient will be cents.

3. When the sum is pence only, annex two ciphers and divide by 72—the quotient will be cents.

4. When the sum is pounds, shillings, &c., reduce it to pence, *then multiply by* 25 and divide by 18—the quotient will be cents, or *proceed by rule the* 3d.

EXAMPLES.

1. Change 100£ Virginia, &c. currency, into federal money.

3)100.000£

Ans. 33333⅓ cts.=$333.33⅓

2. Change 18s. Virginia, &c. currency, into federal money.

6)18s.00

Ans. 300 cents=$3.00

3. Change 36 pence, Virginia, &c. currency, into cents.

72)3600d.

Ans. 50 cents.

5. Reduce 9£ to dollars. Ans. $30.00 cts.

6. Reduce 18d. to cents. Ans. $00.25 cts.

7. Reduce 1£ 2s. 6d. to dollars, &c. Ans. $3.25 cts.

8. Reduce 5£ to dollars, &c. Ans. $16.66⅔ cts.

4. Change 18£ 17s. 3d. Virginia, &c. currency to federal money.

18£
20

377
12

4527
25

18)113175

Ans. 6287½ cts.=$62.87½

CASE 2.

To reduce Federal Money into the currency of any State where the dollar passes for 6s.

RULE.

Reduce the dollars and cents to cents and multiply them by 18, then divide the product by 25, and the quotient will be the answer in pence, which must be reduced to pounds, &c. by Reduction of Sterling Money.

EXAMPLES.

1. Change 333 dolls. 33⅓ cts. to pounds, Virginia currency.

33333⅓ cts.
3

100000=thirds of a cent.
18

3)1800000
25)600000
12)24000 pence.
2,0)200,0

£100 answer.

2. Change 62 dolls. 87½ cts. to pounds, &c. Virginia currency.

6287½ cts.
2

12575=half cents.
18

25)226350
2)9054=half pence.
12)4527 pence.
2,0)37,7,3d.

Ans. £18,17,3d. V a. c.

3. Reduce 3 dols. to shillings. Ans. 18s.
4. Reduce 50 cts. to pence and shillings. Ans. 36d. or 3s.
5. Reduce 25 cents to shillings, &c. Ans. 1s. 6d.
6. Reduce $3.25 cts. to pence, &c. Ans. £1 2s. 6d.
7. Reduce $16.66⅔ cts. to pence, &c. Ans. 1200d. 100s. £5.
8. Reduce 112½ cts. to pence. Ans. 81d.

Case 3.

To reduce the currency of any State where the dollar passes for 7s. 6d. into Federal Money.

Rule.

Increase the given sum in pence by one-ninth part of itself, and the result will be the answer in cents.

Examples.

1. Change 7s. 6d. Pennsylvania, &c. currency to federal money.

9)90d.=7s. 6d.
10

Answer 100 cents=1 dollar.

2. Change 100£ Pennsylvania, &c. currency into federal money?
Ans. $226 66⅔ cents.

3. Change 480£ 19s. 9d. Pennsylvania, &c. into federal money?
Ans. £1282 63⅓ cents.

Case 4.

To reduce Federal Money into the currency of any State where the dollar passes for 7s. 6d.

Rule.

Subtract one-tenth part of the given sum in cents from itself, and the remainder will be pence.

Examples.

1. Change 100 cts. into Pennsylvania, &c. currency.

10)100 cents.
10

Answer 90d. Penn. &c. cur.

2. Change $266 66⅔ cents to Pennsylvania, &c. currency.

266 66⅔
3
10)80000
8000
3)72000
12)24000 pence.
2,0)200,0
£100 answer.

3. Change $1282 63⅓ cents to *pounds*, Pennsylvania, &c. currency. *Ans.* 480£ 19s. 9d.

CASE 5.

To change the currency of any State where the dollar passes for 8s. into Federal Money.

RULE.

1. If the sum be pounds only, annex three ciphers to it, and divide by 4; the quotient will be the answer in cents.

2. Reduce the given sum to shillings, then annex two ciphers, and divide by 8, the quotient will be cents; or, increase the given sum in pence by one 24th part of itself, and the result will be cents.

EXAMPLES.

1. Change 100£ New York, &c. currency, to federal money.

```
      £
   4)100.000
   ---------          $
     250.00 cents=250.
```

2. Change 360£ New York, &c. currency, to federal money. Ans. 900 dollars.

3. Change 412£ 16s. 8d. N. York, &c. currency, to federal money. Ans. $1032 08 cts.

CASE 6.

To change Federal Money into the currency of any State where the dollar passes for 8s.

RULE.

Subtract one twenty-fifth part of the given sum in cents from itself, and the remainder will be pence.

EXAMPLES.

1. Reduce $250 to N. York, &c. currency.

```
  25)250.00
      10 00
  12)240 00 pence.
  20)200.0
    £100 answer.
```

2. Reduce $900 to N. York and North Carolina currency. Ans. £360.

3. Reduce $1032 08 to North Carolina, &c. currency. Ans. 412£ 16s. 8d. N. C. &c. currency.

CASE 7.

To reduce the currency of any State where the dollar passes for 4s. 8d. into Federal Money.

RULE.

Annex two ciphers to the pence in the given sum, and divide by 56, and the quotient will be the answer in cents.

EXAMPLES.

1. Reduce 4s. 8d. S. Carolina & Georgia cur. to federal money.

```
   4s. 8d.
   12
  8)5600
  7)700
   100 cents=1 dollar.
```

2. Reduce 100£ Georgia, &c. currency, into federal money. Ans. $428 57$\frac{1}{7}$ cents.

3. Reduce 11£ 5s. 8d. South Carolina, &c. currency, to federal money.

Ans. $41.21 Am.+

Case 8.

To reduce Federal Money to the currency of any State where the dollar passes for 4s. 8d.

Rule.

Multiply the cents in the given sum by 14, and divide the product by 25; the quotient will be the answer in pence.

Examples.

1. Reduce 100 cts to Georgia cur.

```
          14
      5)1400
       5)280
Answer    56 pence=4s. 8d.
```

2. Reduce 428 dollars 57¼ cents, to pounds, South Carolina currency.

Ans. £100 South Carolina currency.

A General Rule.

To reduce the currency of any State, Kingdom, or Province, in which money is computed by pounds, shillings, pence, and farthings, to the Federal Currency of the United States.

1. Reduce the given sum of current money to shillings, six-pences, groats, three-pences, pence, or farthings, whichsoever may be most convenient, and multiply the result by 1000 for a dividend.

2. Reduce the value of a current dollar in the said State, &c. to the same denomination for a divisor, by which divide the above dividend, and the quotient will be mills. If any thing remain, it may be rejected as useless.

Examples.

1. What is the value of one farthing, Virginia currency, in federal money? Ans. 3 mills,47 decimals of a mill, and 64 rem.

```
s.  d.  qr.     qr.       m.  Answer.
6×12×4=288)1.000(3,47+64
            864
            ----
            136,0
            115 2
            ----
            208 0
            201 6
            ----
              64
```

2. Reduce 19£ 19s. 11¾d. Virginia, &c. currency, to federal money. Ans. $66 66cts. 3m.+56 *remainder.*

3. What is one farthing, sterling money, worth in the federal currency of the United States?

Ans. 4 mills, 6 decimals of a mill, and 64 remainder.

```
s. d.  d.  qrs.  qrs. qr.     m.   Ans.
4  6=54×4=216)1000(4,6+64
              864
              ----
              136,0
              129 6
              ----
                 64
```

4. Reduce 19£ 19s. 11¾d. sterling money of Great Britain, to the federal currency of the United States. Ans. 88 dols. 88cts. 4 mills+56 remainder.

III. OF TROY WEIGHT.

EXAMPLES.

1. In 276 lb. 11 oz. 19 pwts. 23 grs. how many ounces, pennyweights and grains?

lbs.	oz.	pwt.	grs.
276	11	19	23
12			

Ans. 3323 ounces.
20

Ans. 66479 pwts.
24

265939
132958

Ans. 1595519 grains.

2. In 15 lb. 9 oz. 13 pwts. 16 grs. how many ounces, pwts. and grains? Ans. 189ozs. 3793pwts. and 91048 grains.

3. In 1595519 grains of gold, how many pounds, &c.?

24)1595519 grs.

2,0)6647,9 23 grs. over.

12)3323 19 pwts. over.

lb.276 11 oz. 19 pwts. 23grs.

4. In 8lb. 2 oz. 3 pwts. 16 grs. how many grs.? Ans. 47128 grs.

5. In 91048 grains of silver, how many pounds, &c.?
Ans. 15 lb. 9 oz. 13 pwts. 16 grs.

6. In 275520 grains how many pounds, &c.?
Ans. 47 pounds, 10 ounces.

IV. OF AVOIRDUPOIS WEIGHT.

N. B.—The following denominations of Avoirdupois Weight are sometimes used in mercantile transactions, and may be referred to as occasion requires.

	lbs.
A stone of butter	16
A firkin of butter	56
A barrel of butter	224
A stone of beef	8
A barrel of beef	220
A barrel of pork	220
A barrel of flour	196
A barrel of raisins	112
A quintal of rice	100
A quintal of fish	112
A barrel of anchovies	30
A gallon of train oil	7½

	Cwt.	qrs.	lbs.
A puncheon of prunes	10	0	00
A fother of lead	19	2	00
A stone of shot	14	0	00
A stone of iron	14	0	00
A scare of steel	0	0	20
A faggot of steel	1	0	8
A burden of steel	1	2	12
A barrel of gunpowder	1	0	00
A barrel of candles	1	0	8
A firkin of soap	0	2	8
A barrel of soap	2	1	4
A barrel of pot-ash	1	3	4

EXAMPLES.

1. In 33 tons, 16 cwt. 3 qrs. 17 lbs. 12 ozs. how many ozs. and drams?

T.	cwt.	qrs.	lbs.	ozs.
33	16	3	17	12

```
      20
    ----
     676
       4
    ----
    2707
      28
   -----
   21673
   5414
   -----
   75813
      16
  ------
  454890
  75813
  -------
Ans. 1213020 ounces.
          16
  --------
  7278120
 1213020
 --------
Ans. 19408320 drams.
```

See the operation of this question contracted.

T.	cwt.	qrs.	lbs.	ozs.
33	16	3	17	12

```
      20
    ----
     676
     676
    676 .
   676 . .
   . . 101lbs. in 3qrs. 17lbs.
   -----
   75813
      16
  -------
Ans. 1213020 ounces.
          16
  --------
Ans. 19408320 drams.
```

2. In 224768 drs. of silk, how many hundred-weight, qrs. &c.?

```
16)224768 drams.
  16)14048 ounces.
    28)878 pounds.
      4)31 10 lbs.
Ans.     7 cwt. 3 qrs. 10 lbs.
```

3. In 17 pigs of lead, each weighing 4 cwt. 3qrs. how many fothers, &c.

```
                 Cwt. qrs.
                  4   3
                  4
     Cwt. qrs.  | ----
1 fother=19  2  | 19 qrs.
          4     | 17
        ----    | ---Fo.cwt.qrs.
Quarters 78     |323(4 2 3 Ans.
                 312
               4)11 qrs. left.
                 2 3 qrs.
```

4. In 19408320 drs. how many tons, hundreds weight, &c.? Ans. 33 tons, 16 cwt. 3 qrs. 17 lbs. 12 oz.

5. In 2688 lbs. of butter, how many firkins? Ans. 48.

6. In 17 cwt. 1 qr. 6 lbs. of sugar, how many parcels, each 17 lbs. Ans. 144 parcels.

V. OF APOTHECARIES WEIGHT.

EXAMPLES.

1. In 15 lb. 6 oz. 4 drs. 2 sc. 12 grs. of ipecacuanha, how many *grains, and proof?* Ans. 89572.

2. A physician mixed 8 oz. of jalap with 2 oz. 6 dr. 1 sc. 4 grs. of calomel, and is desirous of knowing how many potions of 18 grs. each, are contained in the said composition. Ans. 288.

3. In 89572 grains of medicine, how many pounds, &c.
Ans. 15 lbs. 6 oz. 4 dr. 2 sc. 12 gr.

4. A doctor bought of an Apothecary, 64 gallypots, each containing 5 drachms of medicine, how many potions of 25 grains each are contained therein. Ans. 768 potions.

VI. OF CLOTH MEASURE.

EXAMPLES.

1. In 17 yards, 1 qr. 2 nails, how many nails? Ans. 278 na.
2. In 4712 nails, how many yards, &c.? Ans. 294 yds. 2 qrs.
3. In 47128 nails of Irish linen, how many pieces, each 25 yds.? Ans. 112 pieces, 20 yds. 2 qrs.
4. In 4 pieces of cloth, each 14 yds. how many quarters and nails? Ans. 224 qrs. 896 nails.
5. In 626 English ells, how many yards? Ans. 782 yds. 2 qrs.
6. In 1425 yds. of tapestry, how many Flemish ells? Ans. 1900.
7. In 4975 Flemish ells, how many English ells? Ans. 2985 E. E.
8. In 4599 yds. how many French ells? Ans. 3066.
9. In 54 yds. 3 qrs. 7 inches, how many inches? Ans. 1978 in.
10. In 2897 ins. how many E. ells, &c. Ans. 64 E. E. 1 qr. 8in.

VII. OF DRY MEASURE.

EXAMPLES.

1. In 98 barrels, 4 bu. 3 pks. 1 gal. 3 qts. of Indian corn, how many pints, and proof? Ans. 31678 pints.
2. In 33867 pints of wheat, how many bushels, &c.?
Ans. 529 bus. 0 pks. 1 gal. 1 qt. 1 pt.
3. In 16127 quarts of barley, how many quarters, &c.?
Ans. 62 qrs. 7 bu. 3 pks. 1 gal. 3 qts.

VIII. OF LIQUID MEASURE.

EXAMPLES.

1. In 72 hogsheads of beer, how many barrels? Ans. 108 bls.
2. If a back contain 30 barrels of beer, how many gallons does it hold? Ans. 1080 gallons.
3. In 92 hogshead of ale, how many barrels, kilderkins, firkins, gallons, quarts, and pints?
Ans. 138bls. 276kil. 552fir. 4416gal. 17664qts. 35328pts.
4. In 36408 pints of beer, how many barrels, &c.?
Ans. 126bls. 1fir. 6gals.
5. In 126 gallons of cider, how many bottles, each holding 1½ pints? Ans. 672.

6. In 3 hogsheads of brandy, how many half-anchors?

Ans. 37 half anchors and 4 gallons over.

7. In 1712 gallons of wine, how many pipes?

Ans. 13 pipes, 1 hogshead, 11 gallons.

8. In 8063 gills of wine, how many hogsheads, &c.?

Ans. 3hhds. 62gals. 3qts. 1pt. 3 gills.

IX. OF LAND MEASURE.

Examples.

1. Reduce 17 acres, 3 roods, 10 perches, to perches. Ans. 2850

2. Reduce 17551 perches into acres, &c. Ans. 109A. 2r. 31p.

3. If a piece of ground contain 24 acres, and an inclosure of 17 acres 3 roods be taken out of it, how many perches are there in the remainder? Ans. 1000p.

4. One field contains 7 acres, 1 rood, 36 perches; another 10 acres 32 perches; and a third 12 acres 1 rood. How many shares, of 76 perches in each share, are contained in the whole? Ans. 63.

X. OF LONG MEASURE.

Examples.

1. A degree contains $69\frac{1}{2}$ English or statute miles. How many furlongs, poles, yards, &c. are contained therein?

```
  m.  f.
  69  4
   8
 ---
 556   furlongs.
  40
 ----
2)22240  poles.
     5½
 ------
 111200
  11120
 ------
 122320  yards.
      3
 ------
 366960  feet.
     12
 -------
 4403520  inches.
       3
 --------
 13210560  barleycorns.
```

2. In 1360820 feet, how many miles, furlongs, poles, &c.?

Ans. 257m. 5fur. 33p. $4\frac{1}{2}$y. 2f.

Examine the work carefully.

```
      3)1360820  feet.
        -------
         453606  yards+2 feet.
              2
        -------
5½=11)907212  half yards.
        -------
     4,0)8247,3+9 hf. y.=4½ y.
        -------
        8)2061+33p.
        -------
     257m. 5f. 33p. 4½y. 2f.
```

3. How many times will a wheel of 18 feet 9 inches round turn between Richmond, in Virginia, and Philadelphia, which is 278 miles? Ans. 78284 times, and 180 inches remainder.

4. How many barleycorns will reach round the terraqueous globe, which is 360 degrees, and each degree $69\frac{1}{2}$ English miles?

Ans. 4755801600 barleycorns.

XI. OF SOLID MEASURE.

Examples.

1. How many solid feet and inches are contained in a block of marble that is 6 feet 8 inches long, 4 feet 6 inches wide, and 2 feet 4 inches thick?

	ft.	in.	
	6	8	long.
	4	6	wide.
	24	0	
	2	8	
	3	0	
	0	4	
Square	30	0	measure.
	2	4	thick.
	60	0	
	10	0	
Ans.	70	0	solid feet, and

120960 solid inches.

2. How many cords are contained in a pile of tan-bark that is 30 feet long, 20 feet wide, and 12 feet high? Ans. 56 cords, and 32 feet over.

3. If a pile of wood be 50 feet long, 12 feet wide, and 8 feet high, how many cords are contained therein? Ans. 37 cords, and 64 feet over.

4. How many tons of round timber are contained in 1000 solid feet? Ans. 25 tons.

5. How many tons of hewn timber are contained in 2000 solid feet? Ans. 50 tons.

6. How many tons of hewn timber are contained in 2500 solid feet? Ans.

XII. OF TIME.

Examples.

1. Reduce 12 years, 10 months, 3 weeks, 2 days, 18 hours, 20 minutes, and 44 seconds, into seconds. Ans 403639200 seconds.

2. In 121812 seconds how many days, &c.?

Ans. 1 day, 9 hours, 50 min. 12 sec.

3. How many days, hours, minutes, and seconds have passed since the birth of Christ to Christmas, 1830?

6 hours=¼)1830 years.
365

9150
10980
5490

667950
457½

Answer 668407½ days, 16041780 *hours*, 962506800 *minutes, and* 57750408000 seconds.

4. How many days are there from the 24th of May, 1829, to the 16th day of August, 1830, inclusive?

From the 24th of May, 1829, to the 24th of May, 1830=1 year=365 days.

The balance of May,	8
June,	30
July,	31
August,	16
Answer	450 days.

5. From March 2d to November the 19th following, (inclusive,) how many days?

Ans. 263 days.

XIII. OF MOTION OR CIRCLE MEASURE.

An Example.

In half a year's time the sun makes his progress through 6 signs of the zodiac. How many degrees, minutes, and seconds does the progress amount to? Ans. 180 deg. 10800 min. and 648000 sec.

PROMISCUOUS QUESTIONS.

1. A merchant wishes to change 124£ in gold coins for an equal number of crowns, half-crowns, pistareens, bits, and half-bits. How many of each sort must he have? Ans. He must have 256 of each sort.

Examine the work carefully.

```
                s. d.
A crown       =5  0
A half-crown  =2  6
A pistareen   =1  3
A bit         =0  7½ | £124
A half-bit    =0  3¾ |   20
               ----- |  -----
                9 8¼ |  2480
               12    |    12
               ----  |  -----
               116   | 29760
                 4   |     4
               ----  |  -----
               465   )119040(256.
                        930
                        ----
                        2604
                        2325
                        ----
                         2790
                         2790
                         ----
                         ....
```

2. In 2184 pounds of beef, how many messes, each an equal number of 1, 2, 3, 4, 5, 6, and 7 pounds?

```
1+2+3+4+5+6+7=28
     28)2184(78 messes, ans.
        196
        ---
         224
         224
         ---
         ...
```

3. Divide 112 dollars among A, B, C, and D, so that B may have 5 dollars as often as A gets 3, and C 7 as often as B 5, and D 10 as often as C gets 7. What is the share of each man?

```
       $   $ cts.   $ cts.
     A  3×4.48=13.44 A's pt.
     B  5×4.48=22.40 B's pt.
     C  7×4.48=31.36 C's pt.
     D 10×4.48=44.80 D's pt.
     --  ----         | ------
Divisor25)112.00      | 112.00
          ------      | proof.
Multiplier $4.48      |
```

---

4. A goldsmith has 3 ingots of silver, each weighing 2 lb. 2 oz. 13 pwts. 8 grs. He intends to make them into spoons of 2 oz., cups of 5 oz., salt-cellars of 1 oz., and snuff-boxes of 2 oz., and wants an equal number of each sort. Please to inform him how *many to make.* Ans. 8 of each sort.

5. How many teaspoons will 4560 grains of silver make, allowing each spoon to weigh half an ounce? Ans. 19 spoons.

6. A butcher cut 780 pounds of beef into pieces of $1\frac{1}{2}$, 3, 4, 5, and 6 pounds, and had an equal number of each weight. How many were there? Ans. 40 pieces of each weight.

7. How many bags, each holding $4\frac{1}{2}$ bushels, can be filled out of a garner containing 190 bush. of wheat? Ans. 42 bags, and 1 b. over.

8. How many casks, which will hold 33 gallons each, may be filled out of 5 pipes and 1 hogshead of rum? Ans. 21 casks.

9. A vintner is desirous of drawing off a pipe of Canary wine into bottles containing pints, quarts, and pottles, and of each sort an equal number. How many will he have? Ans. 144.

10. A tract of land containing 1299600 square perches is to be divided into 25 equal parts. How many acres, &c. will there be in each part? Ans. 324A. 3r. 24 perches.

THE SINGLE RULE OF THREE DIRECT.

The Single Rule of Three Direct is composed of three terms, which are always given in the question, to find the fourth term or answer; two of them are called the terms of supposition, and the other one, the term of demand.

A Rule for stating the Questions and the Operation.

1. Set down the term of supposition in the first place; that always moves the question, and is the principal cause of gain or loss.

2. Set down the term of supposition in the second place, that is of the same denomination as the fourth term or answer required.

3. Set down the term of demand, that is, whatever you wish to find the value of, in the third place, and the stating will be completed.

4. If the first and third terms of the stating are of different names, they must be reduced into the same denomination.

5. Reduce the middle term to the lowest denomination mentioned in it, or lower, if the nature of the question should require it.

6. When all the terms of the stating are reduced as above directed, (if necessary)—then multiply the second and third terms together, and divide the product by the first term—the quotient will be the fourth term or answer to the question, and of like name with the middle term or whatever denomination it was reduced to.

7. If there should be any remainder, reduce it to the next denomination below the name of the quotient now obtained, and divide the product by the same divisor as before, and the quotient will be so many of this last denomination—proceed in the same manner with all the remainders, till you have reduced them to the lowest denomination that the middle term admits of, and the several quotients, properly connected, will be the answer.

8. The questions must be proved by reversing the order of the stating, and working it back again, which will produce the middle term of the first stating for the answer, if the work in both statings be correct.

A Poetical Rule.

This Golden Rule has places three,
The first and third must so agree,
That of one name they may remain,
If to the truth you would attain;
The second by third, then multiply,
And divide by the first ingeniously,
Then will your quotient be the same,
That you in second place did frame;
Which by Reduction may be brought
Into whatever name is sought.
Change the stating and backward work,
And that will prove the given quirk.

Examples.

1. If 8 yards of linen cost 5 dollars, what will 2 yards cost at the same rate?

```
  yds. $  yds.              yds. $  c.   yds.
As 8 : 5 :: 2            As 2 : 1.25 :: 8
       2                           8
      ----                       -----
    8)10.00                    2)10.00
      -----                      -----
Ans. $1.25               Ans. $5.00
```

which is equal to the middle term in the first stating; therefore I say the answer is right.

---

2. If 2 pounds of loaf sugar cost 1s. 6½d. what will 24 pounds cost at the same rate?

```
   lbs.  s. d.   lbs.              lbs.   s. d.  lbs.
As 2 : 1 6½ :: 24              As 24 : 18 6 :: 2
       12      37                      12
       --      ---                     ---
       18      168                     222
        2       72                       2
       --      ---                     ---
       37    2)888                   6)444
               ---                     ---
             2)444                    4)74
               ---                     --
           12)222d.                 12)18½d.
               ---                     ---
Answer,     18s. 6d.            Ans.  1s. 6½d.
```

which is equal to the middle term in the first stating; consequently the work is right.

3. If 13 yards of broad cloth cost 70 dollars, what will 27 yards come to, at the same rate?

yds. $ yds
As 13 : 70 :: 27
27
490
140
$ c. m.
13)1890(145.38,4 Ans.
13
59
52
70
65
13)5.00(38 cents.
3 9
1 10
1 04
13)60(4 mills.
52
8 rem.

yds. $ c. yds.
As 27 : 145.38,4 :: 13
13
436152
145384
1889992
8 rem.ad.
3)1890.000
9)630.000
Answer $70 proof.

4. If $2\frac{1}{2}$ pounds of butter cost $15\frac{1}{2}$d. what will 2 cwt. 3 qrs. 12 lbs. cost?

lbs. d. C. qr. lbs.
As $2\frac{1}{2}$: $15\frac{1}{2}$ :: 2 3 12
2 4 4
5 62 11
28
100
22
320
2
640
62
1280
3840
5)39680
4)7936 qrs.
12)1984 pence.
2,0)16,5 4d.
Ans. *£8 5 4d.*

C. qr. lb. £ s. d. lbs.
As 2 3 12 : 8 5 4 :: $2\frac{1}{2}$
4 20
11 165
28 12
100 1984
22 5
320 8)992,0
2 8)124
64,0
Ans. $15\frac{1}{2}$d. proof.

5. If 26 yds. of superfine cloth cost £42, what will 54 yds. come to

```
 yds.   £    yds.
As 26 : 42 :: 54
        54
       ----
       168
       210
       ----  £  s. d.
26)2268(87  4 7¼
   208
   ----
    188
    182
    ---
      6
     20
    ---
26)120(4s.
   104
   ---
    16
    12
    ---
26)192(7d.
   182
   ---
    10
     4
    ---
26)40(¼qr.
   26
   --
   14 rem.
```

```
 yds.   £  s. d.    yds.
As 54 : 87 4 7¼ :: 26
        20
       ----
       1744
         12
       -----
       20935
           4
       -----
       83741
          26
      ------
      502446
     167482
     -------
     2177266
          14 rem. ad.
     -------
   6)2177280
     -------
    9)392880
     -------
     4)40320 qrs.
     -------
    12)10080
     -------
     2,0)84,0
     -------
Answer  £42 proof.
```

6. If $2\frac{1}{4}$ lbs. of tea cost \$3.33⅓ cents, what will 9 lbs. come to?

```
   lbs.   $ cts.   lbs.
As 2¼  :  3.33⅓ :: 9
   4      3        4
   --     ----     --
   9      1000     36
            36
          ----
          6000
         3000
         -----
       9)36000
         -----
       3)4000 thirds
Answer $13.33⅓ cts.
```

```
   lbs.   $ cts.   lbs.
As 9  :  13.33⅓ :: 2¼
   4       3        4
   --     ----     --
   36     4000      9
             9
          -----
        6)36000
          -----
         6)6000
          -----
         3)1000 thirds.
Ans. $3.33⅓ proof.
```

7. If 1lb. of tobacco cost 6¼cts. what will 96lbs. cost? Ans. $6
8. If 1lb. of cheese cost 12½cts. what will 1cwt. cost? Ans. $14
9. If 1cwt. of sugar cost $11.20cts. what will 1lb. cost? Ans. 10cts.
10. If one pair of stockings cost 37½cts., what will 19 dozen pair cost? Ans. $85.50 cts.
11. If a firkin of butter containing 56lbs. cost $4.66⅔cts., what will 1lb. of it come to? Ans. 8⅓cts.
12. If 1lb. of tobacco cost 12½ cents, what will 17cwt. 2qrs. come to? Ans. $245.
13. If 19 dozen pair of shoes cost 85 dollars 50 cents, what will 1 pair cost? Ans. 37½cts.
14. If 15 lbs. of tobacco cost $1.50 cents, what will 478lbs. cost? Ans. $143.40cts.
15. Sold 3cwt. of tobacco at 25cts. per lb. What did it amount to? Ans. $84.
16. Bought 19 chaldrons of coals, at $4.91⅔ cents per chaldron. What did they amount to? Ans. $93.41⅔cts.
17. Sold 3 hogsheads of tobacco, weighing together 15cwt. 1qr. 19lbs. at the rate of 21cts. per lb. What sum did they amount to? Ans. $362.67cts.
18. What will 5 pieces of cloth, each containing 19 yards, cost at the rate of $2.75cts. per yard? Ans. $261.25cts.
19. If an ell of holland cost 75 cents, what is the value of 5 pieces, each containing 12 ells? Ans. $45.
20. How many pounds of bacon can I buy for 24 dollars, at the rate of 12½cts. a pound? Ans. 192lbs.
21. If a man spend $2.25cts. in 4 weeks, how long will $178.31¼cts. last him at that rate? Ans. 6 years 5 weeks.
22. If I expend 6¼cts. a day, what will my expenses amount to in a year? Ans. $22.81¼cts.
23. If 1⅓ yards of silk cost $2.50cts. what will 1qr. and 2 nails of the same cost? Ans. 62½cts.
24. If the price of one acre of land be $18.25 cts. what will 50 acres, 2 roods, and 20 perches come to, at the same rate? Ans. $923.90cts. 6¼ mills.
25. If a bushel of potatoes cost 75cts., what will 56 bushels 3 pecks come to? Ans. $42.56¼cts.
26. If I give $42.56¼cts. for 56 bushels 3 pecks of wheat, what will a peck of it cost? Ans. 18¾cts.
27. How much sugar will $28 dollars pay for, at 8⅓cts. per lb.? Ans. 336lbs.
28. How many yards of cotton cloth can be bought for 12 dollars, at 18¾cts. a yard? Ans. 64yds.
29. A wine merchant bought 3 pipes of wine, containing 120½, 124, and 126¾ gallons, respectively, at 91⅔cts. per gallon. What did they amount to? Ans. 340.31¼cts.

30. A linen draper bought 4 pieces of linen, two of which contained 27½ yards each, and the other two 25¾ yards each, at 62½cts. per yard. What was the whole cost? Ans. $66.56¼cts.

31. How much wheat can be bought for $568.35cts at the rate of $1.35cts. a bushel? Ans. 421 bushels.

32. A merchant bought 12 pieces of cassimere, each containing 12 yds., at $1.75cts. per yd. What did they cost him? Ans. $252.

Examples in Sterling Money.

33. If an ounce of silver cost 5s. 6d., what will be the price of a tankard that weighs 1lb. 10oz. 10pwts. and 4 grs.?

Ans. £6 3s. 9½d.+96.

34. If 7cwt. 3qrs. 14lbs. of loaf sugar cost £25 14s. 6d., what are 8lbs. of the same sugar worth? Ans. 4s. 8d.

35. A merchant bought 14 pieces of drab cloth, each containing 28 yards, at 13s. 6½ per yard. What sum did the whole cost him?

Ans. £265 8s. 4d.

36. A draper bought 35 pieces of cassimere, each containing 12 yards, at 14s. 10d. ¾qrs. per English ell. What did the whole cost him? Ans. £250 5s.

37. A merchant bought 50 pieces of kersey, each containing 34 Flemish ells, at the rate of 8s. 4d. per English ell. What was the total amount? Ans. £425.

38. If one grain of gold be worth 1½d. what are 14lb. 3oz. 8pwts. worth? Ans. £514 4s.

39. A certain man was owing of a merchant £950 10s. but compounded with him for 7s. 9d. in the pound. What sum did the merchant receive, and how much did he lose?

Ans. He received £368 6s. 4½d. and lost £582 3s. 7½d.

40. A gentleman's daily expenses are 18s. 10d. ¾qrs. and he lays up £191 3s. 0d. ¼qr. at the year's end. What is the amount of his annual income? Ans. £536.

THE SINGLE RULE OF THREE INVERSE.

The Single Rule of Three Inverse is composed of three terms, which are given in the question, to find the fourth term or answer; two of them are called the terms of supposition, and the other one, the term of demand.

Rule.

1. State the question and reduce all the terms of the stating (if necessary) precisely in the same manner as in the Single Rule of Three Direct.

2. *Multiply the first and second terms together, and divide the product by the third term*—the quotient thence arising will be the an-

swer, and of like name with the middle term or whatever denomination it was reduced to.

3. The questions must be proved by changing the order of the stating and working it backward as in the Single Rule of Three Direct.

EXAMPLES.

1. If 48 men can build a wall in 24 days, how many men can do it in 192 days?

```
   da.  men.  da.
As 24 : 48 :: 192
        24
       ---
       192
       96
       ---
  192)1152(Ans. 6 men can
      1152 build the wall in
      ----   192 days.
      ....
```

```
   da.   men.  da.
As 192 : 6 :: 24
         6
       ----
      4)1152
       ----
      6)288
       ---
Ans. 48 men, = to
the middle term of
the first stating.
```

2. Suppose 800 soldiers were placed in a garrison, and their provisions were computed sufficient for 2 months. How many soldiers must depart from the garrison, that the provisions may serve the rest 5 months?

```
Mo. Sol.  Mo.
 2 : 800 :: 5
       2
      ---
  5)1600
   ----
    320 subtracted.
from 800
     ---
Ans. 480 soldiers must
```

leave the garrison, because 320 will consume the provisions in 5 months.

```
Mo. Sol. Mo.
5 : 320 :: 2
            5
         ----
       2)1600
         ----
         800 proof.
```

3. How many yards of matting that is half a yard wide, will cover a floor that is 18 feet wide and 30 feet long?

```
   F.W. F.L. In.W.
As 18 : 30 :: 18
        12
       ---
       216
        30
      ----
   18)6480
      ----
    3)360 feet.
      ---
Ans. 120 yards.
```

```
   In.W.  yds.  F.W.
As 18 : 120 :: 18
         18     12
        ---    ---
         96    216
        12
       ----
       2160 yards.
          3 feet.
       ----
  216)6480
      ----
Answer 30 ft. proof.
```

4. How much in length, that is 9 inches wide, will make a square foot? Ans. 16 inches.

5. If I lend my neighbor 100 dollars for 6 months, (allowing 30 days to the month,) how long must he lend me one thousand dollars to requite my kindness? Ans. 18 days.

6. If a man perform a journey in 3 days, when they are 16 hours long, how many days of 12 hours long will he require to perform the same journey? Ans. 4 days.

7. If 100 dollars principal, in 12 months, gain 6 dollars interest, what principal will gain the same interest in 4 months?
Ans. 300 dollars.

8. How many yards of baize, $\frac{3}{4}$ of a yard wide, will line 4 yards of camblet that is 1 yard wide? Ans. $5\frac{1}{3}$ yards.

9. How many yards of tapestry, that is 2 nails wide, will line 4 yards of velvet that is $\frac{3}{4}$ of a yard wide? Ans. 24 yards.

10. If 28s. will pay for the carriage of one hundred weight 150 miles, how far may 6 cwt. be carried for the same money?
Ans. 25 miles.

11. How much in length, that is 3 inches wide, will make a square foot? Ans. 48 inches.

12. A and B joined in company; A had 50£ of stock in the firm for 10 months, and B had his stock in for 8 months. They shared equally of the gain. What was B's stock worth? Ans. 62£ 10s.

13. If the penny loaf weighs 9 oz. when wheat is sold at 6s. 3d. per bushel, how much should it weigh when wheat is worth but 4s. 6d. a bushel? Ans. 12 oz. 10 pwts.

14. If 15s. worth of wine will serve 46 men, when the tun is worth 12£, how many men will the same 15s. worth suffice when the tun is worth but 8£? Ans. 69 men.

15. If 8 boarders drink a barrel of cider in 12 days, how long would it last if 4 more were to join the company? Ans. 8 days.

16. A ship's company of 15 men have bread enough for their voyage, allowing each man 8 ozs. per day, when they pick up a crew of 5 persons in distress, to whom they are willing to give part. What will be the daily allowance of each person? Ans. 6 oz.

OF DIRECT AND INVERSE PROPORTION.

The Rule of Three contains two sorts of proportion, namely, Direct and Inverse, which are only parts of the same general rule; and, in a scientific arrangement, it would be best to consider them so: but a distinction is necessary, because it makes the Rule of Three more intelligible to the young student.

I. Of Direct Proportion.

1. *When more* requires more, or less requires less, the propor-

tion is direct, and the question of course belongs to the Single Rule of Three Direct.

Explanation.

2. More requires more when the third term of the stating is greater than the first, and requires the answer to be more than the second in the same proportion.

3. Less requires less when the third term of the stating is less than the first, and requires the answer to be less than the second in the same proportion.

II. OF INVERSE PROPORTION.

1. When more requires less, or less requires more, the proportion is inverse, and the question of course belongs to the Single Rule of Three Inverse.

Explanation.

2. More requires less when the third term of the stating is greater than the first, and requires the answer to be less than the second in the same proportion.

3. Less requires more when the third term of the stating is less than the first, and requires the answer to be more than the second in the same proportion.

EXAMPLES.

1. If an ingot of silver weighs 36ozs. 10pwts., what is it worth at 83$\frac{1}{3}$ cents per ounce? Ans. \$30 41$\frac{2}{3}$ cents.

2. What quantity of shalloon, that is 3 quarters of a yard wide, will line 7$\frac{1}{2}$ yards of cloth that is 1$\frac{1}{2}$ yards wide? Ans. 15 yds.

3. If a student pay 8 dollars for 5 months' tuition, how much will he have to pay for three months? Ans. \$4.80 cents.

4. A silversmith sold a tankard for \$36 at 87$\frac{1}{2}$ cents per ounce. What did it weigh? Ans. 41ozs. 2pwts. 20grs.+100 rem.

5. How many yards of matting, that is 2 feet 6 inches wide, will cover a floor that is 27 feet long and 20 feet wide? Ans. 72 yds.

6. If a herring and a half cost three half-pence, what will a dozen come to? Ans. 1 shilling.

7. A cistern that holds 252 gallons receives water from a conduit at the rate of 60 gallons per hour, but in the same time discharges 45 gallons through a hole in the bottom. In what space of time will the cistern be full? Ans. 16 hours 48 minutes.

8. If a staff that is 4 feet long give 6 feet of shade on level ground, what is the height of a steeple whose shade is 99 feet at the same time? Ans. 66 feet.

9. How many yards of paper, that is 2$\frac{1}{2}$ feet wide, will cover a wall which is 12 feet long and 9 feet high? Ans. 14yds. 1ft. 2$\frac{1}{4}$in.+6

10. What will 2cwt. 1qr. 19lb. of wool cost, at 8s. 6d. per stone? Ans. 8£ 4s. 6$\frac{1}{4}$d.+10 rem.

11. *If a pint of wine* cost 10d. what will 3 hhds. come to? Ans. 63£

12. If a pipe of Canary wine cost 40£ what will one pint of it cost? Ans. $9\frac{1}{2}+96$ remainder.

13. How many yards of cotton cloth, that is 3 quarters of a yard wide, will line a coverlet that is 9ft. long and 7ft. wide? Ans. $9\frac{1}{3}$yds.

14. If 1 pennyweight of gold be worth 89 cents, what are 8ozs. 10pwts. 16grs. worth? Ans. \$151.$89\frac{1}{3}$ cents.

15. What is the weight of a pea to a steelyard, which, being suspended 39 inches from the center of motion, will equipoise 208lbs. suspended at the draught end 3 quarters of an inch from the said center of motion? Ans. 4 pounds.

16. What weight, suspended 3 quarters of an inch from the fulcrum, will be equipoised by the pea of a steelyard that weighs 4lbs. suspended 39 inches from the said fulcrum, at the other end? Ans. 208 pounds.

17. A lady very fair did herself betake,
Of nice needle work, a carpet rare to make,
With silk and silver wrought most curiously,
With elegant pictures, pleasant to the eye;
Fifteen yards in length the carpet did extend,
Seven quarters wide it was from end to end;
But she's desirous, before the carpet's seen,
To line it with silk five quarters wide, and green.
Now this lady fair desires of you to know
How many full yards will line the carpet so.
Now, sir, only grant the lady-fair's desire,
And she will your talents very much admire.
Answer.—Dear Miss, the question is quite easy done,
It requires yards exactly twenty-one.

18. As I was hunting on the forest grounds,
Up starts a hare before my two gray-hounds;
The dogs being swift on foot did fairly run
Unto her fifteen rods just twenty-one.
The distance that she started up before
Was four-score, sixteen rods just, and no more.
Now this I'd have you unto me declare,
How far they ran before they caught the hare?

$21-15=6$; then, as $6 : 21 :: 96 .. 336$ rods, answer.

This question may be proved by Simple Addition, in the following manner:

$$96+15+15+15+15+15+15+15+15+15+15+15+15+15+15+15+15=336$$
$$00+21+21+21+21+21+21+21+21+21+21+21+21+21+21+21+21=336$$

Or thus: $15\times16+96=336$, and $21\times16=$ the same.

N. B. Sound, that is not interrupted, moves about 1150 feet in one second of time.—See Vyse's Arithmetic, page 68. Mr. *Nicholas Pike says it moves about 1142 feet in a second.—See Pike's Arithmetic, page 370.*

19. How long after the firing of a cannon at Newburyport will the sound be heard at Ipswich, estimating the distance to be 10 miles?

10 miles=52800ft., then: As 1142 feet. : 1 sec. :: 52800 feet. .. 46sec.+268 rem. Answer.

20. If I see the flash of a cannon 1 minute and 3 seconds before I hear the report, how far is she off, supposing the sound to move 1150 feet in a second. Ans. 24150 yards.

21. One evening fair I did espy
A Mediterranean ship pass by,
Whose gallant gunners quickly sped,
And fired a volley from her head.
Now from the time the flash appear'd
Until the sound I plainly heard,
Was minutes two and seconds four,
What was her distance from the shore?

As 1s. : 1142ft. :: 124sec. .. 26min. 1442yds. 2ft. Answer.

The following question is inserted by the request of a friend:

22. A Maypole there was whose height I would know,
The sun shining quite straight, to work I did go;
The length of its shadow, upon level ground,
Was sixty-five feet, when measur'd, I found.
A staff I had by me just five feet in length,
Which gave out a shadow four feet and one tenth.
The height of the Maypole I would gladly know,
And that is the thing you are required to show.

Answered poetically by Mr. James Brown.

Sir, your question to solve I did puzzle my brain,
I read it and tried it, and read it again;
I thought it was sure some profound mystery,
Till at length it turn'd out the plain Rule of Three.
Suppose that a shadow of four feet and one-tenth
Requires a staff which is just five feet in length,
The Maypole, whose shadow was sixty-five feet,
Will be twenty-six yards perpendicular height.
The remainder is full fifteen inches and more,
Which I probably might have mentioned before;
But I thought if Goliah would want such a rod,
When we cut it we'd leave a good stump on the sod.
Now, sir, if my answer does please you aright,
I am your humble servant, and bid you good night.

THE DOUBLE RULE OF THREE DIRECT.

The Double Rule of Three Direct is composed of five terms, which are given in the question to find the answer; three of them are called the terms of supposition, and the other two the terms of demand.

A RULE FOR STATING THE QUESTIONS, &c.

1. Set down the term of supposition in the first place, which is the principal cause of gain or loss, interest or demand, increase or decrease, action or passion.

2. Set down the term of supposition in the second place, that signifies the time in which the gain is acquired, the loss sustained, the action performed, or the distance from one place to another.

3. Set down the term of supposition in the third place; that designates the gain or loss, interest or action performed, which must be of like name with the answer required.

4. Place the two terms of demand in a line with the terms of supposition, so that the first and fourth terms of the stating may be of like name; also, the second and fifth, and the stating will be completed.

5. If the first and fourth terms of the stating be of different denominations, reduce them both to the same, and do likewise by the second and fifth terms.

6. Reduce the middle term to the lowest denomination mentioned in it, or lower, if the nature of the question require it.

7. When all the terms of the stating are reduced as above directed, (if necessary,) then multiply the first and second terms together for a divisor, and the three remaining terms for a dividend; the quotient thence arising will be the answer required to the question, and of like name with the middle term, or whatever denomination it was reduced to.

8. Questions may be wrought by two single statings in direct proportion, which may serve as proof to the compound stating; or, the operation may be proved by changing the order of the compound stating and working backward, which will produce the middle term of the first stating for the answer, if the work in both be right.

9. When a question is wrought by two single statings, the answer to the first will be the middle term of the second one.

10. The operation, in many questions, may be contracted by working with the aliquot parts instead of the whole terms. The aliquot parts are found by dividing a multiplying term by a dividing term, or a dividing term by a multiplying term.

EXAMPLES.

1. If 24 bushels of provender serve 16 horses 12 days, how many bushels will serve 48 horses 32 days?

```
  H.  D.   B.    H.  D.
If 16×12 : 24 :: 48×32
   12      48
   ---    ---
   192    192
           96
         ----
         1152
           32
         ----
         2304
        3456
       -----
  192)36864(192 bush.
      192      Answer.
      ----
      1766
      1728
      ----
       384
       384
       ---
       ...
```

```
  H.  D.    B.     H.  D.
If 48×32 : 192 :: 16×12
      32     16
     ---   ----
      96   3072
    144      12
    ----  -----
    1536 )36864(24b. proof.
          3072
          ----
          6144
          6144
          ----
          ....
```

The operation contracted.

```
  H.  B.   B.    H.  D.
If 16×12 : 24 :: 48×32
            8  =  4× 2
Ans.      192 bushels.

  H.  D.    B.     H.  D.
If 48×32 : 192 :: 16×12
       2     4        6
             6
Ans.        24 bush. proof.
```

The same question wrought by two statings.

```
  H.   B.   H.
If 16 : 24 :: 48
        48
       ---
       192
       96
    -------
 16)1152(72b.
    112
    ---
     32
     32
     --
     ..
```

```
  D.   B.   D.
If 12 : 72 :: 32
        32
       ---
       144
      216
     -----
  12)2304
     ----
Ans. 192 bushels as before.
```

2. If 7qrs. of malt are sufficient for a family of 7 persons 4 months, how many quarters are enough for 42 persons 12 months?

```
   1  2    3     4   5
   P. mo. qrs.   P.  mo.
If 7×4 : 7 :: 42×12
               3
Answer       126 quarters.
```

3. If $56.33⅓cts. be the wages of 16 men for 8 days, what sum will 32 men earn in 24 days?

```
   M.  D.    $  c.      M.  D.
If 16×8 : 56.33⅓ :: 32×24
                    6 = 2× 3
Answer 338.00
```

4. If a carrier receive $8 for the carriage of 3cwt. 150 miles, how much ought he to have for the carriage of 7cwt. 3qrs. 14lbs. 50 miles?

```
   1     2    3     4          5
  cwt.  mi.   $   cwt. qrs. lbs.
If 3×150  :  8 :: 7   3   14×50
   4             4
   —             —
   12            31
   28            28
   —             —
   336           262
    150          62
   —————         —
   50400         882
                   8
                 7056
                   50
     504,00)3528,00(7$ Ans.
            3528
            ....
```

The operation contracted.

```
cwt.  mi.   $   cwt. qrs. lbs.  mi.
If 3×150 :  8 :: 7   3    14×50
                           8
3×3=....9)63   0    0
Ans.   $7 as before.
```

5. If 100£ principal gain 6£ interest in 12 months, how much will 245£ principal gain in 8 months?

```
   £ p.  mo.  £ int.  £ p.  mo.
If 100×12  :  6  ::  245×8
     12                 8
   ————              ————
   1200              1960
                        6
                    ———— £  s.
             1200)11760(9  16 ans.
                  10800
                  —————
                    960
                     20
                  —————
             1200)19200(16
                  1200
                  ————
                  7200
                  7200
                  ————
                  ....
```

6. If 100 dollars principal gain 6 dollars interest in one year, what will $333.33⅓ cents principal gain in 20 weeks?

```
   $ p.     w.  $ int. $ p.       w.
If 100.00×52 : 6 :: 333.33⅓×20
     3                  3
  ——————            ———————
  300.00            1000.00
      52                 20
  ——————            ————————
   60000            20000.00
  150000                   6
  ——————          ——————————  $ c.
  15600.00        )120000.00(7.69 Ans.
                   109200 00
                  ——————————
                   1080000,00
                   936000 0
                  ——————————
                   144000 00
                   140400 00
                  ——————————
       Remainder    3600 00
```

```
   $ p.      w.    $ c.     $       w.
If 333.33⅓×20 : 7.69 :: 100.00×52
      3                      3
  ———————                ——————
  1000.00                300.00
       20                    52
  ————————               ————————
  20000.00               15600.00
                          769
                         ————————
                         14040000
                         936.....
                        1092......
                           360000
                         ——————————  $
        20000.00)120000.0000(6pf.
                 120000 00
                 —————————
```

7. A certain man lent out £86 to receive interest for the same, and when it had continued 8 months he received for principal and interest together, the sum of £88 17s. 4d. I demand at what per cent. per annum he received interest?

$$
\begin{array}{lccccc}
 & & \pounds & s. & d. & \\
 & & 88 & 17 & 4 & \\
 & & 86 & 00 & 0 & \\
\pounds\ mo. & & \hline & & & \pounds\quad mo. \\
\text{If } 86\times 8 : & & 2 & 17 & 4 :: & 100\times 12 \\
\quad 8 & & 20 & & & 12 \\
\hline 688 & & 57 & & & 12)1200 \text{ pence.} \\
 & & 12 & & & \hline \\
 & & \hline & & & 2,0)10,0 \\
 & & 688 & & & \hline
\end{array}
$$

Answer £5 per cent. per annum.

8. I demand the interest of 250 dollars for 73 days, at 6 dollars per cent. per annum?

$$
\begin{array}{lccccc}
 & \$\quad D. & & \$ & & \$\quad D. \\
\text{If} & 100\times 365 & : & 6 & :: & 250\times 73 \\
 & 5 & & & & 6 \\
 & \hline 500 & & & & 5,00)15,00 \\
 & & & & & \hline
\end{array}
$$

Answer $3

9. If 200lbs. be carried 40 miles for 50 cents, what sum will pay for the carriage of 20200lbs. 60 miles? Ans. $75.75 cents.

10. If 180 bushels of oats be sufficient for 18 horses 20 days, how many bushels will serve 60 horses 36 days? Ans. 1080 bush.

11. If a regiment of 936 soldiers eat 312 quarters of wheat in 84 days, how many quarters will suffice an army of 11088 soldiers 56 days? Ans. 2464 quarters.

12. I demand the interest of 2240 dollars for 7 years, at $6 per cent. per annum. Ans. $940.80 cents.

13. If a wagoner charge 7 dollars for the wagonage of 700 lbs. 120 miles, how much must I pay him for the wagonage of 4200lbs. 240 miles? Ans. $84.

14. What is the interest of 669 dollars for 3 years and 9 months, at $5 per cent. per annum? Ans. 125.43\frac{3}{4}$ cents.

15. If a family of 12 persons use 6 barrels of flour in 3 months, how many barrels will they use in a year if 6 more be taken into the family? Ans. 36 barrels.

16. If a man borrowed $1070 on interest, and at the expiration of 18 months have to pay $1262.60cts., at what rate per cent. per annum, was the interest computed? Ans. $12 per cent.

17. If 8 reapers received £3 4s. for 4 days' work, what sum will pay 48 reapers for 16 days' work? Ans. £76 16s.

18. If a family of 9 people expend $450 in 5 months, how much will be sufficient to maintain them 8 months, if 5 people more come into the family? Ans. $1120.

19. What is the interest of 654 dollars for 164 days, at $6 per cent. per annum? Ans. $17.63cts.+41 rem.

20. A certain usurer put out $250 to receive interest for the same, and when it had continued 9 months, he received for principal and interest the sum of $261.25cts At what rate per cent. per annum, was the interest calculated? Ans. 6 per cent.

21. What is the interest of $947 for 294 days, at $6 percent. per annum? Ans. $15.76cts. 7m+125 rem.

22. I demand the interest on $1333.33⅓cts. for 1 week, at $6 per cent. per annum? Ans. $1.53cts. 8m.+72.

23. If 7 men use 98lbs. of flour in 14 days, how many barrels will be sufficient for 28 men 56 days? Ans. 8 barrels.

24. A man and his wife, having wrought 3 days, received $2.25 cts. in payment; how much must they have for 10 days, when their two sons help them? Ans. $15.

THE DOUBLE RULE OF THREE INVERSE.

The Double Rule of Three Inverse is a compound of Direct and Inverse Proportion, and is composed of five terms, which are given in the question to find the answer—three of them are called the terms of supposition, and the other two the terms of demand.

A Rule for Stating the Questions, &c.

1. Set the term of supposition in the first place, that is of the like name with the answer required to the question.

2. Set the term of supposition in the second place, that signifies the time in which the gain is acquired, the loss sustained, or the work, &c. performed.

3. If the answer to the question be required in days, &c.—then, set the days in the first place, and their length (if given) in the second. See the 5th question.

4. The agents that perform the work or the commodity to be removed, &c. may occupy the second place in the statings.*

5. Set the term of supposition in the third place, that designates the gain or loss, the work performed, the distance travelled, the wages earned, or the price paid for the removal of any commodity, &c.

6. Set the two terms of demand in a line with the three terms of supposition, so that the second and fourth terms of the stating may be of one name; also, the third and fifth, and the stating will be completed.

* *Examine the third* and *sixth questions* carefully.

7. Multiply the third and fourth terms together for a divisor and the three remaining ones for a dividend—the quotient thence arising will be the answer required, and of like name with the first term in the stating, or whatever it was reduced to.

8. The questions may be proved by reversing the order of the stating and working backward, which will produce the first term of the first stating, if the work in both be right.

9. Questions may be answered by two single statings, one of which will be direct proportion and the other inverse, and the answer to the first stating will always be the middle term of the second one.

10. Whenever a dividing term can be divided by a multiplying term, or a multiplying term by a dividing term, the operation may be abbreviated by using the quotients thence arising, instead of the whole terms.

Examples.

1. If 7 men can reap 84 acres of wheat in 12 days, how many men can reap 100 acres in 5 days?

1 2 3 4 5
M. D. A. D. A.
If $7 \times 12 : 84 \times 5 :: 100$
12 5 84
84 420)8400(20 ans.
840
...0

M. D. A. A.
If $20 \times 5 : 100 \times 12 :: 84$
5 12 100 m.
100 1200)8400(7
8400
....

See the work abbreviated.
M. D. A. D. A.
As $7 \times 12 : 84 \times 5 :: 100$
7 Ans. 20 men.

2. If 4 reapers earn \$4 in 3 days, how many men will earn \$32 in 16 days?

R. D. \$ D. \$
As $4 \times 3 : 4 \times 16 :: 32$
2
Ans 6 men. Prove the work.

3. If \$5 be the wages of 8 men for 3 days, how many days must 20 men work for \$50.

D. M. \$ M. \$
As $3 \times 8 : 5 \times 20 :: 50$
4 2 10
Ans. 12 days. Prove the work.

4. If 100£ principal gain 6£ interest in 12 months, what principal will gain 20£ interest in 8 months?

£ p. mo. £ int. mo. £ int.
As $100 \times 12 : 6 \times 8 :: 20$
12 6 1200
1200 48)24000
Ans. 500£

5. If a footman travel 240 miles in 12 days when they are 12 hours long, in how many days can he travel 720 miles when they are 16 hours long?

D. H. mi. H. mi.
As $12 \times 12 : 240 \times 16 :: 720$
12 3
16)144
$9 \times 3 = 27$ days, Ans.

6. If 200lbs. be carried 40 miles for 50cts. how far may 20200lbs. be carried for $75.75cts?

mi. lbs. c. lbs. $ c.
∴ 40 × 200 : 50 × 20200 :: 75.75
101
5 4
505)30300
Ans. 60*mi.*

7. If 30 men perform a piece of work in 20 days, how many men will accomplish another piece of work, four times as large, in one fifth part of the time?

M. D. PW. D. PW.
∴ 30 × 20 : 1 × 4 :: 4
30
Ans. 600 men.

8. A person having engaged to remove 8000cwt. of merchandize a certain distance in 9 days, and with 18 horses in 6 days he removed 4500cwt. how many horses will be required to remove the rest in the remaining 3 days?

cwt.
8000
4500
H. D. cwt. D.
∴ 18 × 6 : 4500 × 3 :: 3500
2 250 14
2
Ans. 28 horses.

9. If 8 men can build a wall 20 feet long, 6ft. high and 4ft. thick, in 12 days, in what time can 24 men build another one 200 feet long, 8ft. high, and 6ft. thick?

ft. ft.
20 long 200 long
6 high 8 high
120 1600
4 thick 6 thick
D. M. M.
∴ 12 × 8 : 480 × 24 :: 9600
4 3 20
4
Ans. 80 days

10. If twenty dogs for thirty groats,
Go forty weeks to grass;
How many hounds for 60 crowns
May winter in that place?

D. W. cro. W. cro.
∴ 20 × 40 : 2 × 12 :: 60
20
20
400
5
Ans. 2000 hounds.

OF DIRECT AND INVERSE PROPORTION.

A RULE TO FIND WHETHER THE PROPORTION IS DIRECT OR INVERSE.

1. Let the principal cause of gain or loss, interest or demand, increase or decrease, &c. occupy the first place in the stating.

2. Let the term which signifies the time or the distance of one place from another, occupy the second place in the stating.

3. Let the term which denotes the gain or loss, the action performed, or the price paid for removing any commodity, occupy the third place in the stating.

4. Place the two terms of demand under those of the same name in the supposition, and a blank place will occur under that term, which is of like name with the answer.

5. If the blank should fall under the third term, the proportion is direct—therefore multiply the first and second terms together for a divisor, and the other three for a dividend.

6. If the blank should fall under the first or second term, the proportion is inverse—therefore multiply the third and fourth terms together for a divisor, and the other three for a dividend; and the quotient, in either case, will be the answer required.

NOTE 1. In the 1st example, the blank place will occur under 100 acres, which is the the third term; therefore the proportion is *direct;* consequently, move the 7 men and 12 days into the 4th and 5th places of the stating, and proceed by *direct proportion* to find the answer.

NOTE 2. In the 2d example, the blank place will occur under 36 days, which is the 2d term; therefore, the proportion is *inverse;* consequently, move the 18 horses and 180 bushels into the 4th and 5th places of the stating, and proceed by *inverse proportion* to find the answer.

PROMISCUOUS EXAMPLES.

1. If 20 men mow 100 acres of grass in 5 days, how many acres can 7 men mow in 12 days? Ans. 84 acres.

2. If 60 horses can eat 1080 bushels of oats in 36 days, in how many days will 18 horses eat 180 bushels? Ans. 20 days.

3. If 500 dollars principal gain 20 dollars interest in 8 months, at what rate per cent. per annum was the interest computed?
Ans. at $6 per cent. per annum.

4. If 60 horses can eat 1080 bushels of oats in 36 days, how many horses will eat 180 bushels in 20 days? Ans. 18 horses.

5. If 32 men earn 338 dollars in 24 days, in what time will 16 men earn $56.33⅓ cents? Ans. 8 days.

6. If I pay 84 dollars for the carriage of 4200lbs. 240 miles, how far may 700lbs. be carried for 7 dollars? Ans. 120 miles.

7. What sum will gain $33.75 cents in 9 months, at 6 dollars per cent. per annum? Ans. 750 dollars.

8. A wall which is to be built 27 feet high has been raised 9 feet in 6 days by 12 men; how many men must be employed to finish the work in 4 days? Ans. 36 men.

9. If 5 men can make 150 pair of shoes in 20 days, how many pair can 15 men make in 60 days? Ans. 1350 pair.

THE COMPARISON OF WEIGHTS AND MEASURES.

The operations are performed by the Single Rule of Three Direct.

EXAMPLES.

1. If $1\frac{1}{2}$ yards in Virginia be equal to one ell in France, how many yards in Virginia are equal to 120 ells in France?

ell.		yds.		ells.
1	:	$1\frac{1}{2}$	::	120
		2		
		3		
		120		
		2)360		

Ans. 180 yards.

2. If 6s. 8d. in Virginia be worth one French crown, what sum in Virginia is equal to 320 Fr. crowns? Ans. 106£ 13s. 4d.

3. If 24 yards at Richmond, in Virginia, make 32 ells in Flanders, how many ells in Flanders are equal to 120 yards in Richmond? Ans. 160 ells.

4. If $2\frac{1}{2}$ yards in Philadelphia make 2 ells in London, how many yards in Philadelphia are equal to 60 ells in London? Ans. 75 yards.

5. If 60 pounds in America make 56 pounds at Amsterdam, how many pounds in America are equal to 350 pounds at Amsterdam? Ans. 375 pounds.

6. If 112lbs. in America be equal to 106 lbs. at Lyons, how many lbs. at Lyons are equal to 1 ton in America? Ans. 2120lbs.

7. If 100 English ells make 125 yards in the United States, how many English ells are equal to 1000 yards in the United States? Ans. 800 English ells.

CONJOINED PROPORTION,*

OR, THE RULE OF CONJUNCTION.†

Conjoined Proportion, or the Rule of Conjunction, is a comparison of the coins, weights, or measures, of several countries in the same question; or, it is the joining of many proportions together, and by the relation which several *antecedents* have to their *consequents*, the proportion between the first *antecedent* and the last *consequent* is discovered, as well as the proportion between the others in their several respects.

CASE 1.

When it is required to find how many of the first sort of coin, weight, or measure, mentioned in the question, are equal to a given quantity of the last sort.

RULE.

Place the *antecedents* on the left hand and the *consequents* on the right, and let the last number stand on the left hand; then, multiply *all the numbers* in the left hand column together for a dividend,

*See Thomas Dilworth's Arithmetic. †See William Gordon's Arithmetic.

and those in the right hand column for a diviser: the quotient thence arising will be the answer required.

N. B.—Equal antecedents and consequents cancel each other; the others may be divided by any number that will divide both without a remainder, and the quotients used in their stead.

EXAMPLES.

1. Suppose 100 yards of America make 100 yards of England, and 100 yards of England 50 canes of Thoulouse, and 100 canes of Thoulouse 160 ells of Geneva, and 100 ells of Geneva 200 ells of Hamburg; how many yards of America are equal to 379 ells of Hamburg?

Antecedents. *Consequents.*
100 yards of America=100 yards of England.
100 yards of England=50 canes of Thoulouse.
100 canes of Thoulouse=160 ells of Geneva.
100 ells of Geneva=200 ells of Hamburg.
379 ells of Hamburg.

100×100×100×100×379=37900000000 dividend
100×50×160×200=. . . . . 160000000 divisor
=236 yds. 3 qrs. 2 na. Answer.

See the operation abbreviated.

	Antecedents.	*Consequents.*	*Ant.*	*Con*
	100 yards of A.=	100 yards of E.	0	0
50)	100 yards of E.=	50 canes of T.	2	1
20)	100 canes of T.=	160 ells of G.	5	8
100)	100 ells of G. =	200 ells of H.	1	2
	379 ells of Hamburgh.			

379
5
8)1895
236yds. 3qrs. 2na. answer.

2. If 20lbs. at Richmond, in Virginia, make 25lbs. at Antwerp, and 150lbs. at Antwerp make 180lbs. at Leghorn, how many pounds at Richmond, in Virginia, are equal to 144lbs. at Leghorn?
Ans. 96lbs. at Richmond, in Virginia.

3. If 12lbs at Boston make 10lbs. at Amsterdam, and 100lbs. at Amsterdam 120lbs. at Paris, how many pounds at Boston are equal to 80lbs. at Paris? Ans. 80lbs. at Boston are=80lbs. at Paris.

4. If 25lbs. at London make 22lbs. at Nuremburg, and 88lbs. at Nuremburg 92lbs. at Hamburg, and 46lbs. at Hamburg 49lbs. at Lyons, how many pounds at London are equal to 98lbs. at Lyons?
Ans. 100 pounds.

5. If 100lbs. English make 95lbs. Flemish, and 19lbs. Flemish 25lbs. at Bologna, how many pounds English are equal to 50lbs. at *Bologna?* Ans. 40lbs. English.

6. If 6 braces at Leghorn make 3 ells English, and 5 ells English 9 braces at Venice, how many braces at Leghorn are equal to 45 braces at Venice? Ans. 50 braces at Leghorn.

7. If 3 dozen pair of gloves are equal in value to 2 pieces of ribbon, and 3 pieces of ribbon to 7 dozen of points, and 6 dozen of points to 2 yards of Flanders lace, 3 yards of Flanders lace to 81 shillings, how many dozen pair of gloves may be bought for 28 shillings? Ans. 2 dozen pairs.

Case 2.

When it is required to find how many of the last sort of coin, weight, or measure, mentioned in the question, are equal to a given quantity of the first sort.

Rule.

Place the numbers alternately, as before, beginning on the left hand with the *antecedents*, and let the last number stand on the right hand; then multiply all the numbers in the left hand column together for a divisor, and those on the right for a dividend; the quotient thence arising will be the answer required.

Examples.

1. If 20 braces at Leghorn=10 vares at Lisbon, and 40 vares at Lisbon=80 braces at Lucca, how many braces at Lucca are=100 braces at Leghorn?

Antecedents.	*Consequents.*
20 braces are	=10 vares.
40 vares are	=80 braces.
—	100
8,00	)800,00
Ans.	100 br. at Lucca.

Contracted.

10)20=10	2	1
40)40=80	1	2

100 braces as before.

2. If 12lbs. at Boston=10lbs. at Amsterdam, and 100lbs. at Amsterdam=120lbs. at Paris, how many pounds at Paris are=to 80 pounds at Boston? Ans. 80lbs. at Paris.

3. If 10lbs. at London are=9lbs. at Amsterdam, and 90lbs. at Amsterdam=112lbs at Thoulouse, how many pounds at Thoulouse are=50lbs. at London? Ans. 56lbs.

4. If 96lbs. at Richmond, in Virginia, are=144lbs. at Leghorn, and 180lbs. at Leghorn=150lbs. at Antwerp, how many pounds at Antwerp are=20lbs. at Richmond, in Virginia? Ans. 25lbs.

5. If 40lbs. English are=50lbs. at Bologna, and 25lbs. at Bologna=19lbs. Flemish, how many pounds Flemish are=100 pounds English? Ans. 95lbs. Flemish.

6. If 50 braces at Leghorn are=45 braces at Venice, and 9 braces at Venice=5 English ells, how many English ells are equal to *6 braces at Leghorn?* Ans. 3 English ells.

THE RULES OF PRACTICE.

Practice is a short way of finding the value of any quantity of goods, or other mercantile articles, when the price of one integer and the number you wish to find the value of, are both known. The questions may be proved by varying the parts, by Compound Multiplication, or by the Single Rule of Three Direct.

THE TABLES.

Aliquot parts of a dollar, or 100 cents.

cents.	parts.
1	= $\frac{1}{100}$
1¼	= $\frac{1}{80}$
1⅔	= $\frac{1}{60}$
2	= $\frac{1}{50}$
2½	= $\frac{1}{40}$
3⅓	= $\frac{1}{30}$
4	= $\frac{1}{25}$
4⅙	= $\frac{1}{24}$
5	= $\frac{1}{20}$
6¼	= $\frac{1}{16}$
8⅓	= $\frac{1}{12}$
10	= $\frac{1}{10}$
12½	= $\frac{1}{8}$
16⅔	= $\frac{1}{6}$
20	= $\frac{1}{5}$
25	= $\frac{1}{4}$
33⅓	= $\frac{1}{3}$
50	= $\frac{1}{2}$

Aliquot parts of 1d.

1 qr.	= $\frac{1}{4}$
2 qrs	= $\frac{1}{2}$

Aliquot parts of 1s.

D.	parts.
1	= $\frac{1}{12}$
1½	= $\frac{1}{8}$
2	= $\frac{1}{6}$
3	= $\frac{1}{4}$
4	= $\frac{1}{3}$
6	= $\frac{1}{2}$

Aliquot parts of a pound sterling.

s.	d.	
1		= $\frac{1}{20}$
1	3	= $\frac{1}{16}$
1	4	= $\frac{1}{15}$
1	8	= $\frac{1}{12}$
2	0	= $\frac{1}{10}$
2	6	= $\frac{1}{8}$
3	4	= $\frac{1}{6}$
4	0	= $\frac{1}{5}$
5	0	= $\frac{1}{4}$
6	8	= $\frac{1}{3}$
10	0	= $\frac{1}{2}$

Aliquot parts of 1cwt.

lbs.	
56	= $\frac{1}{2}$
28	= $\frac{1}{4}$
16	= $\frac{1}{7}$
14	= $\frac{1}{8}$
8	= $\frac{1}{14}$
7	= $\frac{1}{16}$

Aliquot parts of 1 ton.

cwt.	
10	= $\frac{1}{2}$
5	= $\frac{1}{4}$
4	= $\frac{1}{5}$
2½	= $\frac{1}{8}$
2	= $\frac{1}{10}$

Aliquot parts of a year.

mo.	yr.
6	= $\frac{1}{2}$
4	= $\frac{1}{3}$
3	= $\frac{1}{4}$
2	= $\frac{1}{6}$
1	= $\frac{1}{12}$

Aliquot parts of a mo.

days.	
15	= $\frac{1}{2}$
10	= $\frac{1}{3}$
6	= $\frac{1}{5}$
5	= $\frac{1}{6}$
3	= $\frac{1}{10}$
2	= $\frac{1}{15}$

I. OF PRACTICE IN FEDERAL MONEY.

CASE 1.

When the price of an integer is less than a dollar, and also an aliquot part of it.

RULE.

Divide the given number by the said aliquot part, and the quotient will be the answer in dollars—but if there should be a remainder, proceed with it according to the rule given in Division of Federal Money.

EXAMPLES.

c.			c.		
1	$\frac{1}{100}$	9975 at 1 cent each.	1⅔	$\frac{1}{60}$	9842 at 1⅔ cents each.
		\$99.75 cents. Ans.			\$164.03⅓ cents. Ans.
1¼	$\frac{1}{80}$	9876 at 1¼ cents each.	2	$\frac{1}{50}$	9839 at 2 cents each.
		\$123.45 cents. Ans.			\$196.78 cents. Ans.

c.			c.		
2½	1/40	9956 at 2½ cents each. $248.90 cents. Ans.	10	1/10	7832 at 10 cents each. $783.20 cents. Ans.
3⅛	1/32	9834 at 3⅛ cents each. $327.80 cents. Ans.	12½	1/8	6786 at 12½ cents each. $848.25 cents. Ans.
4	1/25	9999 at 4 cents each. $399.96 cents. Ans.	16⅔	1/6	7683 at 16⅔ cents each. $1280.50 cents. Ans.
4⅙	1/24	9998 at 4⅙ cents each. $416.58⅓ cents. Ans.	20	1/5	5678 at 20 cents each. $1135.60 cents. Ans.
5	1/20	9675 at 5 cents each. $483.75 cents. Ans.	25	1/4	2766 at 25 cents each. $691.50 cents. Ans.
6¼	1/16	9832 at 6¼ cents each. $614.50 cents. Ans.	33⅓	1/3	2918 at 33⅓ cents each. $972.66⅔ cents. Ans.
8⅓	1/12	8999 at 8⅓ cents each. $749.91⅔ cents. Ans.	50	1/2	1239 at 50 cents each. $619.50 cents. Ans.

Case 2

When the price of an integer is less than a dollar, and not some aliquot part of it.

Rule.

Find the aliquot parts of the given price contained in one dollar, by which divide the given number of things, and the sum of the quotients will be the answer in dollars, &c. Parts may be taken out of parts very conveniently, as may be seen in the following

Examples.

What will 56 yards cost, at 93¾ cents per yard?

c.		yds.
50	½	56
25	½	28 . . . .=the value at 50 cents.
12½	½	14 . . . .=the value at 25 cents.
6¼	½	7 . . . .=the value at 12½ cents.
		3.50 . .=the value at 6¼ cents.
93¾		$52.50 cents. Ans.

5	$\frac{1}{20}$	1234 at $5\frac{1}{2}$ cents.
$\frac{1}{2}$	$\frac{1}{10}$	61.70
		6.17
		$67.87 cents. Ans.

5	$\frac{1}{20}$	4321 at $7\frac{1}{2}$ cents.
$2\frac{1}{2}$	$\frac{1}{2}$	216.05
		$108.02\frac{1}{2}$
		$\$324.07\frac{1}{2}$ cents. Ans.

10	$\frac{1}{10}$	2006 at $11\frac{1}{4}$ cents.
$1\frac{1}{4}$	$\frac{1}{8}$	200.60
		$25.07\frac{1}{2}$
		$\$225.67\frac{1}{2}$ cents. Ans.

10	$\frac{1}{10}$	2075 at 15 cents.
5	$\frac{1}{2}$	207.50
		103.75
		$311.25 cents. Ans.

10	$\frac{1}{10}$	2910 at $17\frac{1}{2}$ cents.
5	$\frac{1}{2}$	291
$2\frac{1}{2}$	$\frac{1}{2}$	145.50
		72.75
		$509.25 cents. Ans.

$12\frac{1}{2}$	$\frac{1}{8}$	3746 at $18\frac{3}{4}$ cents.
$6\frac{1}{4}$	$\frac{1}{2}$	468.25
		$234.12\frac{1}{2}$
		$\$702.37\frac{1}{2}$ cents. Ans.

20	$\frac{1}{5}$	4632 at 24 cents.
4	$\frac{1}{5}$	926.40
		185.28
		$1111.68 cents. Ans.

$33\frac{1}{3}$	$\frac{1}{3}$	1295 at $41\frac{2}{3}$ cents.
$8\frac{1}{3}$	$\frac{1}{4}$	$431.66\frac{2}{3}$
		$107.91\frac{2}{3}$
		$\$539.58\frac{1}{3}$ cents. Ans.

50	$\frac{1}{2}$	468 at $91\frac{2}{3}$ cents.
$33\frac{1}{3}$	$\frac{1}{3}$	234
$8\frac{1}{3}$	$\frac{1}{12}$	156
		39
		$429 Ans.

25	$\frac{1}{4}$	2911 at $31\frac{1}{4}$ cents.
$6\frac{1}{4}$	$\frac{1}{4}$	727.75
		$181.93\frac{3}{4}$
		$\$909.68\frac{3}{4}$ cents. Ans.

25	$\frac{1}{4}$	2181 at $37\frac{1}{2}$ cents.
$12\frac{1}{2}$	$\frac{1}{2}$	545.25
		$272.62\frac{1}{2}$
		$\$817.87\frac{1}{2}$ cents. Ans.

25	$\frac{1}{4}$	3719 at $43\frac{3}{4}$ cents.
$12\frac{1}{2}$	$\frac{1}{2}$	929.75
$6\frac{1}{4}$	$\frac{1}{2}$	$464.87\frac{1}{2}$
		$232.43\frac{3}{4}$
		$\$1627.06\frac{1}{4}$ cents. Ans.

$33\frac{1}{3}$	$\frac{1}{3}$	3761 at $66\frac{2}{3}$ cents.
$33\frac{1}{3}$	$\frac{1}{3}$	$1253.66\frac{2}{3}$
		$1253.66\frac{2}{3}$
		$\$2507.33\frac{1}{3}$ cents. Ans.

50	$\frac{1}{2}$	5555 at $83\frac{1}{3}$ cents.
$33\frac{1}{3}$	$\frac{1}{3}$	2777.50
		$1851.66\frac{2}{3}$
		$\$4629.16\frac{2}{3}$ cents. Ans.

Case 3.

When the price of an integer is more than one dollar, but less than two.

Rule.

1. Let the given number of integers stand for so many dollars.
2. Take parts of it, with the cents, &c. as in the following cases.
3. Add these parts and the given number together, and the sum will be the answer in dollars, &c.

Examples.

25	¼	175 at \$1.51½ cents.
20	⅕	43.75
5	¼	35.00
1	⅕	8.75
½	½	1.75
		0.87½
		\$265.12½ cents. Ans.

391 at \$1.06¼ cents.
\$415.43¾ cents. Ans.

793 at 1.08⅓ cents.
\$859.08⅓ cents. Ans.

864 at \$1.10¼ cents.
\$952.56 cents. Ans.

999 at \$1.12½ cents.
\$1123.87½ cents. Ans.

871 at \$1.16⅔ cents.
\$1016.16⅔ cents. Ans.

987 at \$1.18¾ cents.
\$1172.06¼ cents. Ans.

1111 at \$1.25 cents.
\$1388.75 cents. Ans.

1011 at \$1.28 cents.
\$1294.08 cents. Ans.

1773 at \$1.31¼ cents.
\$2327.06¼ cents. Ans.

1776 at \$1,44 cents.
\$2557.44 cents. Ans.

901 at \$1.74 cents.
\$1567.74 cents. Ans.

875 at \$1.81¼ cents.
\$1585.93¾ cents. Ans.

Case 4.

When the price of an integer is more than two dollars.

Rule.

1. Multiply the given number of integers by the dollars, and take parts of it, with the cents, &c. in the price of one integer, as in the foregoing cases.
2. Add the product by the dollars and the said parts together, and that sum will be the answer in dollars, &c.

EXAMPLES.

50	½	89 at \$2.87½ cents.
		2
		178
25	½	44.50
12½	½	22.25
		11.12½
		\$255.87½ cents. Ans.

78 at \$3.56¼ cents.
\$277.87½ cents. Ans.

68 at \$4.66⅔ cents.
\$317.33⅓ cents. Ans.

65 at \$5.67½ cents.
\$368.87½ cents. Ans.

83 at \$6.75 cents.
\$560.25 cents. Ans.

58 at \$7.81¼ cents.
\$453.12½ cents. Ans.

49 at \$8.83⅓ cents.
\$432.83⅓ cents. Ans.

37 at \$9.93¾ cents.
\$367.68¾ cents. Ans.

CASE 5.

When the given quantity and the price of an integer are both of several denominations.

RULE.

1. Multiply the price by the number of integers in the said quantity.
2. Take parts of the said price, with the given parts of the integer.
3. Add these parts and the above product together, and that sum will be the answer required in dollars, cents, &c.

EXAMPLES.

		72cwt. 3qrs. 27lbs. of sugar, at
2 qrs.=	½	\$10.50 cents per cwt.
		72=the number of integers.
		2100
		7350
		756.00=the value of 72 cwt.
1 qr.=	½	5.25=the value of 2 qrs.
16lbs.=	⅐	2.625=the value of 1 qr.
8lbs.=	½	1.500=the value of 16 lbs.
2lbs.=	¼	750=the value of 8 lbs.
1lb. =	½	1875=the value of 2 lbs.
		9375=the value of 1 lb.

Answer, \$766.40,625=\$766.40cts. 6¼m.

	\$ cts.	Answers. \$ cts.
2. 37cwt. 2qrs. 14lbs of cheese at	20.10 per cwt.?	756.26¼.
3. *5cwt. 2 qrs. 10lbs.* of tallow at	11.20 per cwt.?	62.60.

			Answers.
	$ cts.		$ cts.
4. 70cwt. 2qrs. 14lbs. of tobacco, at	9.00	per cwt.	635.62½.
5. 4cwt. 1qr. 16 lbs. of iron, at	7.70	per cwt.	33.82½.
6. 7cwt. 0qr. 16lbs. of soap, at	6.30	per cwt.	45.00.
7. 12cwt. 3qrs. 26lbs. of tobacco, at	14.00	per cwt.	181.75.
8. 10cwt. 3qrs. 19lbs. of sugar, at	12.60	per cwt.	137.58¾.
9. 13lb. 10oz. 12pwts. of silver, at	10.50	per lb.	145.77½.
10. 144lbs. 12oz. of butter, at	00.16	per lb.	23.16.
11. 83yds. 2qrs. of cloth, at	10.50	per yd.	876.75.
12. 64yds. 3qrs. of drab, at	2.25	per yd.	145.68¾.
13. 476 acres, 3ro. 28 perches, at	9.00	per acre.	4292.32½.
14. 953 acres, 3ro. 16 perches, at	4.50	per acre.	4292.32½.
15. 240 acres, 2ro. 20 perches, at	15.50	per acre.	3729.68¾.
16. 420 acres, 1ro. 26 perches, at	16.80	per acre.	7062.93.

Case 6.

To find the value of any number of pounds, feet, &c., at any given rate per cent., that is, by the hundred.

Rule.

1. Point off two figures at the right hand of the given number of pounds, feet, &c. for decimals.
2. Multiply the said given number by the dollars, and take parts of it with the cents, &c., as in the foregoing cases.
3. Add these parts to the above product, and the sum will be the answer.

Examples.

1. How much will 1278 feet of plank cost, at $1.37½ cents per 100 feet?

25	¼	12.78
		1
		12.78
12½	½	3.19½
		1.59¾

Ans. $17.57¼ cents.

2. What will 876 pounds of beef cost, at $5.62½ cents per 100 pounds?

50	½	8.76
		5
		43.80
12½	¼	4.38
		1.09½

Ans. $49.27½ cents.

	Answers.
3. What will 250lbs. of bacon cost, at $8.33⅓cts. per C.?	$20.83⅓.
4. What will 1560lbs. of sugar cost, at $11.37½cts. per C.?	$177.45.
5. *What will 3784lbs.* of tobacco cost, at $7.87½cts. per C.?	$297.99.
6. *What will 1674lbs.* of pork cost, at $5.33⅓cts. per C.?	$89.28.

Case 7.

Having the rate per cent. and the sum of money to be laid out both given, to find how many pounds, feet, &c. may be bought with the said sum, at the rate proposed.

Rule.

1. Reduce the sum of money to be laid out to the lowest denomination mentioned, and multiply it by 100 for a dividend.

2. Reduce the given rate per cent. to the same denomination for a divisor, by which divide the said dividend, and the quotient will be the answer.

Examples.

1. How many feet of plank may we bought with $17.57½cts. at the rate of $1.37½cts. per 100ft.

```
       $17.57½ cents.
             4
       ------
1.37½ |70.29
  4   |   100
----- |------
5.50  )7029.00(1278ft. Ans.
       ......
```

2. How many pounds of bacon can I buy for $20.83⅓ cents, at the rate of $8.33⅓ cents per 100 pounds?

```
 8.33⅓ |$20.83⅓ cents.
   3   |    3
------ |------
 25.00 ) 6250.00(250lbs. Ans.
         ------
         ......
```

3. How many pounds of beef can I buy for $49.27½ cents, at the rate of $5.62½ cents per C.? Ans. 876lbs.

4. How much sugar can I buy for $177.45 cents, if I give at the rate of $11.37½ cents for 100 pounds? Ans. 1560lbs.

5. What quantity of tobacco can be purchased for $297.99cts., at the rate of $7.87½ cents per C.? Ans. 3784lbs.

Case 8.

To find the amount of any number of shingles, &c. which are commonly made, or bought and sold, at a certain rate per thousand.

Rule.

Point off three figures at the right hand of the given number for decimal places, and proceed as in case the sixth.

Examples.

1. What will 4864 shingles come to, at $2.75cts. per thousand?

```
50 | ½ |  4.864
   |   |      2
   |   | ------
   |   |  9.728
25 | ½ |  2.432
   |   |  1.216
         ------
```

Ans. $13.37,6

2. What will 3596ft. of boards cost, at $6.25 cents per M.? Ans. $22.47½ cents.

3. What will the laying of 56480 bricks cost, at $5.45cts. per M.? Ans. $307.81 cents, 6 mills.

4. What will 42420 pine shingles cost, at $3.25cts. per M.? Ans. $137.86cts. 5m

II. OF PRACTICE IN STERLING MONEY.

Case 1.

When the price of an integer is less than a penny.

Rule.

1. Take such a part or parts of the given quantity as the price of an integer is of a penny, for the answer in pence.

2. If a remainder should occur in any example, proceed with it according to the general rule given under Compound Division.

Examples.

qr.	d.	
¼	¼	7612 at ¼qr. each.
	12	1903 pence.
	2,0	15,8.7d.
		7£ 18s. 7d. Ans.
½	½	6812 at ½d. each.
	12	3406 pence.
	2,0	28,3.10d.
		14£ 3s. 10d. Ans.
½	½	4712 at ¾qrs. each.
¼	½	2356 1178
	12	3534 pence.
	2,0	29,4.6d.
		14£ 14s. 6d. Ans.
½	½	9995 at ¾qrs. each.
¼	½	4997½ 2498¾
	12	7496¼
		624.8d.
		31£ 4s. 8d. ¼qr. Ans.

8765 at ¼qr. each.
9£ 2s. 7d. ¼qr. Ans.

9875 at ½d. each.
20£ 11s. 5½d. Ans.

6789 at ¾qrs. each.
21£ 4s. 3d. ¾qrs. Ans

9999 at ¼qr. each.
10£ 8s. 3d. ¾qrs. Ans

9873 at ½d. each.
20£ 11s. 4½d. Ans.

4567 at ¾qrs. each.
14£ 5s. 5d. ¼qr. Ans.

3999 at ½d. each.
8£ 6s. 7½d. Ans.

7519 at ¾qrs. each.
23£ 9s. 11d. ¼qr. Ans

Case 2.

When the price of an integer is more than a penny, but less than *a shilling*.

RULE.

Find the aliquot parts of the given price contained in a shilling, by which divide the given number of integers, and the sum of the quotients will be the answer in shillings, &c.

2. It is frequently more convenient to take parts of parts than parts of the whole. If there should be any remainder, reduce it to the next lower denomination, and so on, as before directed.

EXAMPLES.

d.	s.	
1	$\frac{1}{12}$	7612 at 1d. each.
	2,0	63,4.4d.
		31£ 14s. 4d. Ans.
1	$\frac{1}{12}$	8612 at 1¼d. each.
¼	¼	717.8d.
		179.5
	2,0	89,7.1
		44£ 17s. 1d. Ans.
1½	⅛	4121 at 1½d. each.
	2,0	51,5.1½d.
		25£ 15s. 1½d. Ans.
1	$\frac{1}{12}$	1861 at 1¾d. each.
½	½	155.1d.
¼	½	77.6½
		38.9¼
	2,0	27,1.4¾
		13£ 11s. 4¾d. Ans.
2	⅙	4761 at 2d. each.
	2,0	79,3.6d.
		39£ 13s. 6d. Ans.
2	⅙	6181 at 2¼d. each.
¼	⅛	1030.2d.
		128.9¼
	2,0	115,8.11¼
		57£ 18s. 11¼d. Ans.

d.	s.	
2	⅙	1218 at 2½d each.
½	¼	203.0d.
		50.9
	2,0	25,3.9
		12£ 13s. 9d. Ans.
2	⅙	8012 at 2¾d. each.
½	¼	1335. 4d.
¼	½	333.10
		166.11
	2,0	183,6.1d.
		91£ 16s. 1d. Ans.
3	¼	7612 at 3d. each.
	2,0	190,3
		95£ 3s. Ans.
3	¼	6128 at 3¼d. each.
¼	$\frac{1}{12}$	1532
		127.8d.
	2,0	165,9.8
		82£ 19s. 8d. Ans.
3	¼	6180 at 3½d. each.
½	⅙	1545
		257.6d.
	2,0	180,2.6
		90£ 2s. 6d. Ans.

d.	s.	
3	¼	7812 at 3¾d. each.
¾	¼	1953
		488.3d.
	2,0	244,1.3
		122£ 1s. 3d. Ans.
4	⅓	8120 at 4d. each.
	2,0	270,6.8d.
		135£ 6s. 8d. Ans.
3	¼	7000 at 4¼d. each.
1	⅓	1750
¼	¼	583. 4d.
		145.10
	2,0	247,9.2
		123£ 19s. 2d. Ans.
4	⅓	6001 at 4½d. each.
½	⅛	2000.4d.
		250.0½
	2,0	225,0.4½
		112£ 10s. 4½d. Ans.
4	⅓	7121 at 4¾d. each.
½	⅛	2373.8d.
¼	½	296.8½
		148.4¼
	2,0	281,8.8¾
		140£ 18s. 8¾d. Ans.
4	⅓	7181 at 5d. each.
1	¼	2393.8d.
		598.5
	2,0	299,2.1d.
		149£ 12s. 1d. Ans.

d.	s.	
4	⅓	8121 at 5¼d each.
1	¼	2707
¼	¼	676.9d.
		169.2¼
	2,0	355,2.11¼
		177£ 12s. 11¼d. Ans.
4	⅓	6128 at 5½d. each.
1	¼	2042.8d.
½	½	510.8
		255.4
	2,0	280,8.8
		140£ 8s. 8d. Ans.
4	⅓	6100 at 5¾d. each.
1	¼	2033.4d.
½	½	508.4
¼	½	254.2
		127.1
	2,0	292,2.11
		146£ 2s. 11d. Ans.
6	½	3011 at 6d. each.
	2,0	150,5.6d.
		75£ 5s. 6d. Ans.
4	⅓	7610 at 6¼d. each.
2	½	2536.8d.
¼	⅛	1268.4
		158.6½
	2,0	396,3.6½d.
		198£ 3s. 6½d. Ans.

CASE 3.

When the price of an integer is more than one shilling but less than two.

RULE.

Let the given quantity stand for shillings, and take parts of it with so much of the price as is more than one shilling, and add them to the said given quantity, and that sum will be the answer in shillings, &c

EXAMPLES.

qr.	s.	
$\frac{1}{4}$	$\frac{1}{48}$	486 at $12\frac{1}{4}$d. each.
		10 $1\frac{1}{2}$d.
	2,0	49,6 $1\frac{1}{2}$
		24£ 16s. $1\frac{1}{2}$d. Ans.
$\frac{1}{2}$d.	$\frac{1}{24}$	594 at $12\frac{1}{2}$d. each.
		24 9d.
	2,0	61,8 9
		30£ 18s. 9d. Ans.
$\frac{3}{4}$qr.	$\frac{1}{16}$	518 at $12\frac{3}{4}$d. each.
		32 $4\frac{1}{2}$d.
	2,0	55,0 $4\frac{1}{2}$
		27£ 10s. $4\frac{1}{2}$d. Ans.
1d.	$\frac{1}{12}$	1234 at 13d. each.
		102 10d.
	2,0	133,6 10
		66£ 16s. 10d. Ans.

1281 at $13\frac{1}{4}$d. each.

70£ 14s. $5\frac{1}{4}$d. Ans.

6100 at $13\frac{1}{2}$d. each.

343£ 2s. 6d. Ans.

1210 at $13\frac{3}{4}$d. each.

69£ 6s. $5\frac{1}{2}$d. Ans.

1234 at 14d. each.

71£ 19s. 8d. Ans.

1271 at $14\frac{1}{4}$d. each.

75£ 9s. $3\frac{3}{4}$d. Ans.

6120 at $14\frac{1}{2}$d. each.

369£ 15s. Ans.

1210 at $14\frac{3}{4}$d. each.

74£ 7s. $3\frac{1}{2}$d. Ans.

1260 at 15d. each.

78£ 15s. Ans.

1612 at $15\frac{1}{4}$d. each.

102£ 8s. 7d.

1210 at $15\frac{1}{2}$d. each.

78£ 2s. 11d. Ans.

7612 at $15\frac{3}{4}$d. each.

499£ 10s. 9d. Ans.

6100 at 16d. each.

406£ 13s. 4d. Ans.

7121 at $16\frac{1}{4}$d. each.

482£ 3s. $0\frac{1}{4}$d. Ans.

1218 at $16\frac{1}{2}$d. each.

83£ 14s. 9d. Ans.

8100 at $16\frac{3}{4}$d. each.

565£ 6s. 3d. Ans.

4128 at 17d. each.

292£ 8s. Ans.

Case 4.

When the price of an integer is any even number of shillings under twenty.

Rule.

Multiply the given number of integers by half the price, and double the first figure of the product for shillings, and the rest of the product will be pounds.

Examples,

What will 86 bushels of wheat cost, at 6s. per bushel?

86 bushels.
3 shillings.

Ans. 25£ 16s.

86 bushels.
6 shillings.

2,0)51,6

£25 16s. Proof.

	s.		£	s.
1.	486 at 2 each.	Ans.	48	12
2.	769 at 4 each.		153	16
3.	593 at 6 each.		177	18
4.	456 at 8 each.		182	8
5.	185 at 10 each.		92	10
6.	367 at 12 each.		220	4
7.	171 at 14 each.		119	14
8.	296 at 16 each.		236	16
9.	193 at 18 each.		173	14
10.	98 at 12 each.		58	16

Case 5.

When the price of an integer is any odd number of shillings under twenty.

Rule.

Multiply the given number of integers by the whole price, and divide the product by 20, and the quotient will be the answer.

Examples.

	s.		£	s.
1.	999 at 1 each.	Ans.	49	19
2.	426 at 3 each.		63	18
3.	399 at 5 each.		99	15
4.	298 at 7 each.		104	6
5.	953 at 9 each.		428	17
6.	299 at 11 each.		164	9
7.	601 at 13 each.	Ans.	390	13
8.	191 at 15 each.		143	5
9.	121 at 17 each.		102	17
10.	837 at 19 each.		795	3
11.	242 at 11 each.		133	2
12.	89 at 19 each.		84	11

Case 6.

When the price of an integer is shillings and pence.

Rule.

1. If the shillings and pence be any aliquot part of a pound, it *may be done at* once by taking such a part of the given number as

the price of an integer is of a pound, and the result will be the answer in pounds, shillings, &c.

EXAMPLES.

s. d.			s. d.		
1.3	$\frac{1}{16}$	986 at 1s. 3d. each. 61*l*. 12s. 6d. Ans.	3.4	$\frac{1}{6}$	275 at 3s. 4d. each. 45*l*. 16s. 8d. Ans.
1.4	$\frac{1}{15}$	846 at 1s. 4d. each. 56*l*. 8s. Ans.	6.8	$\frac{1}{3}$	684 at 6s. 8d. each. 228*l*. Ans.
1.8	$\frac{1}{12}$	769 at 1s. 8d. each. 64*l*. 1s. 8d. Ans.	1.8	$\frac{1}{12}$	875 at 1s. 8d. each. 72*l*. 18s. 4d. Ans.
2.6	$\frac{1}{8}$	186 at 2s. 6d. each. 23*l*. 5s. Ans.	2.6	$\frac{1}{8}$	695 at 2s. 6d. each. 86*l*. 17s. 6d. Ans.

2. If the shillings and pence be not some aliquot part of a pound, or if there be shillings, pence, and farthings, multiply the given number by the shillings, and take parts of it, with the pence, &c., which must be added to the product by the shillings, and that sum will be the answer in shillings, &c.

EXAMPLES.

d.		
3	$\frac{1}{4}$	126 at 9s. 3d. each.
		9
		1134 product.
		31 6d.
	2,0	116,5 6
		58*l*. 5s. 6d. Ans.

86 at 6s. 10½d. each.
29*l*. 11s. 3d. Ans.

98 at 12s. 4½d. each.
60*l*. 12s. 9d. Ans.

31 at 4s. 9d. each.
7*l*. 7s. 3d. Ans.

73 at 7s. 6d. each.
27*l*. 7s. 6d. Ans.

70 at 7s. 4¾d. each.
25*l*. 17s. 8½d. Ans.

55 at 4s. 8½d. each.
12*l*. 18s. 11½d. Ans.

17 at 17s. 4¼d. each.
14*l*. 15s. 0¼d. Ans.

12 at 13s. 10½d. each.
8*l*. 6s. 6d. Ans.

148 at 11s. 9½d. each.
87*l*. 5s. 2d. Ans.

76 at 10s. 10¾d. each.
41*l*. 8s. 1d. Ans.

83 at 19s. 11¾d. each.
82*l*. 18s. 3¼d. Ans.

Case 7.

When the price of an integer is pounds, or pounds, shillings, &c.

Rule.

Multiply the given number of integers by the pounds, and take such a part or parts of it as the rest of the price is of a pound, which must be added to the product by the pounds, and that sum will be the answer in pounds, shillings, &c.

Examples.

s.		
10	½	49 at 4*l.* 17s. 6d. each.
		4
		196
5	¼	24 10s.
2 6d	½	12 5
		6 2 6d.
		238*l.* 17s. 6d. Ans.

26 at 4*l.* 8s. each.
114*l.* 8s. Ans.

49 at 3*l.* 7s. 6d. each.
165*l.* 7s. 6d. Ans.

36 at 5*l.* 13s. 9d. each.
204*l.* 15s. Ans.

47 at 7*l.* 10s. each.
352*l.* 10s. Ans.

26 at 11*l.* 14s. 9½d. each.
305*l.* 4s. 7d. Ans.

17 at 9*l.* 15s. 6d. each.
166*l.* 3s. 6d. Ans.

16 at 3*l.* 6s. 7½d. each.
53*l* 6s. Ans.

47 at 3*l.* 3s. 4d. each.
148*l.* 16s. 8d. Ans.

20 at 4*l.* 13s. 4d. each.
93*l.* 6s. 8d. Ans.

17 at 2*l.* 6s. 8d. each.
39*l.* 13s. 4d. Ans.

Case 8.

When the given quantity and the price of an integer are both of several denominations.

Rule.

1. Multiply the price by the number of integers in the said quantity. 2. Take parts of the said price with the given parts of the integer. 3. Add the said parts and the above product together, and that sum will be the answer required in pounds, shillings, &c.

Examples.

qrs.		
		17cwt. 3qrs. 19lbs. of sugar, at 2*l.* 2s. 6d. per cwt.
2	½	17
		36 2 6
1	½	1 1 3
16lb.	$\frac{1}{7}$	0 10 7½
2	⅛	0 6 0¾ +3 rem.
1	½	0 0 9 +3 rem.
		0 0 4½
		38 1 6¾+ Ans.

12cwt. 3qrs. 16lbs. of tobacco, at 4*l.* 12s. per cwt.
Ans. 59*l.* 6s. 1¼d.+

12cwt. 2qrs. 14lbs. of sugar, at 3*l.* 14s. per cwt.
Ans. 46*l.* 14s. 3d.

4cwt. 1qr. 16lbs. of tobacco, at 3*l.* 12s. per cwt.
Ans. 15l. 16s. 3½d.+5.

TARE AND TRET.

Tare and Tret are allowances made by the seller to the buyer on various kinds of mercantile commodities.

Tare is an allowance made for the weight of the hogshead, barrel, box, or bag, &c. which contains the commodity.

Tret is an allowance of 4lbs. in every 104lbs. for waste, dust, &c.

Cloff is an allowance of 2lbs. in every 3cwt. for the turn of the scale.

Gross weight is the whole weight of the goods, together with the weight of the hogshead, barrel, box, or bag, &c. which contains them, including ropes, canvass, and other coverings.

Suttle is when part of the allowance is deducted from the gross.

Neat weight is the weight of the goods, after the several allowances are deducted from the whole gross weight.

Case 1.

When the tare is so much in the whole gross weight.

Rule.

Subtract the tare from the whole gross weight, and the remainder will be the neat weight required.

Examples.

1. The gross weight of a certain hogshead of sugar is 13cwt. 1qr. 16lbs.; the tare is 1cwt. 3qrs. 24lbs. What is the neat weight?

Ans. 11cwt. 1qr. 20lbs.

2. The gross weight of a certain hogshead of tobacco is 1691lbs. and the hogshead weighs 107lbs. What is the neat weight?

Ans. 1584lbs.

Case 2.

When there are several hogsheads, each containing the same weight.

Rule.

Multiply the weight of one hogshead, or bag, &c. by the number of hogsheads, or bags, &c. and the product will be the whole gross weight, from which subtract the given tare, and the remainder will be the neat weight.

Examples.

1. What is the neat weight of 12 hogsheads of sugar, each weighing 6cwt. 2qrs. 17lbs. gross; the whole tare being 8cwt. 3qrs. 14lbs.?

Ans. 70cwt. 3qrs. 22lbs.

2. What is the neat of 4 casks of indigo, each weighing 4cwt. 2qrs. 18lbs. gross; the whole tare being 1cwt. 0qrs. 26lbs.?

Ans. 17cwt. 1qr. 18lbs.

Case 3.

When there are different quantities of gross weight, and also of tare.

Rule.

Add the gross weight into one sum, and the tare into another; then subtract the whole tare from the whole gross, and the remainder will be the neat weight.

Examples.

1. What is the neat weight of 3 barrels of sugar, weighing as follows, viz:

	Cwt.	qr.	lbs.		qrs.	lb.
No. 1 gross	3	1	2	Tare	2	24
2	3	2	1		2	24
3	5	1	12		3	16
	12	0	15	c. 2	1	8
	2	1	8			
Ans.	9	3	7 neat weight.			

2. What is the neat weight of 5 hogsheads of tobacco, weighing as follows, viz:

	lbs.		lbs.
No. 1 gross	1600	tare	102
2	1720		110
3	1410		90
4	1524		100
5	1806		120
	8060	—	522=7538 lbs. neat

Case 4.

When the tare is so much per hogshead, barrel, bag, &c.

Rule.

Multiply the number of hogsheads, or barrels, &c. by the tare per hogshead, barrel, &c. and the product will be the whole tare, which must be subtracted from the whole gross weight, and the remainder will be the neat weight required.

Examples.

1. What is the neat weight of 5 hds. of tobacco, each weighing 14cwt, 2qrs. 17lbs. gross, the tare being 100lbs. per hd.?

```
      14
      14
     14.
    14..
     73lbs=2qrs. 17lbs.
    ----
    1641
       5
    ----
lbs. 8205=whole gross wt.
100×5=509
    ----
Ans. 7705lbs. neat weight.
```

2. In 241 barrels of figs, each weighing 3qrs. 19lbs. gross, tare 10lbs. per barrel, how many lbs. neat? Ans. 22413lbs.

3. What is the neat weight of 17 bags of cotton, each weighing 2cwt. 3qrs. 4lbs, tare 9lbs. per bag, and what is it worth at 12½cts. per lb.? Ans. 45cwt. 3qrs. 27lbs. neat weight, and it is worth $643.87½ cents.

4. At $1.25 cents per lb. what will 3 chests of hyson tea come to, weighing gross 96lbs. 97lbs. and 101lbs. respectively, the tare being 20lbs. per chest?

Ans. $292.50 cents.

5. In 70 bales of Smyrna silk, each weighing 317lbs. gross, the tare per bale being 16lbs. how many pounds neat weight?
Ans. 21070lbs.

6. What is the neat weight of 30 bales of Cyprus silk, each weighing 249lbs. gross, tare per bale 14lbs.? Ans. 7050lbs.

Case 5.

When the tare is at so much per hundred weight.

Rule.

Divide the whole gross weight by such aliquot parts as the tare is of 1cwt. and the sum of the quotients will be the whole tare—then subtract the whole tare from the whole gross weight, and the remainder will be the neat weight required.

Examples.

1. What is the neat weight of 9 hds. of tobacco, each weighing 6cwt. 2qrs. 12lbs. gross; tare 17lbs. per cwt.?

		Cwt.	qrs.	lbs.	
		6	2	12	
				9	
16	$\frac{1}{4}$	59	1	24	=whole gross wt.
1	$\frac{1}{16}$	8	1	27+	added.
		0	2	3+	
		9	0	2	=whole tare.
		50	1	22	neat wt. Ans.

2. In 173cwt. 3qrs. 17lbs. gross; tare 16lbs. per hundred weight, how much neat weight?
Ans. 149cwt. 0qrs. 7lbs.

3. What is the neat weight of 7 barrels of pot ash, each 201lbs. gross; tare 10lbs. per cwt.?
Ans. 1281lbs. 6 ozs.

4. In 12 butts of currants, each 7cwt. 1qr. 10lbs. gross; tare 16 lbs. per cwt., how much neat wt.?
Ans. 75cwt. 1qr. 26lbs. 14ozs.

5. What will a hogshead of raisins weighing 9cwt. 2qrs. gross; tare 12lbs. per cwt. amount to at 23cts. per lb.? Ans. $218.50cts.

6. What is the neat weight of 36 kegs of figs, each weighing 2 cwt. 0qrs. 12lbs. gross; tare 14lbs. per cwt. and what will they amount to at 12½cts per lb.?
Ans. 66cwt. 1qr. 14lbs. neat, and the amount is $929.25cts.

Case 6.

When tret is allowed after the tare is deducted.

Rule.

Divide the suttle weight by 26, and subtract the quotient from the same; the *remainder* will be neat.

EXAMPLES.

1. In 247cwt. 2qrs. 15lbs. gross; tare 28lbs. per cwt. and tret 4 lbs. in every 104lbs., how much neat weight?

lbs.		Cwt.	qrs.	lbs.	
28	$\frac{1}{4}$	247	2	15	gross.
		61	3	17	12ozs. tare, subtracted.
4	$\frac{1}{26}$	185	2	25	4=suttle weight.
		7	0	16	0=tret, subtracted.
		178	2	9	4 neat weight. Ans.

2. What is the neat weight of 5hds. of sugar, each 10cwt. 1qr. 20lbs. gross; tare 3qrs. 25lbs. per hd. and tret 4lbs. per 104lbs.?

Ans. 45cwt. 1qr. 24lbs.

CASE 7.

When tare, tret, and cloff are allowed.

RULE.

Deduct the tare and tret as before, then divide the suttle by 168, and the quotient will be the cloff, which must be deducted from the suttle, and remainder will be the neat.

EXAMPLES.

1. What is the neat weight of 5hhds. of sugar, each weighing 10cwt. 1qr. 20lbs. gross—tare 3 qrs. 25lbs. per hhd.—tret 4lbs. per 104lbs.—and cloff 2lbs. in every 3cwt.? Ans. 45cwt. 0qrs. 22lbs. rejecting all remainders after the pounds. Examine the work carefully.

```
            cwt. qrs. lbs.
             10   1   20
                       5
q. lbs.      52   0   16=gross wt.
3 25×5=       4   3   13=the tare.
        26)47   1    3
              1   3    7=the tret.
lbs.
2=1/168)45   1   24=the suttle.
          00   1   02=the cloff.
Ans.      45   0   22 neat weight.
```

2. What is the neat weight of a hogshead of tobacco, containing 1784lbs. gross—tare 7lbs. per cwt.—tret and cloff as usual?

Ans. 1600lbs. by rejecting the remainders after pounds, and 1598lbs. 10ozs, by reducing the remainders, &c.

```
                  lbs.
7lbs.=1/16)1784 gross.
            111=the tare.
4lbs.=1/26)1673
            ··64=the tret.
2lbs.=1/168)1609=the suttle.
            ···9=the cloff.
Ans.        1600 neat weight.
```

3. What is the neat weight of 29 bags of cotton, each containing 350lbs. gross—tare 8lbs. per cwt.—tret and cloff as usual; and what is it worth at 12½ cents per pound? Ans. 9010lbs. neat weight, and $1126.25 in value.

4. What is the neat weight of 9hhds. of tobacco, each weighing 1058lbs. gross—tare 30lbs. per hogshead—tret and cloff as usual; and what will it amount to at 10 cents per lb., after deducting $171 95 cents for duties and other charges? Ans. 8845lbs. neat weight, and it will amount to $712.55 cents, after paying the duties and charges aforementioned.

SIMPLE INTEREST.

Interest is the premium or compensation allowed by the borrower to the lender for the use of a certain sum of money, relative to which there are four particular terms, namely:

1. The principal, or sum of money lent out for interest.
2. The time for which interest is to be calculated on the principal.
3. The rate per cent., which, according to a law of Virginia, is six dollars for the use of one hundred dollars one year, and in the same proportion for any sum of money for a shorter or longer space of time.
4. The amount, which is the sum of the principal and its interest, when they are added together.

Case 1.

To find the interest of any given sum for one year.

General Rule.

Multiply the principal by the rate per cent., and point off the two right hand figures of the dollars in the product; the result will be the interest of the given sum for one year, in dollars, cents, &c. The figures on the right hand of the mills are so many decimal parts of a mill, and consequently may be rejected as of no account.

Examples.

1. What is the interest of 275 dollars for one year, at $6 per cent. per annum?

$275 principal.
6 rate per cent.

$16.50 cents. Answer.

2. What is the interest of $275 25 cents for one year, at $6 per cent. per annum?

$275.25 cents.
6

$16.51,50 Answer.

3. What is the interest of $275 66⅔ cents for one year, at $6 per cent. per annum?

$275.66⅔ cents.
6

$16.54,00 Ans.

4. What is the interest of $275 87½ cents for one year, at $6 per cent. per annum?

$275.87,5
6

$16.55,250 Ans.

5. What will a bond of $756 65 cents amount to in one year, at $6 per cent. per annum?

$756.65 cents.
6

45.39,90 interest } added
756.65=principal }

Ans. $802.04,9=amount.

6. What will a bond of $327 82½ cents amount to in one year, at $6 per cent. per annum?

$327.82,5
6

19.66,950 int. } added.
327.82,5=prin. }

Ans. $347.49,45=amount.

7. What is the interest of $987.25 cents for one year, at $4 per cent. per annum? Ans. $39.49 cents.

8. What is the interest of $987.25 cents for one year, at $5 per cent. per annum? Ans. $49.36cts. 2½mills.

9. What is the interest of $987.25 cents for one year, at $6 per cent. per annum? Ans. $59.23cts. 5 mills.

10. What is the interest of $1121.32cts. for one year, at $7 per cent. per annum? Ans. $78.49cts. 2m.+4 tenths of a mill.

Case 2.

When there is a fraction, as ¼, ½, ¾, &c. given with the dollars in the rate per cent.

Rule.

Multiply the principal by the dollars of the rate per cent., as in the foregoing case; then add ¼, ½, or ¾, &c. of the said principal to the product, and point off the two right hand figures of the dollars in that sum; the result will be the answer in dollars, cents, &c.

Examples.

1. What is the interest on $956 75 cents for 1 year, at $5¾ per cent. per annum?

[Examine the operations carefully.]

```
 p.c.   $   c.
 ½=½)956.75
          5
    -------
    4783.75
¼=½) 478.375
     239.1875
    ---------
```

Ans. $55.01,3125 decimals.

2. What is the interest on $1269 84 cents for 1 year, at $6⅔ per cent. per annum?

```
    $   c.
⅓)1269.84
        6
  -------
  7619.04
   423.28=⅓
   423.28=⅓
  -------
```

Ans. $84.65,60

3. What is the interest of \$823 for one year, at \$$4\frac{1}{2}$ per cent. per annum? Ans. \$34.97cts. 7m.+5.

4. What is the interest of \$321 for one year, at \$$5\frac{1}{2}$ per cent. per annum? Ans. \$17.65cts. 5m.

5. What is the interest of \$497 for one year, at \$$6\frac{1}{4}$ per cent. per annum? Ans. \$31.06$\frac{1}{4}$cts.

Case 3.

To find the interest of any given sum for several years.

Rule.

Find the interest of the given sum for a year by one of the preceding cases, then multiply the said interest for a single year by the number of years given in the question, and that product will be the answer required.

Examples.

1. What is the interest of \$789 $16\frac{2}{3}$ cents for 3 years, at \$6 per cent. per annum?

```
      $   cts.
     789.16⅔
           6
    ————————
Interest $47.35,00 for 1 year.
            3
    ————————
Ans. $142.05cts. for 3 years.
```

2. What will a bond for \$567 3125 decimals of a dollar amount to in 8yrs., at \$6 per cent. p. a.?

```
           $567.3125 decimals.
                   6
          ——————————
Interest  $34.03,8750  for 1yr.
                   8
          ——————————
Interest  $272.31,000  for 8yrs.
Principal  567.31,25   added.
          ——————————
Amount    $839.62,25   required.
```

3. What is the interest due on a bond of \$1251.75 cents for 4 years, at \$6 per cent. per annum? Ans. \$300.42 cents.

4. What is the amount of a note for \$11.44 cents, which has been due 5 years, computing interest at \$6 per cent. per annum? Ans. \$14.87cts. 2m.

5. Suppose a bond for \$573.25 cents to be at interest for 3 years, what will it amount to in that time at \$6 per cent. per annum? Ans. \$676.43cts. 5m.

Case 4.

When the given time is any number of months, weeks, or days, more or less than a year, or any number of years.

Common Rule.

1. Find the interest of the given sum for one year, by case the first, &c.

2. If the given time be less than one year, take such a part or parts of the yearly interest as the time proposed is of a year, and the sum of those *parts* (if more than one) will be the answer *required*.

3. If the given time be more than one year, but less than two, take parts of the yearly interest with the time that is more than a year, and add them to the said yearly interest, and that sum will be the answer.

4. If the given time be more than two years, multiply the interest of one year by the number of years given, then add such a part or parts of the said yearly interest to the product, as the months, &c. are of a year, and that sum will be the answer sought.

OR, BY THE SINGLE RULE OF THREE DIRECT.

As the months, weeks, or days in a year,
Are to the interest of the given sum for one year;
So are the months, weeks, or days in the given time,
To the interest required.

EXAMPLES.

1. What is the interest of \$286 for 11 months and 28 days, at \$6 per cent. per annum? Ans. \$17.06cts. 4m.+.65, &c.

Examine the work.

	\$286		
	6		
6mo. $=\frac{1}{2}$	\$17.16=the interest for one year.		
4 $=\frac{1}{3}$	8.58	=interest for	6 months.
1 $=\frac{1}{4}$	5.72	=	4
15days$=\frac{1}{2}$	1.43	=	1
10 $=\frac{1}{3}$	71,5m.	=	15 days.
2 $=\frac{1}{5}$	47,666+	=	10
1 $=\frac{1}{2}$	9,533+	=	2
	4,766+	=	1

Ans. \$17.06,465= int. for 11mo. 28 days.

2. What is the amount of a bond for \$365 for 1 year, 8mo. and 22 days, at \$6 per cent. per annum? Ans. \$402.83cts. 8m.+&c.

	\$365		
	6		
6mo. $=\frac{1}{2}$	\$21.90	=interest for	1 year.
2 $=\frac{1}{3}$	10.95	=	6mo.
20days$=\frac{1}{3}$	3.65	=	2mo.
2 $=\frac{1}{10}$	1.21,666+	=	20 days.
	.12,166+	=	2 days.

\$37.83,832=int. for 1yr. 8mo. 22d.
365.00=principal added.

Ans. 402.83,8=amount required.

3. What is the interest of $3000 for 8 months, at $6 per cent. per annum? Ans. $120.

4. What is the interest of $7500 for 4 months, at $7 per cent. per annum? Ans. $175.

5. What is the interest of $422 for 16 weeks at $4½ per cent. per annum? Ans. $5.84 cents 3 mills+.

6. What is the interest of $523.50 cents for 4 years and 3 mo. at $5¼ per cent. per annum? Ans. $116.80cts. 5m.+9375 decimals.

7. What is the amount of a bond for $256 which has been due 1 year and 6 months, reckoning interest at $5¾ per cent. per annum? Ans. $278.08cts.

8. What sum will $312.50cts. amount to in 4 years and 9 mo., calculating interest at $6¼ per cent. per annum?
Ans. $405.27cts. 3 mills+4375 decimals.

9. What sum will discharge a bond of $221.75cts. which has been due 3 years, 7mo. and 6 days, computing lawful interest?
Ans. $269.64cts. 8 mills.

Case 5.

When the time is months only.

Rule.

1. Multiply the whole principal by half the time or half the principal by the whole time, and point off the two right hand figures of the dollars in the product as before, and the result by either operation will be the interest required, at 6 per cent. per annum.

2. Multiply the whole principal by the whole time and divide the product by 2—then point off the two right hand figures of the dollars in the quotient, and the result will be the same answer as by either of the above operations.

3. To find the interest of any sum, at any other rate per cent. Take aliquot parts of the interest at 6 per cent. and add or subtract as the case may require.

Examples.

1. What is the interest of $1284 for 4 months, at $6 per cent. per annum?

1st. method	2nd. method.	3d. method.
1284	2)1284	1284
2	642	4
	4	2)5136
Ans. $25.68cts.	Ans. $25.68cts.	Ans. $25.68

2. What is the int. of $3000 for 8mo. at $6 p.c.p.a.? Ans. $120.00c.
3. What is the int. of $375 for 9mo. at $6 p.c.p.a.? Ans. $16.87½c.
4. What is the int. of $287 for 11mo. at $6 p.c.p.a.? Ans. $15.78½c.
5. What is the int. of $630 for 18mo. at $6 p.c.p.a.? Ans. $56.70c.
6. *What is the int. of* $7342 for 16mo. at $6 p.c.p.a.? Ans. $587.36c.

Case 6.

When the time is days only.

Rule.

1. When the principal is any number of dollars, multiply it by the given number of days and divide the product by 6, and the quotient will be the interest of the given sum in mills, at 6 per cent. per annum—or, you may multiply the principal by one sixth part of the days, and the result will be the same thing.

2. The interest of any number of dollars for 60 days, at 6 per cent. per annum, is exactly the same number of cents. For any number of days, more or less than 60, take aliquot parts of the interest for 60 days, and add or subtract as the case may require.

Examples.

1. What is the int. of \$1542 for 90 days, at \$6 p. c. p. a.?

\$1542=principal.
90
6)138780
Ans. 23130 mills.

\$1542=principal.
15=$\frac{1}{6}$ part of 90 days.
Ans. 23130 mills, as before.
Consequently the ans. is \$23. 13 cents both ways.

The same question by the second rule.

days. \$ cts.
30=$\frac{1}{2}$)15.42=int. for 60 days.
7.71= 30
Ans. \$23.13c. as before for 90ds.

2. What is the int. of \$3942 for 50 days, at \$6 p. c. p. a.?
Ans. \$32.85cts.

Examine the work carefully.

\$3942
50
6)197100
32850ms.

3)\$3942
$8\frac{1}{3}$
31536
.1314
32850ms.

days. \$
10=$\frac{1}{6}$)39.42
6.57
\$32.85 cents.

The student may now proceed by the easiest and shortest method.

	\$		\$ cts.
3. What is the interest of	3084	for 54 days?	Ans. 27.75.6m.
4. What is the interest of	1425	for 30 days?	Ans. 7.12½.
5. What is the interest of	100	for 20 days?	Ans. 0.33⅓
6. What is the interest of	17	for 105 days?	Ans. 0.29¾.
7. What is the interest of	1	for 15 days?	Ans. 0.00 2½m.
8. What is the interest of	44	for 80 days?	Ans. 0.58⅔.
9. What is the interest of	1000	for 5 days?	Ans. 0.83⅓.
10. What is the interest of	2000	for 15 days?	Ans. 5.00.

CASE 7.

To find the sum due on a bond or note which has one or more partial payments entered on the back of it.

RULE.

1. Find the interest on the principal sum, from the day the bond, &c. became due, to the day on which the first payment was made, then add the said interest to the principal, and from that sum subtract the amount of the first payment, and the balance will be a new principal.

2. Find the interest on the new principal, from the day the first payment was made to the day on which the second payment was made—then add the said new principal and its interest together, and from that sum subtract the amount of the second payment and the residue will be another new principal, with which proceed as before directed, and so on till you get through all the payments endorsed on the back of the bond, &c. But never calculate the interest on a greater sum than the residue of the principal after the interest is extinguished.

3. Find the amount of the balance due from the time the last payment was made to the day on which the bond is to be paid in full, and that amount will be the sum required to discharge the whole debt.

4. To find the length of time from the day the bond or note became due to the day on which the first payment was made, and from one payment to another, the student must refer to a rule given in Subtraction of Time.

EXAMPLES.

1. A man by his note dated January 1st, 1828, promised to pay another one the sum of $100, with interest from the date, at $6 per cent. per annum, on which the following payments are endorsed—

1. Received on the 1st July, 1828, the sum of	$10	in part of within.
2. 1st Jan. 1829,	$20	
3. 1st May, 1829,	$30	
4. 1st Sept. 1829,	$40	

What sum will discharge the above note on the 1st day of January, 1830? Ans. $8.41cts. 6m.+

To interest on $100 from January 1st, 1828, -	$100
Until the 1st July following, (6 months,) -	3
Amount, - - - - - - - - -	103
July 1st, 1828, received in part, - - -	10
The balance due, or new principal, - -	=93
To interest on $93 from the 1st of July, 1828, until the 1st January, 1829, (6 months,)	2.79cts.

The second amount, - - - - - -	95.79
January 1st, 1829, received in part, - -	20.00
The balance due, &c. - - - - - -	75.79
To interest on $75.79 cents from January 1st, 1829, until the 1st of May following, (4mo.)	=1.51,5m.+
The third amount, - - - - - -	77.30,5
May 1st, 1829, received in part, - - -	30.00,0
The balance due, &c. - - - - - -	47.30,5
To interest on $47.30cts. from May 1st, 1829, until the 1st of September following (4mo.)	=00.94,6
The fourth amount, - - - - - -	48.25,1
September 1st 1829, received in part, - -	40.00,0
	8.25,1
To interest on $8.25cts. from September 1st, 1829, until the 1st January, 1830, (4mo.)	0.16,5

Ans. $8.41,6m.

2. A holds B's bond for $500, dated on the 15th March, 1828, which is not payable till the expiration of one year after the date, on which the following credits are entered, viz.—Received, November 15th, 1829, $100; May 15th, 1830, $200; July 15th, 1830, $220. What sum will discharge the said bond on the 15th of January, 1831, allowing interest at $6 per cent. per annum?

Ans. $15.37cts. 3m.+7, &c.

Case 8.

To find the time in which any sum of money will double itself, at any given rate per cent. per annum, and also to find the said rate.

Rule.

1. Divide 100 by the given rate per cent. and the quotient will be the number of years, &c. in which any sum of money will double itself at the said given rate per cent.

2. If the time be required in months or days, multiply the months or days in a year by 100, and divide the product by the rate, and the quotient will be the answer.

3. Divide 100 by the number of years in which any sum will double itself, and the quotient will be the rate per cent.

4. Multiply the months or days in a year by 100, and divide the product by the months or days in which any sum will double itself, and the quotient will be the rate per cent.

Examples.

1. In how many days will any sum of money double itself, at 6 per cent. per annum?

360 days.
100

6)36000

mo. y. m.

Ans. 6000 days=200=16 8

2. If a sum of money double itself in 6000 days, what is the rate per cent. per annum?

3600 days.
100

6,000)36,0000

Ans. 6 per cent. per annum.

3. In what time will a sum of money double itself at $4 per cent. per annum? Ans. 300 months, or 25 years.

4. In what time will a sum of money double itself at $5 per cent. per annum? Ans. 240 months, or 20 years.

Case 9.

To compute the interest on accounts current, &c.

Rule.

1. Multiply the principal by the number of days it bears interest, and set the product in a column drawn for the purpose; then subtract the first payment from the said principal, and multiply the residue by the number of days between the first and second payments; set the product under the former one, and proceed, in the same manner, to multiply each respective balance by the number of days it bears interest, and divide the sum of the products by 6, and the quotient will be the interest required in mills, at 6 per cent. per annum.

2. For any other rate per cent. take aliquot parts of the interest at 6 per cent. and add or subtract, as the case may require.

Examples.

1. On the 1st of January, 1829, I lent my friend Timothy Trusty $100; on the 14th, I lent him $110 more, and on the 20th he paid me $162; February 3d, I lent him $95; on the 10th, he paid me $90; on the 16th, I lent him $186, and on the 26th, he paid me $70; March 1st, I lent him $250; on the 3d, he paid me $270, and on the 13th, $143 more, and on the 20th, we made a final liquidation of our accounts. What was the amount of principal and interest due on the said account at the day of settlement?

Ans. $7.63 cents 6 mills. Examine the work very carefully.

1829.			$		days.	products.
January	1st,	Lent,	100	on interest for	13	=1300
	14th,	Lent,	110			
			210		6	=1260
	20th,	Received,	162			
		Due,	48		14	= 672
February	3d,	Lent,	95			
			143		7	=1001
	10,	Received,	90			
			53		6	= 318
	16,	Lent,	186			
			239		10	=2390
	26,	Received,	70			
			169		3	= 507
March	1,	Lent,	250			
			419		2	= 838
	3,	Received,	270			
			149		10	=1490
	13,	Received,	143			
	20,	the time of adjustment	=6	due.	7	= 42

Interest due at 6 per cent.=1.63cts. 6 mills. 6)9818

Ans. The whole amount=$7.63cts. 6 mills. 1636 mills.

2. I have given my friend Peter Truepay, a cash credit for 1000 dollars, in consequence of which, on the 12th of May, I paid his bill for 250 dollars; on the 27th, I paid his draft for 280 dollars; June 1st, he gave me a check on the Virginia Bank for 290 dollars; July 17th, he paid me 70 dollars; August the 20th, he drew on me for 750 dollars; on the 31st, he paid me $500; September 15th, he drew on me for 135 dollars, and on the 3d of October for 175 dollars more; on the 29th, he paid me 250 dollars, and on the 3d of November, 125 dollars more; on the 12th, he drew on me for 375 dollars, and on the 18th, for 125 dollars more; January 1st, he paid me 290 dollars; on the 20th, 210 dollars more, and on the 1st of March following he demands a settlement. What sum will remain due to me on that day, computing the interest according to law?

Ans. $378.52cts. 3$\frac{1}{3}$ mills

			$	days.	products.
May	12	Paid his bill for	250	15	= 3750
	27	Paid his draft for	280		
			530	5	= 2650
June	1	Received in part	290		
			240	46	=11040
July	17	Received in part	70		
			170	34	= 5780
August	20	Paid his draft for	750		
			920	11	=10120
	31	Received in part	500		
			420	15	= 6300
September	15	Paid his draft for	135		
			555	18	= 9990
October	3	Paid his draft for	175		
			730	26	=18980
	29	Received in part	250		
			480	5	= 2400
November	3	Received in part	125		
			355	9	= 3195
	12	Paid his draft for	375		
			730	6	= 4380
	18	Paid his draft for	125		
			855	44	=37620
January	1	Received in part	290		
			565	19	=10735
	20	Received in part	210		
March	1	Principal due =	355	40	=14200

The principal due=$355.00 6)141140

23.52cts. $3\frac{1}{3}$ mills. 23523$\frac{1}{3}$m.

Ans. The amount=$378.52cts. $3\frac{1}{3}$ mills.

CASE 10.

When the principal is given in English, or Sterling money.

RULE.

1. *Multiply the principal by the rate per cent.—then divide th*

product by 100, and the quotient will be the interest of the given sum for one year.

2. In other respects proceed by the foregoing rules.

Examples.

1. What will 256£ 15s. 9d. amount to in 3 years, 9 mo. 18 days, computing the interest at 5¾£ per cent. per annum?

Ans. 312£,17s.10d. ¾qrs.+

p.c.	£	s.	d.
½=½)	256	15	9
			5¾
	1283	18	9
¼=½)	128	7	10½
	64	3	11¼
	£14.76	10	6¾
	20		
	15.30		
	12		
	3.66		
	4		
	2.67		

mo. yr.	£	s.	d.	
6=½)	14	15	3½+.67 rem.	=int. for 1 year.
			3	=number of years.
	44	5	11.+.01	=int. for 3 years.
3=½)	7	7	7¾+.335	=int. for 6 months.
18d.=⅕)	3	13	9¾+.6675	=int. for 3 months.
	0	14	9.+.7335	=int. for 18 days.
	56	2	1¾+.7460	=whole int.
	256	15	9.	=principal added.
Ans.	£312	17	10¾qrs.	

2. What is the amount of £400 for 2 years, at £5½ p. c. p. a.? Ans. £444.

3. What is the amount of £57 17s. 8d. for 3 months, at £6 p. c. per annum? Ans. £58 15s. 0d. ¼+

4. What will £300 amount to in 5 years 10mo. at £4¾ p. c. p. a.? Ans. £383 2s. 6d.

Application.

1. M holds a bond on P for $268.54cts. dated on the 12th day of January, 1828 (it being leap year) and payable on the 6th day of September following, with lawful interest from the date; please to inform me how much money will discharge the debt?

Ans. $279.19cts.2ms.+

		days.
January	has	31
Subtract		12
Remainder		19
February		29
March		31
April		30
May		31
June		30
July		31
August		31
September		6
The days	=	238

$ c.
268.54
238
2148 32
8056 2
53708
6)63912.52
10.652+
268.54,
Ans. $279.19, 2m.+.

2. W owes me \$60, with lawful interest from 16th May, 1829, subject to a credit of \$54.75cts. on the 31st of August following; what sum will pay up the residue on the 31st of January, 1830?

Ans. \$6.48cts.+

3. On the 20th February, 1828, X gave me his bond for \$164, payable at the end of 4 months; on the 30th of September following he paid me \$90, and on the 12th of March, 1830, he paid me \$65 more; what sum will discharge the residue on the 1st day of January, 1831? Ans. \$19.45cts.+

INSURANCE, COMMISSION, AND BROKERAGE.

A commission is an allowance of so much per cent. to a person called a commissioner, factor, correspondent, or broker, for assisting merchants and others in buying and selling goods, and receiving and paying out money.

Rule.

1. When the commission is one or more than one per cent., find the interest of the given sum, without respect to time, by the first or second case of Simple Interest, and the result will be the amount of the commission required.

2. When the commission is less than one per cent., take such a part or parts of the interest at one per cent. as the given rate per cent. is of a pound or dollar.

3. When the commission is to be deducted before the money is laid out, say—As 100, with the rate per cent. added to it,
Is to the said rate per cent.,
So is the given sum
To the commission required.

Examples.

1. My commission merchant in Richmond has sold 184 barrels of my flour at \$10.50 cents per barrel; what sum must he transmit to me, after deducting his commission of \$2½ per cent.?

```
50c.=½)184 barrels.
       10
     ----
     1840
       92
     ----
  ½)$1932=amount of the flour.
       2½=rate per cent.
     ----
     3864
      966
     ----
   $48.30cts.=the commission, and
```

\$1932—\$48.30cts.=\$1883.70cts., the sum to be transmitted.

2. What will be the annual premium for insurance on a house against loss from fire, valued at $3500, the rate being $$\frac{3}{4}$ per c.?

$$
\begin{array}{c|c|l}
\frac{1}{2} & \frac{1}{2} & \$35.00 = \text{the int. at 1 p. c.} \\
\hline
\frac{1}{4} & \frac{1}{2} & 17.50 \\
 & & 8.75 \\
\hline
\end{array}
$$

Ans. $26.25cts.=annual prem.

3. I have sent $1008 to my factor at Philadelphia for him to lay out in merchandise, after deducting his commission of $5 per cent.; what sum will remain to be laid out?

$$
\begin{array}{l}
\$105 : \$5 :: \$1008 \\
5 \\
105\overline{)5040}(48\$ = \text{his commission, and}
\end{array}
$$

$1008−$48=$960 the sum to be laid out.

4. My correspondent in Liverpool has purchased goods for me to the amount of 394£ 15s. 8d. sterling; what will his commission come to at £2½ per cent.? Ans. 9£ 17s. 10½d.+8.

5. An auctioneer has sold goods for me amounting to $4968, and demands $½ per cent. for his trouble; what sum must I pay him? Ans. $24.84cts.

6. What will be the premium for insuring a ship and cargo from Boston to Amsterdam, valued at $37800, the rate of insurance being $4½ per cent.? Ans. $1701.

7. What is the insurance of an East-India ship and cargo, valued at 14813£ 15s. sterling, at 15¾£ per cent.? Ans. 2333£ 3s. 3¾d.

8. If a broker sell property amounting to $2864.46cts., what sum may he demand for brokerage at $1.50cts. per cent.? Ans. $42.96cts. 6m.+9.

9. What sum may a broker demand for selling merchandise to the amount of $6700, at 25cts. per cent.? Ans. $16.75cts.

COMPOUND INTEREST.

Compound Interest arises by calculating interest upon interest as it becomes due at the end of each year.

Rule.

1. Find the amount of the given principal for the first year by Simple Interest, and that amount will be the principal for the second year.

2. Find the amount of the principal for the second year, and this amount will be the principal for the third year.

3. Proceed on as above directed till you have found the amount of the principal for the last year given in the question, from which *subtract* the *given* principal, and the remainder will be the *compound interest* required.

Examples.

1. What will \$500 amount to in 3 years, at 6 per cent. per annum, compound interest?

p.c.	\$
$5=\frac{1}{20}$	500=the given principal.
$1=\frac{1}{5}$	25
	5
$5=\frac{1}{20}$	530=the amt. for 1st yr.
$1=\frac{1}{5}$	26.50
	5.30
$5=\frac{1}{20}$	561.80=amt. for 2d yr.
$1=\frac{1}{5}$	28.09
	5.61,8m.
Ans.	\$595.50,8m.=amt. 3d yr.

2. What is the compound interest of \$500 for 4 years, at \$6 per cent. per annum?
Ans. \$131.23cts. 8m.+48.

3. What is the compound interest of \$3475 for 3 years, at \$6 per cent. per annum?
Ans. \$663.78cts.+06.

4. What is the compound interest of \$1152 for 6 years, at \$5 per cent. per annum?
Ans. \$391.79cts.+0178.

EQUATION OF PAYMENTS.

Equation is a method of reducing several stated times, at which money is payable, to one mean or equated time.

Common Rule.

Multiply each payment by the time before it becomes due, then add the several products together, and divide the total sum by the whole debt; the quotient thence arising will be the equated time required.

Proof.

The interest of the whole debt payable at the equated time, at any given rate per cent., will be equal to the interest of the several payments for their respective times at the same rate.

Examples.

1. A owes B \$2400, of which 480 are to be paid presently, 960 at the end of 5 months, and the rest at the end of 10 months, but they agree to make one payment of the whole, and wish to know that time.

\$		*Products.*
480 ×	0 =	0000
960 ×	5 =	4800
960 ×	10 =	9600
2400		)14400(6 months. Answer.
		14400

2. P owes Q \$420, which will be due 6 months hence, but P is willing to pay \$60 now, provided he can have the rest forborne a longer time; it is agreed on—the time of forbearance is therefore required.

\$420
60
———
\$ m.
420 : 6 :: 360
6
———
360)2520(7 months. Ans.
2520

3. G is indebted to M in a certain sum, which is to be discharged at 4 separate payments, that is $\frac{1}{4}$ at 2mo. $\frac{1}{4}$ at 4mo. $\frac{1}{4}$ at 6mo. and $\frac{1}{4}$ at 8mo. but they agree to make one payment of the whole; the equated time is therefore required. Ans. 5 months.

4. N bought a quantity of goods on credit, and agreed to pay $\frac{1}{3}$ of the debt every 3mo. until the whole should be discharged, but he afterwards consented to pay it all at once; the equated time is therefore required. Ans. 6 months.

5. W owes Z a certain sum of money, $\frac{1}{2}$ of which is to be paid in hand, $\frac{1}{4}$ at the expiration of 4mo. and the residue at the termination of 8 mo.; I demand the equated time for the payment of the whole debt. Ans. 3 months.

6. X gave his bond to Y for \$600, payable at the termination of 8mo. but he is willing to pay Y \$200 presently, provided he can have the residue forborne a longer time, to which Y consents; the time of forbearance is therefore demanded. Ans. 12 months.

REBATE OR DISCOUNT.

Discount is an allowance made for the payment of any sum of money before it becomes due, and is the true difference between the original debt and its present worth in cash.

The present worth of any sum, or debt, due some time hence, is so much present money, which, being put out to interest for that time at the given rate rate per cent. per annum, would amount precisely to the original debt.

Rule.

1. Say—As the months or days in a year
   Are to the given rate per cent.;
   So is the time proposed
   To its interest at that rate.
2. Add the said interest to \$100, or £, and call that sum the amount.
3. Say—As the above amount
   Is to \$100, or pounds;
   So is the given sum or debt,
   To the present worth required.
4. Subtract the present worth from the given sum, and the remainder will be the discount required.

Proof.

Find the amount of the present worth at the given rate per cent. per annum, and for the time proposed, which will be equal to the original debt, if the work is right.

N. B. It is believed by many that the interest of the debt for the *time before it becomes* due, at the stipulated rate per cent. per an-

num, is the proper sum to be discounted for the prompt payment thereof. But the following statement and the two first examples will clearly demonstrate the contrary.

The Statement.

A owes B the sum of \$560, payable 2 years hence, for the prompt payment of which, B discounted the interest for the time at \$6 per cent. per annum, and therefore received \$492.80cts. which he immediately lent to C for the same time, and at the same rate per cent. at the termination of which C paid him \$551.93cts. 6 mills, which is \$8.06cts. 4 mills less than he would have received from A if the money had remained in his hands till it became due. But, if B had allowed A the true discount only, he would have received \$500, which, being put out to interest for 2 years at \$6 per cent. per annum, will amount precisely to \$560, the original debt.

Examples.

1. D owes E the sum of \$595.20cts. payable at the expiration of 3 years, but he is willing to make prompt payment if E will discount the interest for 3 years, at \$8 per cent. per annum, to which E has consented. How much did he lose by allowing the interest instead of the true discount? Examine the work carefully.

```
    yr.  $   yrs.
As  1 :  8 :: 3
         3
        ---
        24=the interest of $100 for 3 years.
       100
       ---
      $124=the amount of $100 for 3 years.

              $     $      $ cts.     $ cts.
Now I say—As 124 : 100 :: 595.20     595.20=given sum.
                             100
                          --------
                      124)59520.00(  480.00=p. worth.
                          496        -------
                          ---       $115.20=discount.
                           992
     $ c.                  992
   595.20                  ---
        8                   .000
   -------
 $47.61,60=the yearly interest.
        3
 ---------
$142.84,80=the interest for 3 years which E discounted.
 115.20,00=the true discount.
 ---------
Ans. $27.64, 8 mills.=the sum lost by Mr. E.
```

2. What is the difference between the interest of \$1200 for 12 years, at \$5 per cent. per annum, and the true discount of the same sum for the said time and rate p. c. p. a.? Ans. \$270.

```
          yr.  $   yrs.
1.—As     1  : 5 :: 12
                 12
                 --
                 60=the interest of $100 for 12 years.
                100
                ---
                160=the amount of $100 for 12 years.
        $      $        $  $1200=the given sum.
2.—As  160 : 100 :: 1200 .. 750=the present worth.
                            ----
                            $450=the true discount.
           $
          1200=the principal.
             5
          -----
         60.00=the interest of 1200 for 1 year.
            12
        -------
       $720.00=the whole interest for 12 years.
        450.00=the true discount.
        ------
         $270=the difference required.
```

3. What is the present worth of \$2652, one half of which is due at the end of 4 months, and the other half at the end of 8 months, discounting at \$6 per cent. per annum? Ans. \$2575.

```
mo.   $
4=⅓)6=int. of $100 for 1 year.
    -
    2=int. of $100 for 4 mo.
  100
  ---
As 102 : 100 :: 1326 .. 1300=p.w.

mo.   $
6=½)6=yearly interest.
2=⅓)3
     1
     -
     4=int. of $100 for 8 mo.
   100                  $       $
   ---
   104 : 100 :: 1326 .. 1275=p.w.
```

And $ \overset{\$}{1275}+\overset{\$}{1300}=\overset{\$}{2575}$ the whole present worth required.

4. What sum must I discount for the prompt payment of \$2119 50 cents, which is due 2 years hence, at \$5.50 cents per cent. per annum? Ans. \$210.04cts. 1 m.+5100 rem.

5. How much ready money must I have for a bond of \$810, due at the end of 3 months, discounting at \$5 per cent. p. a.? Ans. \$800

6. What is the present worth of \$312, one half of which is due in 3 months, and the other half in 6 months, allowing discount at \$6 per cent. per annum? Ans. \$305.15cts.+.

7. *I have sold* a tract of land for \$2400, one third of which is to

be paid at the end of 1 year, and the residue in two equal annual payments thereafter; what is the present worth of each payment and of the whole debt, allowing discount at $10 per cent. per annum, for prompt payment? Ans. The first payment is $727.27 cents+; the second $666.66⅔cts.; the last $615.38cts.+; and the whole present worth is $2009.31⅔cts.+

8. A man having incautiously sold property on a credit of two years, amounting to $1200, and being suspicious that the obligor will fail before the debt becomes due, is willing to allow a discount of $100 per cent. per annum, for the prompt payment of the residue; what sum ought he to receive? Ans. $400.

BARTER.

Barter is the exchanging of one commodity for another, and teaches merchants and others to proportion the value of their respective commodities so that neither of the parties concerned may sustain any loss.

Rule.

Find the value in money of that commodity whose quantity is given, by the shortest method; then find what quantity of the other commodity may be bought with the said value, at the rate proposed.

Examples.

1. What quantity of sugar, at 12½ cents per lb. must be given in barter for 1200lbs. of rice, at 6¼ cents per lb.?

$$
\begin{array}{rr}
\tfrac{1}{4}) & 1200\text{lbs.} \\
 & 6\tfrac{1}{4} \\
\hline
 & 7200 \\
 & 300 \\
\hline
\end{array}
$$

$75.00cts.=the value of the rice.

$$
\begin{array}{lcccr}
 & \textit{cts.} & & \textit{lb.} & \textit{\$ cts.} \\
\text{As} & 12\tfrac{1}{2} & : & 1 & :: 75.00 \\
 & 2 & & & 2 \\
\hline
 & 25 & & &)150.00 \\
\hline
 & & & \text{Ans.} & 600\text{lbs.}
\end{array}
$$

2. How much tea, at 120cts. per lb. must be bartered for 600 lbs. of coffee, at 20cts. per lb.? Ans. 100lbs.

3. How much wheat, at 125 cents per bushel, must be bartered for 50 bushels of rye, at 70cts. per bushel? Ans. 28 bushels.

4. How much sugar, at 10cts. per lb., must be given in barter for 20cwt. of tobacco, at $10 per cwt.? Ans. 2000lbs.

5. How much sherry wine, at 87½ cents per gallon, must be given for 750gals. of Lisbon wine, at 37½cts. per gal.? Ans. 321³⁄₇gals.

6. How much rye, at 70 cents per bushel, will countervail 400 bushels of corn, at 87½ cents per bushel? Ans. 500 bushels.

7. How much Madeira wine, at 162½ cents per gallon, must I receive for 325 bushels of corn, at 40cts. per bush.? Ans. 80gals.

8. A has linen at 40 cents per yard, ready money, but in barter he will have 50 cents per yard; B has broadcloth at $4 cash per

yard. At what price per yard must B rate his cloth to make it equivalent to A's bartering price; and how many yards of linen must A give B for 6 yards of his cloth? Ans. B's cloth must be rated at $5 per yard, and A must give him 60 yards of linen for 6 yards of his cloth.

```
cts.  cts.    cts.
40  : 50  ::  400
              400
          -------
         40)20000
          -------
Ans. $5.00=B's barter-
           ing price.
```

```
                          6 yards.
                          5 dollars.
cts.      yd.            ----
50   :    1      ::       30
          30
       -------
       50)30.00
       -------
Ans.   60 yards of linen.
```

9. C has corn at $62\frac{1}{2}$ cents per bushel, ready money, but in barter he will have 75 cents per bushel; D has wheat worth $87\frac{1}{2}$ cents per bushel in cash. At what price per bushel must D rate his wheat to make it equivalent to C's bartering price? Ans. $1.05c.

10. D has wheat at $1.25 cents per bushel, ready money, but in barter he will have $1.50 cents per bushel; E has cotton at 20cts. per lb. ready money. What price must the cotton be rated at in barter, and how much must E give D for 100 bushels of wheat? Ans. The cotton must be 24cts per lb., and E must give D 625lbs. for 100 bushels of wheat.

11. H had 41cwt. of iron, at $6 per cwt., for which I gave him $146 in money, and the rest in pork, at 8 cents per pound. How much pork did H receive from me? Ans. 1250lbs.

12. Y had 608 yards of cloth, at $2.33⅓cts. per yard, for which Z gave him $418.66⅔cts. in money and 35cwt. 2qrs. 24lbs. of beeswax; what was the beeswax rated at by the pound? Ans. 25cts.

```
                      33⅓=⅓)608 yards.
                              2 dollars.
                          ------
                          1216
                           202.66⅔
                          ------
                        $1418.66⅔cts.=the price of the cloth.
                           418.66⅔ paid in money.
                          ------
cwt. qrs. lbs.
35    2   24      =4000)1000.00  cts.
                          ------
                  Ans.       25 cents.
```

13. V gave 630 bushels of oats, at 33⅓cts. per bushel, to W for 42 yards of cloth; what was the cloth by the yard? Ans. $5.

14. B had a pipe of Madeira wine worth $1.87½cts. per gal., for which W gave him a pipe of French brandy and $15.75cts. in money. *What was the brandy* valued at by the gallon? Ans. $1.75cts.

15. M bartered 40 yards of linen, at \$1.18$\frac{3}{4}$cts. per yard, with N for 28$\frac{1}{2}$lbs. of tea, at \$1.62$\frac{1}{2}$cts. per lb. Which of them must pay the balance, and how much? Ans. N must pay M \$1.18$\frac{3}{4}$cts.

$$\begin{array}{lll}
 & \$\ \ cts. & \\
\frac{1}{2}) & 1.62\frac{1}{2} & \\
 & \quad\ 4 & \\
\hline
 & 6.50 & \\
 & \quad\ 7 & \\
\hline
 & 45.50 & \\
 & \quad 81\frac{1}{4} & \\
\hline
 & \$46.31\frac{1}{4} &
\end{array}
\qquad
\begin{array}{rl}
cts. & \\
12\frac{1}{2}=\frac{1}{8}) & 40 \text{ yards.} \\
6\frac{1}{4}=\frac{1}{2}) & 5 \\
 & 2.50 \\
\hline
 & 47.50 = \text{the value of M's linen.} \\
 & 46.31\frac{1}{4} = \text{the value of N's tea.} \\
\hline
\end{array}$$

Ans. \$1.18$\frac{3}{4}$cts. to be paid in money.

16. P bartered 20cwt. of cheese, at 12$\frac{1}{2}$cts. per pound, with Q for 70yds of broadcloth at \$4 a yard. Which of them was in debt, and how much? Ans. Neither of them.

17. T bartered 56 bushels of wheat, at \$1.37$\frac{1}{2}$cts. a bushel, with S for 144 bushels of corn at 50cts. a bushel. Which was indebted, and how much? Ans. S was \$5 in debt to T.

18. L has linen at 70 cents a yard, in ready money, but in bartering he will have 75 cents a yard, for a quantity of which B gave him broadcloth at \$6 a yard that cost \$5.41$\frac{1}{3}$ cents a yard in cash. Which of them had the advantage in bartering, and how much linen must L give B for 50 yards of his cloth? Perform the work in full and study it carefully.

1. As $\overset{yd.}{1} : \overset{\$}{6} :: \overset{yds.}{50} \cdot\cdot \overset{\$}{300}$=the value of B's 50 yards of cloth.

2. As $\overset{cts.}{75} : \overset{yd.}{1} :: \overset{\$}{300} \cdot\cdot \overset{yds.}{400}$ of linen that L must give to B.

3. As $\overset{cts.}{70} : \overset{cts.}{75} :: \overset{\$\ cts.}{5.41\frac{1}{3}} \cdot\cdot \overset{\$\ cts.}{5.80}$=the bartering price of B's cloth; consequently he had the advantage of L by 20cts. in the yard.

19. A has 100 yards of cotton shirting at 20 cents a yard, in ready money, but in barter he will have 25 cents a yard, which he exchanges with C for buttons at 12$\frac{1}{2}$ cents a dozen, that cost him only 10 cents a dozen in ready money. How many dozen of buttons must C give A for his linen, and which of them had the best bargain, and by how much?

1. As $\overset{yd.}{1} : \overset{cts.}{25} :: \overset{yds.}{100} \cdot\cdot \overset{\$}{25}$=the bartering price of A's shirting.

2. As $\overset{cts.}{12\frac{1}{2}} : \overset{doz.}{1} :: \overset{\$}{25} \cdot\cdot \overset{doz.}{200}$ of buttons.

3. As $\overset{yd.}{1} : \overset{cts.}{20} :: \overset{yds.}{100} \cdot\cdot \overset{\$}{20}$=the cash price of A's shirting.

4. As $\overset{doz.}{1} : \overset{cts.}{10} :: \overset{doz.}{200} \cdot\cdot \overset{\$}{20}$=the cash price of C's buttons; consequently they traded exactly even.

20. K had 74 sheep, at \$1.75cts. each, for which L gave him \$30 in money and the rest in corn at 50cts. per bushel. How much corn did K receive of L? Ans. 199 bushels.

LOSS AND GAIN.

Loss and Gain is a rule by which we may easily discover what sum is gained or lost in the buying and selling of goods and all other sorts of mercantile property. It likewise teaches us how to rise or fall in the price of property, so as to gain or lose any particular sum, by the integer, in the whole quantity, or so much per cent., that is by the hundred.

Case 1.

To find how much is gained or lost by the whole quantity.

Rule.

Find the difference between the buying and selling prices by the integer, then multiply that difference by the given number of integers, and the product will be the gain or loss, according to the tenor of the question. Or, find the price which the whole quantity cost, and the price for which it was sold, then subtract the least amount from the greatest, and the remainder will be the whole gain or loss required, as before.

Examples.

1. If I buy 144 yards of cotton shirting at 20cts. a yard, and sell it for 25cts. a yard, what sum do I gain in the whole?

cts.
25=the selling price.
20=the buying price.

5cts.=the gain per yd.
144 yards.

Ans. $7.20=whole gain.

cts. 25=¼)144yds.
$36

cts. 20=⅕)144yds.
28.80

28.80cts.

Ans. $7.20cts. by second rule.

2. If I buy deals at 20 cents apiece, and sell them at 17cts. apiece, what sum do I lose by 120 dozen?

20 cents.
17 cents.

deal.
As 1 : 3 :: 120 dozen.
12

1440
3 cents.

Ans. $43.20 cents.

3. Bought 18cwt. of tobacco at $9,50cts. per cwt., and retailed it out at 12½cts. per lb., what sum did I gain in the whole?

112lbs.=1cwt.
18

12½=⅛)2016 pounds in all.

$252 sold for.
$ cts.
9.50×18=171=the cost.

Ans. $81=whole gain.

4. If I buy a ton of iron for $120 dollars, and retail it out at 6¼ cents a pound, what is the clear gain? Ans. $20.

5. A tavern-keeper bought a barrel of French brandy containing 32 gallons, at $2.25cts. a gallon, and retailed it at 25 cents by the half pint; how much did he gain in all? Ans. $56.

6. A fruiterer bought 10 dozen of oranges at 2 for 16cts., and 10 dozen more at 3 for 21 cts., and sold them at 5 for 40cts.; whether did he gain or lose, and how much? Ans. He gained $1.20cts.

CASE 2.

To find how much is gained or lost by laying out a given sum.

RULE.

Find the difference between the buying and selling prices of one article by Subtraction: then say—

As the price that the said article cost,
Is to the difference found as above directed;
So is the sum given to be laid out
To the whole gain or loss required.

EXAMPLES.

1. If I lay out $1000 in hats, at $4 each, and sell them at$4.50 cents each, what do I gain by the speculation?

$ *cts.*
4.50=the selling price.
4.00=the buying price.

$4 : 50 :: $1000
1000

4)500.00

Ans. $125=the whole gain.

2. A merchant laid out $120 in coffee at 12½cts. per lb., and sold it at 18¾ cents per lb.; how much did he gain by the bargain? Ans. $60.

3. A tobacconist purchased a quantity of tobacco amounting to $254, at 10 cents per lb., which he sold at 8 cents per pound, in consequence of its being damaged; how much did he lose by the transaction. Ans. $50.80cts.

4. A grocer bought $80 worth of sugar at 8cts. per lb., which he sold out at 12½cts. per lb.; how much did he gain? Ans. $45.

CASE 3.

To find how much is gained or lost per cent.

RULE.

Find the difference between the buying and selling prices of one article or thing by Subtraction: then say—

As the price that the said article cost
Is to the said difference;
So is $100, or pounds,
To the whole gain or loss per cent.

EXAMPLES.

1. A tobacconist bought a quantity of tobacco at 15cts. per lb. and afterward sold it for 20 cents per lb.; how much did he gain per cent. by the speculation?

cts.
20=the selling price.
15=the buying price.

As 15 : 5 :: 100 dollars.
100

15)5.0000

Ans. 33.33\frac{1}{3}$cts.=the whole gain per cent.

2. If hats be bought for $5 apiece and afterward sold for $6 12$\frac{1}{2}$cts. apiece, what is the gain per cent? Ans. $22.50cts. p. c.

3. If I buy cloth at $2.50cts. a yard and sell it for $2.25cts. a yard, what will be the loss per cent? Ans. $10 per cent.

4. A grocer bought pepper at 12$\frac{1}{2}$cts. per lb. and sold it for 10$\frac{1}{2}$ cents per lb.; how much did he lose per cent.? Ans. $16 p. c.

5. If a yard of silk be purchased for $1.20cts. and sold for $1 50cts., what is the gain per ct.? Ans. $25 per cent.

6. If a trader gain 37$\frac{1}{2}$cts. on a dollar, how much will he gain per cent.? Ans. $37.50cts. per cent.

CASE 4.

When the given gain or loss is at a certain rate per cent., to find how much is gained or lost in the whole quantity.

RULE.

As 100 dollars, or pounds,
Is to the given gain or loss;
So is the price of the whole
To the whole gain or loss.

EXAMPLES.

1. If I buy 12cwt. 2qrs. 14lbs. of sugar at $10 per cwt. and sell it at $20 per cent. advance, what will be the neat profit on the whole quantity at that rate?

qrs. 2=$\frac{1}{2}$ | $ 10=price of 1cwt.
12cwt.

120
lbs. 14=$\frac{1}{4}$ | 5
1.25

$ 100 : $ 20 :: 126.25=whole cost.
20

1,00)2525,00
Ans. $25.25cts.

2. If I sell 500 deals at 15cts. apiece, and by so doing lose $9 per cent., how much do I lose in the whole quantity at that rate? Ans. $6.75cts.

3. A grocer bought 1250lbs. of coffee at 16cts. per lb. and sold it immediately at $25 per cent. advance; how much did he gain by the transaction? Ans. $50.

4. If I sell 120 reams of paper at $2.50cts. a ream, and thereby lose $12.50cts. per cent., what will be the loss in the whole quantity at the same rate? Ans. $37.50cts.

CASE 5.

To find how a thing must be sold to gain or lose a given rate per cent.

RULE.

As 100 dollars is to the given price, so is 100 dollars, with the gain added or loss subtracted, to the gaining or losing price. The operation may be performed more concisely by taking parts of the given price, and adding or subtracting them, as the case requires.

EXAMPLES.

1. A tobacconist bought a quantity of tobacco at 15cts. per lb.; at what rate per lb. must he sell it to gain \$33.33$\frac{1}{3}$cts. per ct.?

$$
\begin{array}{lll}
 & & \$100.00 \\
 & & 33.33\tfrac{1}{3} \\
\hline
\overset{\$}{100} : & \overset{cts.}{15} :: & 133.33\tfrac{1}{3} \\
3 & & 3 \\
\hline
300 & & 400.00 \\
 & & 15 \\
\hline
 & & 3,00)60,00. \\
\end{array}
$$

Ans. 20 cents per lb.

By Practice.

$$\overset{\$\ cts.}{33.33\tfrac{1}{3}} = \overset{p.c.}{\tfrac{1}{3}})\ \overset{cts.}{15} = \text{given price.}$$
$$5$$

Ans. 20 cents, as before.

2. A merchant bought a piece of cloth at \$2.50cts. a yd., which not proving so good as he expected, he is willing to lose \$17$\frac{1}{2}$ per cent. by the sale of it; how must he sell it by the yard?

$$
\begin{array}{lll}
 & & \$100.00 \\
 & & 17.50 \\
\hline
\overset{\$}{100} : & \overset{\$\ cts.}{2.50} :: & 82.50 \\
 & & 2.50 \\
\hline
 & & 412500 \\
 & & 16500 \\
\hline
\end{array}
$$

Ans. \$2.06,25m.

$$
\begin{array}{l|l}
\overset{\$}{10} = \tfrac{1}{10} & \overset{\$\ cts.}{2.50} = \text{given price.} \\
\hline
5 = \tfrac{1}{2} & .25 \\
2\tfrac{1}{2} = \tfrac{1}{2} & 125 \\
 & \quad 625 \\
\hline
 & .4375 \text{ deducted.} \\
\end{array}
$$

Ans. \$2.06,25, as before.

3. If I buy a piece of linen at 75cts. a yard, at what price must I sell it by the yard to gain \$50 per cent.? Ans. \$1.12$\frac{1}{2}$cts.

4. If I buy a yard of silk velvet for \$4.62$\frac{1}{2}$cts. and sell it at \$100 per cent. advance, what is the amount? Ans. \$9.25cts.

5. I have a quantity of cork, which is worth \$4 a barrel, but I am willing to lose \$10 per cent. in the sale of it; what is the selling price? Ans. \$3.60cts. by the barrel.

CASE 6.

How to find the prime cost of any commodity, when there is a certain sum gained or lost per cent.

RULE.

As \$100, with the gain per cent. added, or the loss per cent. subtracted, is to the *selling price*, so is \$100 to the prime cost.

EXAMPLES.

1. A tobacconist sold a quantity of tobacco at 20cts. per lb., and by so doing gained $33.33⅓ cents per cent.; what did it cost him by the pound?

```
   $ cts.
 100.00
  33.33⅓
 -------    $      $ cts.
 133.33⅓ : 20 :: 100.00
       3               3
 -------          ------
  400.00          300.00
                      20
                  ------
          4,00)60,00.00
```

Ans. 15cts. per lb.

2. A merchant sold a piece of cloth at $2.06¼cts. a yard, and thereby lost $17½ per cent.; what did the cloth cost him by the yard?

```
 $ cts.
 100.00
  17.50
 ------                    $
  82.50 : 2.0625 :: 100
                   100
         ---------- $ cts.
   82.50)206.2500(2.50  Ans.
         ----------
            ...
```

3. If I sell a piece of linen at $1.12½cts. a yard, and thereby gain $50 per cent., what was the prime cost? Ans. 75cts.

4. If I sell a yard of silk velvet for $9.25cts., and thereby gain $100 per cent., how much did it cost me? Ans. $4.62½cts.

5. I have sold a parcel of corn at $3.60cts. per barrel, and thereby have lost $10 per cent.; what did it cost by the barrel? Ans. $4.

CASE 7.

Having the whole quantity and its prime cost given, to find the retailing price, when a certain sum is gained or lost thereon.

RULE.

As the whole quantity is to its prime cost, with the gain added or the loss subtracted, so is any part of the given quantity to the price for which it must be sold.

EXAMPLES.

1. A draper bought 112 yards of superfine broadcloth for $500; I demand how he must sell it per yard to gain $256 on the whole quantity?

```
       $
      500=the prime cost.
      256=sum to be gained.
      ---
yds.
112 : 756 :: 1yd.
        1

112)756.00(6$ 75cts. Ans.
```

2. A linen draper bought 4 pieces of Irish linen, each containing 26yds., for $104, but it proved to be so much damaged that he is willing to lose $39 in the price of the whole quantity; how much did he sell it for by the yd. after making the above deduction?

```
   yds.   $
    26   104
     4    39
   ---   ---
As 104 : 65 :: 1yd.
              1

         104)65.00(62½cts. Ans.
```

3. An ordinary keeper bought a pipe of French brandy at $1.37½ cents a gallon; he paid $2.25cts. for the pipe, $3.50cts. for cartage, and 11 gallons leaked out on the way home; how must he sell it by the gallon to gain $22.25cts. on the whole quantity?

Ans. $1.75 per gallon.

4. A grocer bought 40 gallons of molasses at 50 cents a gallon, and by some accident 6 gallons leaked out on the way home; what must the retailing price be, by the gallon, to gain $7.20cts. on the whole quantity? Ans. 80cts. per gallon.

Case 9.

Having the whole quantity and its prime cost given—to find the retailing price of any part of it, when a certain rate per cent. is to be gained or lost thereon.

Rule.

1. Say—As $100 is to the prime cost of the commodity, so is $100, with the gain per cent. added, or the loss per cent. subtracted, to the gaining or losing price of the whole quantity. The operation may be abbreviated by taking parts of the prime cost, and adding or subtracting them, as may be required.

2. Say—As the whole quantity is to its gaining or losing price, found as above directed, so is any part of the given quantity to the retailing price for which it must be sold.

Examples.

1. A wholesale merchant bought 48 pieces of fine Irish linen, at $18 a piece, and sold 18 of the best pieces at $24 a piece, and 12 more of them at $21.75cts. a piece; at what rate by the piece must he sell the rest to gain $25 per cent. on the whole quantity?

```
                           $18
                            48 pieces.
                           ---
                           144
                           72
          p. c.            ---
          $25=¼)864=the prime cost.
                216=the interest at 25 p. c.
Pieces.         ----
  48            1080=the whole amount.
 ----           ----
 18 at $24.00cts.=432=price of 18 pieces } added.
 12 at $21.75cts.=261=price of 12 pieces }
 ----             ----
Subtract 30 and    693 from 48 and $1080.
 ----             ----
As 18      :      387 :: 1 piece .. $21.50ct. Ans.
```

2. A draper bought 100 yards of cloth for \$190; how must he sell it per yard to gain \$15 p. c. on the whole quantity?

$$\begin{array}{l} \qquad\quad \$100 \\ \qquad\qquad 15 \\ \$ \quad \$ \quad \overline{} \\ 100 : 190 :: 115 \\ \qquad 115 \\ yds. \quad \text{——} \quad yd. \\ 100 : 218.50 :: 1 .. \$2.18\tfrac{1}{2}c. \text{ Ans.} \end{array}$$

3. If I buy 1200lbs. of sugar at \$12 per cent., how must I sell it by the pound to gain \$50 p. c. on the whole? Ans. 18c. per lb.

4. Bought 12 pieces of white cloth at \$21.66⅔cts. a piece, and paid \$3.33⅓cts. per piece for dying it; how much must each piece be sold for to gain \$20 p. c. on the whole? Ans. \$30.

5. If a bag of cotton, weighing 8cwt. 0qrs. 20lbs. cost \$45.55cts., how must it be sold by the lb. to lose \$8 per cent.? Ans. 4cts. 5m.+

Case 9.

If, when a thing is sold at a given price, there is so much gained or lost per cent.—to find how much would be gained or lost per cent. if it were sold at another given price.

Rule.

1. Say—As the first given price is to \$100, with the gain per cent. added, or the loss per cent. subtracted, so is the other given price to its gain or loss per cent.

N. B. If the answer exceeds \$100, the excess will be the gain per cent., but if it be less than \$100, the deficiency will be the loss per ct.

Examples.

1. If I sell tobacco at 20 cts. a lb. and thereby gain \$33.33⅓c. per cent., what will be the gain or loss per cent. if I sell it at 15cts. per lb.? Ans. Nothing.

$$\begin{array}{l} \qquad\qquad \$100.00c. \\ \qquad\qquad\quad 33.33\tfrac{1}{3} \\ \overset{c.}{\text{As } 20} : 133.33\tfrac{1}{3} :: \overset{c.}{15} \\ \qquad\qquad\qquad 3 \\ \qquad\qquad 400.00 \\ \qquad\qquad\quad .15 \\ \qquad 20)60.00\,00 \\ \qquad\quad 3)300.00 = \text{thirds.} \end{array}$$

Ans. \$100... consequently I neither gain nor lose, because there is no excess nor deficiency.

2. If I sell linen at 66⅔cts. per yd. and thereby gain \$10 per cent. what will I gain or lose per cent. if I sell it at 58⅓cts. per yard?

$$\begin{array}{l} \qquad\qquad \$100 \\ \qquad\qquad\quad 10 \\ \text{As } \overset{c.}{66\tfrac{2}{3}} : 110 :: \overset{c.}{58\tfrac{1}{3}} \\ \qquad\quad 3 \qquad\qquad\quad 3 \\ \qquad 200 \qquad\qquad 175 \\ \qquad\qquad\qquad\qquad 110 \\ \qquad\qquad 200)192.5000 \end{array}$$

Ans. \$96.25cts. which is \$3.75cts. less than \$100, consequently the loss per cent. is exactly \$3.75cts.

3. If by selling wine at \$1.50 cents per gallon, I lose \$12 per cent.; what shall I gain or lose by selling 4 gallons of the same for 75cts.? Ans. I shall lose \$1 per cent.

4. If by selling cassimer at \$2.25cts. a yd. I gain \$12½ per cent. what shall I gain or lose by selling 8 yds. of the same for \$24?

Ans. I shall gain \$50 p. c.

Case 10.

To find the gain per cent. per annum, when the commodity is bought for ready money and sold on credit.

Rule.

The operation in questions belonging to this case is performed by the Double Rule of Three Direct, as in the following example; or by two single statings.

Examples.

1. A merchant bought a parcel of goods for \$60 ready money, and sold them immediately on 4 months credit for \$83.33⅓cts.; how much did he gain per cent. per annum?

```
                  $  cts.
                  83.33⅓ =the credit price.
                  60.00  =the cash price.
                  ------
   $  mo.
  60×4 :  23.33⅓ :: $100×12mo.
   4          3         12
  ---      -----     ----
  240      7000      1200
              1200
        ----------
       240)84000.00
        ----------
         3)350.00=thirds.
```

Ans. \$116.66⅔cts. per annum.

2. A tobacconist bought a quantity of tobacco for \$120 in cash, which he sold immediately for \$150 on a credit of 6 months; what did he gain per cent. per annum? Ans. \$50 p. c. p. a.

Case 11.

When a given rate per cent. is lost by the sale of a single thing, or by any quantity of merchandise, and the seller ought to have gained another given rate per cent., to find how much the said thing, or quantity of merchandise, was sold under its just value.

Rule.

1. As 100, with the loss per cent. subtracted from it, is to 100, with the gain per cent. added to it; so is the selling price of the thing, &c. to the sum for which it ought to have been sold.

2. Subtract the selling price of the thing from the sum for which it ought to have been sold, and the remainder will be the answer required.

EXAMPLES.

1. A merchant sold a parcel of goods for 63£, and by so doing lost 17£ per cent., whereas he ought to have gained 20£ per cent. by the transaction; how much under their just value were they sold?

100£ 100£
17 20

As 83 : 120 :: 63£ to 91£ 1s. 8d. and
91£ 1s. 8d−63£=28£ 1s. 8d. Answer.

2. A merchant sold a quantity of goods for $600, and by so doing lost $20 per cent., whereas he ought to have gained $25 per cent. by trading fairly; how much under their just value were the goods sold? Ans. $337.50cts.

CASE 12.

To find the quantity of any thing contained in a box or bag, &c. when the buying and selling prices are both given, and a certain rate per cent. is gained or lost by the transaction.

RULE.

As the selling price is to the quantity for which it paid, so is the buying price per box or bag, &c. with the gain per cent. added, or the loss per cent. subtracted, to the quantity contained in the said box or bag, &c.

EXAMPLES.

1. A grocer bought 100 boxes of prunes, at $2.10cts. per box, and sold them at $3.50cts per cwt., and by doing so he gained $25 per cent.; how many pounds were contained in each box, one with another? Ans. 84lbs. or 3qrs. of a cwt.

p.c. $ cts.
$25=¼)2.10=cost per box
.525=gain per b.

3.50 : 112 :: 2.625
112

3.50)294.000(84lbs. Ans.
2800

1400
1400

Another method.

$ cts.
2.10=cost per box.
100 boxes.

$25=¼)210.00 whole cost.
52.50 whole gain.

$ cts. lbs.
3.50 : 112 :: 262.50 .. 8400 lbs.
& 8400÷100=84lbs., as before.

2. A merchant bought 80 bags of cotton at $37.50cts per bag, and sold it at $15 per 100lbs., and by doing so he gained $20 per cent.; how many pounds were contained in each bag, what were the buying and selling prices of each pound, what was the whole gain, and the gain on each pound? Ans. Each bag contained 300lbs.; the cotton cost 12½cts. per lb. and was sold at 15cts. per lb.; the whole gain was $600, and the gain on each pound 2½cts.

FELLOWSHIP.

Fellowship is a rule by which the accounts of merchants and other persons trading in partnership are so adjusted, that each man may have his equitable share of the profit, or sustain his proportional part of the loss, in a just ratio to his share of the joint stock, together with the time of its continuance in trade. Legacies are adjusted by this rule, when there is a deficiency of assets, and the effects of bankrupts divided among their creditors.

Case 1.

When the several shares composing the joint stock of the whole company are continued in trade an equal term of time.

Rule.

As the whole sum of the joint stock
Is to the whole gain or loss,
So is each man's share in the stock
To his just share of the gain or loss.

Proof.

Add the several shares of the gain or loss together, and the total sum will be equivalent to the whole gain or loss, if the operation is right.

Examples.

1. Three merchants, namely, A, B, and C, join in company; A put in \$140, B \$300, and C \$160; their clear gain is \$120. What is each partner's share of it, in proportion to his share in the joint stock? Ans. A must have \$28, B \$60, and C \$32.

Examine the work carefully.

```
  $
140=A's share in stock.
300=B's share in stock.
160=C's share in stock.
---
600=the whole sum.
```

```
    $     $     $            $     $     $            $     $     $
As 600 : 120 :: 140      As 600 : 120 :: 300      As 600 : 120 :: 160
              140                      300                      160
             ----                  -------                     ----
             4800              6,00)360,00                     7200
              120                  -------                      120
             ----              Ans. $60=B's sh.                ----
      6,00)168,00                   28=A's sh.          6,00)192,00
             ----                   32=C's sh.                 ----
  Ans. $28=A's share.              $120=proof.       Ans. $32=C's sh.
```

2. Divide the number 360 into four parts, which shall be in proportion to each other, as 3, 4, 5, 6. Ans. 60, 80, 100, and 120.

$3+4+5+6=18.$

As 18 : 360 :: 3	As 18 : 360 :: 4	As 18 : 360 :: 5	As 18 : 360 :: 6
3	4	5	6
18)1080	18)1440	18)1800	18)2160
60	80	100	120

3. It is required to divide $120 between A and B, so that A may have ½ and B ⅓ of the whole sum; what is each man's part in the above proportion?

½ of $120 = $60 and
⅓ of 120 = 40

The sum = 100, which is deficient by $20.

Wherefore, I say—1st. As $100 : $120 :: $60 .. $72 A's part.
2d. As 100 : 120 :: 40 .. 48 B's part.

$120 = proof.

4. Three men, namely, A, B, and C, joined in partnership; A put in $1200, B $4800, and C $2000; they gained $800. Please to inform me how much each partner must receive of the gain?
Ans. A must receive $120, B 480, and C 200.

5. Three men, namely, D, E, and F, joined in company; D put in $2880, E $11520, and F $4800, and by misfortune lost $1920. What sum must each man sustain of the loss?
Ans. D must lose $288, E 1152, and F 480.

6. A, B, and C freighted a ship with 108 tuns of wine, of which A had 48, B 36, and C 24, and by reason of bad weather they were obliged to cast 45 tuns overboard; how much must each man sustain of the loss? A must lose 20 tuns, B 15, and C 10.

7. P, Q, and R have rented a pasture containing 360 acres at $140 per annum, of which P holds 90, Q 120, and R 150 acres; please to tell me how much each tenant must pay in proportion to the land he holds? Ans. P must pay $60, Q 80, and R 100.

8. Four men, namely, D, E, F, and G, traded together with a stock of $800, and in two years time they gained twice as much and $40 more—D's stock was $140, E's $260, and F's $300; I demand G's stock, and what each man gained by trading? Ans. G's stock was $100; D gained $287, E $533, F $300, and G $205.

9. Three neighbors, namely, X, Y, and Z, bought a grindlestone in partnership for $4.50cts. of which X agreed to pay ½, Y ⅓ and Z ¼; please to tell me how much each one had to pay of the debt in the *said proportion?* Ans. X had to pay $2.07cts. 6m.+900 rem. Y *$1.38cts. 4m.+600* rem. and Z $1.03cts. 8m.+450 rem.

10. A merchant is indebted to S $168, to T $960, to V $337.50 cents, but upon his decease his estate is found to be worth only $983.28 cents; how must it be divided among his creditors?

		$ c. m.	Remainders.
Ans. S	must have	112.71,9+	134550
T		644.11,3+	119850
V		226.44,6+	38700
		2=	293100 divided
	Proof	983.28,0...	146550 by

Case 2.

When the several shares composing the joint stock of the whole company are continued in trade an unequal term of time.

Rule.

1. Multiply each man's share in the joint stock of the whole company by the time of its continuance in trade.

2. Say—As the sum of the several products
Is to the whole gain or loss
So is each particular product
To its share of the gain or loss.

The operation may be proved as in case the first.

Examples.

1. Three merchants, namely, A, B, and C, traded together; A put in $120 for 9 months, B $100 for 16 months, and C $100 for 14mo., and they gained $150; what is each man's share in proportion to his stock and time?

	$	mo.	products.
A put in	120×	9=	1080
B	100×	16=	1600
C	100×	14=	1400
The whole sum=			4080

		$		$		$ c. m.
As 4080	:	150	::	1080	..	39.70,5+3600
4080	:	150	::	1600	..	58.82,3+2160
4080	:	150	::	1400	..	51.47,0+2400
						2=8160
						$150.00,0 proof.

2. Three merchants namely, D, E, and F, joined in partnership for 12 months; D put in at first $873.60cts. and 4 months after that time he put in $96 more; E put in at first $979.20cts. and at the

end of 7 months he took out $206.40cts.; F put in at first $355.20cts. and 3 months afterwards he put in $206.40cts. and 5 months after that time he put in $240 more, and at the termination of 12mo. their clear gain is found to be $3446.40 cents; what is each partner's share thereof, in proportion to his stock, and the time of its continuance in trade?

		$ c.		m.	products	
1st.	D put in	873.60	×	4	=3494.40	
2d.	Add	96.00				=11251.20=D's product.
		969.60	×	8	=7756.80	
1st.	E put in	979.20	×	7	=6854.40	
2d.	Subtract	206.40				=10718.40=E's product.
		772.80	×	5	=3864.00	
1st.	F put in	355.20	×	3	=1065.60	
2d.	Add	206.40				
		561.60	×	5	=2808.00	= 7080.00=F's product.
3d.	Add	240.00				
		801.60	×	4	=3206.40	29049.60=whole sum.

	$ c.			$ c. m.	remainders.	
As 29049.60 :	3446.40 ::	11251.20..		1334.82,6+	336000	
29049.60 :	3446.40 ::	10718.40..		1271.61,4+	1570560	Ans.
29049.60 :	3446.40 ::	7080.00..		839.96,0+	998400	
					1=2904960	
				$3446.40,0=proof.		

3. A with a capital of $400 commenced merchandising on the 1st day of January 1829, and being successful in business, he took in B as a partner on the 1st day of March following, with a capital of $500—three months afterwards they admitted C as a third partner, with a capital of $600—on the 1st of January, 1830, they dissolved partnership, and found their neat gain to be $700; what is each man's share of it, in proportion to his stock and time?

Ans. A must receive $240, B 250, and C 210.

4. A, B, and C traded in company, A put in $1000 for 5 months; B, $1200 for 8 months, C $1500 for 6 months, and D $2100 for 4 months—by misfortune on sea they lost goods to the amount of $1600; please to inform me how much each one must sustain of the damage? Ans. A must lose $250, B 480, C 450 and D 420.

5. B, C, and D hold a pasture in common, for which they pay $90 per annum—in this pasture B had 45 oxen for 8 weeks, C had 25 for 12 weeks, and D 34 for 10 weeks; what must each one pay of the rent?

Ans. B had to pay $32.40cts. C $27 and D $30.60cts.

VULGAR FRACTIONS.

1. A vulgar fraction is a part or so many parts of a unit or one, and is represented by two numbers, one of which is placed above the other, with a line drawn between them, thus: $\frac{1}{4}$ one-fourth, $\frac{1}{8}$ one-eighth, $\frac{6}{7}$ six-sevenths, $\frac{7}{8}$ seven-eighths, &c.

2. The upper number is called the numerator, and shows the part or parts that is expressed by the fraction, and is the remainder in division.

3. The lower number is called the denominator, and shows how many parts are contained in the unit, and is the divisor in division.

4. Vulgar Fractions are proper, improper, single, compound, or mixed.

5. A proper fraction is when the numerator is less than the denominator, as $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{8}$, $\frac{7}{8}$, &c. &c.

6. An improper fraction is when the numerator is equal to or exceeds the denominator, as $\frac{4}{4}$, $\frac{12}{12}$, $\frac{8}{7}$, $\frac{9}{5}$, &c. &c.

7. A single fraction is a simple expression, denoting any part or number of parts into which the unit may be divided, as $\frac{1}{2}$, $\frac{2}{8}$, $\frac{7}{8}$, &c.

8. A compound fraction is the fraction of a fraction, as $\frac{1}{2}$ of $\frac{2}{3}$, of $\frac{7}{8}$, &c.

9. A mixed number is composed of a whole number and a fraction, as $7\frac{1}{2}$, $8\frac{2}{3}$, $16\frac{3}{4}$, &c. &c.

10. A mixed fraction has another fraction annexed either to its numerator or denominator, as $\frac{42\frac{7}{8}}{49}$, or $\frac{73}{131\frac{4}{5}}$, &c.

11. The common measure of two or more numbers, is that number which will divide all of them without any remainder.

12. The common multiple of two or more numbers, is such a number that may be divided by any one or all of them without a remainder; and if it be the least number which can be so divided, it is called their least common multiple. Thus—30, 45, 60, and 75, are common multiples of 3, 5, and 15, but 15 is the least common multiple of 3, 5, and 15.

Problem 1.

To find the greatest common measure of any two given numbers.

Rule.

Divide the greater by the less, and that divisor by the remainder following, and so on, till nothing be left, and the last divisor will be the greatest common measure that will divide both the given numbers.

N. B.—If the common measure is found to be unity, or 1, the given numbers are prime to each other, and therefore *incommensurable.*

EXAMPLES.

1. What is the greatest common measure of 172 and 268?

```
172)268(1
    172
    ---
     96)172(1
         96
         ---
         76)96(1
            76
            --
            20)76(3
               60
               --
               16)20(1
                  16
                  --
                   4)16(4
                     16
                     --
                      0
```

In the above example, 4 is the only divisor that leaves no remainder, wherefore it is the greatest common measure, and will divide both the given numbers without any remainder.

```
4)172  and  268
  ---       ---
   43 proof  67
```

2. What is the greatest common measure of 72 and 96?

```
72)96(1
   72
   --
   24)72(3
      72
      --
      00
```

In the second example, 24 is the greatest common measure that will divide both the given numbers without any remainder. Try it.

3. What is the greatest common measure of 288 and 1728? Ans. 288 is the greatest common measure, because it leaves no remainder.

PROBLEM 2.

To find the greatest common measure of more than two numbers?

RULE.

1. Find the greatest common measure of any two of the given numbers by the first problem.

2. Divide any one of the other given numbers by the common measure now found, and so on as in the first problem, till a second common measure is found, with which proceed in the same manner through all the given numbers, and the least common measure will divide them without a remainder.

EXAMPLES.

1. What is the greatest common measure of 522, 918, & 1998?

```
918)1998(2        54)522(9
    1836             486
    ----             ---
     162)918(5        36)54(1
         810             36
         ---             --
         108)162(1       18)36(2
             108             36
             ---             --
              54)108(2       ..
                 108
                 ---
```

In this example, 54 is the greatest common measure of 918 and 1998; next I divide 522 by the common measure 54, and the first divisor that leaves no remainder is 18, wherefore 18 is the greatest common measure of the three given numbers.

```
18)522   918   1998
   ---   ---   ----
    29    51    111 proof.
```

2. What is the greatest common measure of 83, 191, and 573?

```
83)191(2
   166
   ---
    25)83(3
       75
       --
        8)25(3
          24
          --
           1)8(8
             8
             -
             .
```

In this example, the common measure is found to be 1, wherefore the given numbers are prime to each other, and of course incommensurable, because they are indivisible by any common divisor.

3. What is the greatest common measure of 147, 189, 231, and 273? Ans. 21.

Problem 3.

To find the least common multiple of two or more given numbers.

Rule.

1. Divide the given numbers by any divisor that will divide any two or more of them without a remainder, and set the quotients and indivisible numbers in a line underneath.

2. Divide the second line as before, and so on, till there are no two numbers left that can be divided by one divisor.

3. Multiply all the divisors and the last line of quotients continually together, and their product will be the common multiple required, and may be divided by any one or all the said given numbers without a remainder.

N. B.—The foregoing problems will be found very useful in reducing vulgar fractions to their lowest terms, and finding divisible numbers in the rules of position, &c.

Examples.

1. What is the least number that can be divided by the nine digits without a remainder?

```
3) 1 , 2 , 3 , 4 , 5 , 6 , 7 , 8 , 9
   ---------------------------------
4) 1 , 2 , 1 , 4 , 5 , 2 , 7 , 8 , 3
   ---------------------------------
2) 1 , 2 , 1 , 1 , 5 , 2 , 7 , 2 , 3
   ---------------------------------
   1 , 1 , 1 , 1 , 5 , 1 , 7 , 1 , 3
```

$4 \times 3 \times 2 \times 5 \times 7 \times 3 = 2520$. Ans.

2. What is the least common multiple of 3, 4, 8, 12? Ans. 24.

3. What is the least number that can be divided 12, 15, 18, 24, and 36, without remainders? Ans. 360.

4. What is the least common multiple of 3, 5, 8, 10, and 12? Ans. 120.

5. What is the least common multiple of 7, 14, and 21? Ans. 42.

REDUCTION OF VULGAR FRACTIONS.

Reduction of Vulgar Fractions is the changing of them from one form or denomination to another, in order to prepare them for addition, subtraction, multiplication, and division.

CASE 1.

To reduce vulgar fractions to their lowest terms.

RULE.

Divide both terms of the fraction by their greatest common measure, and the quotients will be the terms of the fraction required; or, divide the terms of the fraction by any number that will divide them both without a remainder, and divide the quotients in the same manner, and so on, till no number more than 1 will divide them, and the fraction will be in its lowest terms.

EXAMPLES.

1. Reduce $\frac{144}{240}$ to its lowest terms.

```
144)240(1
    144
    ---
     96)144(1
         96
         ---
         48)96(2
            96
            ---
```

c.m.$=48)\frac{144}{240}=\frac{3}{5}$ the answer.

2. Reduce $\frac{315}{630}$ to its lowest terms. Ans. $\frac{1}{2}$.

3. Reduce $\frac{1188}{1386}$ to its lowest terms. Ans. $\frac{6}{7}$.

4. Reduce $\frac{48}{272}$ to its lowest terms. Ans. $\frac{3}{17}$.

5. Reduce $\frac{192}{576}$ to its lowest terms. Ans. $\frac{1}{3}$.

6. Reduce $\frac{252}{364}$ to its lowest terms. Ans. $\frac{9}{13}$.

CASE 2.

To reduce fractions which have different denominators to others, retaining the same value, that shall have one common denominator.

RULE.

Multiply each numerator by all the denominators but its own, for its respective new numerator; then multiply all the denominators together for a common denominator.

NOTE.—This case and case the 1st prove each other.

EXAMPLES.

1. Reduce $\frac{7}{8}$, $\frac{9}{10}$, and $\frac{11}{12}$ to a common denominator.

$$\left.\begin{array}{r} 7\times10\times12=840 \\ 9\times\ 8\times12=864 \\ 11\times\ 8\times10=880 \end{array}\right\}\text{New numerators.}$$

$8\times10\times12=960$ the common denominator; consequently the new fractions are $\frac{840}{960}$, $\frac{864}{960}$, and $\frac{880}{960}$, which, being reduced to their *lowest terms*, make $\frac{7}{8}$, $\frac{9}{10}$, and $\frac{11}{12}$. *Prove it.*

2. Reduce $\frac{5}{6}$ and $\frac{7}{8}$ to a common denominator. Ans. $\frac{40}{48}$ and $\frac{42}{48}$.

3. Reduce $\frac{1}{2}$, $\frac{2}{3}$, $\frac{5}{6}$, and $\frac{7}{8}$ to a common denominator. Ans. $\frac{144}{288}$, $\frac{192}{288}$, $\frac{240}{288}$, and $\frac{252}{288}$.

4. Reduce $\frac{3}{5}$, $\frac{1}{2}$, $\frac{1}{9}$, and $\frac{6}{7}$ to a common denominator. Ans. $\frac{378}{630}$, $\frac{315}{630}$, $\frac{70}{630}$, and $\frac{540}{630}$.

5. Reduce $\frac{4}{9}$, $\frac{7}{11}$, $\frac{6}{7}$, and $\frac{1}{2}$ to a common denominator. Ans. $\frac{616}{1386}$, $\frac{882}{1386}$, $\frac{1188}{1386}$, & $\frac{693}{1386}$.

Case 3.

To reduce a mixt number to an improper fraction.

Rule.

Multiply the whole number by the denominator of the fraction, and add the numerator to the product for a new numerator, under which place the given denominator, and the result will be the fraction required.

Examples.

1. Reduce $27\frac{2}{9}$ to an improper fraction.
$27 \times 9 + 2 = \frac{245}{9}$ the answer.

2. Reduce $28\frac{22}{24}$ to an improper fraction. Ans. $\frac{694}{24}$.
3. Reduce $12\frac{17}{19}$ to an improper fraction. Ans. $\frac{245}{19}$.
4. Reduce $7\frac{3}{4}$ to an improper fraction. Ans. $\frac{31}{4}$.
5. Reduce $183\frac{5}{21}$ to an improper fraction. Ans. $\frac{3848}{21}$.

To express a whole number fractionwise, put 1 for its denominator, thus $\frac{5}{1}$, &c.

Case 4.

To reduce an improper fraction to a whole or mixt number.

Rule.

Divide the numerator by the denominator, and the quotient will be whole or mixt number required.

Note.—This case and case the 3d prove each other.

Examples.

1. Reduce $\frac{694}{24}$ to a mixt number.

```
24)694(28 22/24 Ans.
   48
   ---
   214
   192
   ---
    22 remainder.
```

2. Reduce $\frac{245}{9}$ to a mixt number. Ans. $27\frac{2}{9}$.
3. Reduce $\frac{12}{12}$ to a whole number. Ans. 1.
4. Reduce $\frac{96}{24}$ to a whole number. Ans. 4.
5. Reduce $\frac{245}{19}$ to a mixt number. Ans. $12\frac{17}{19}$,

---

Case 5.

To reduce a compound fraction to a single one.

Rule.

1. Multiply all the numerators together for a new numerator.
2. Multiply all the denominators together for a new denominator.
3. Like numbers in the numerators and denominators may be *cancelled or omitted* in the operation.

Examples.

1. Reduce $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{7}{8}$ to a single fraction.

$\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}\times\frac{4}{5}\times\frac{7}{8}=\frac{168}{960}$, or $\frac{7}{40}$. Ans. $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{7}{8}=\frac{7}{40}$, as before.

In the second operation, the numerators 2, 3, 4, and the denominators 2, 3, 4, being equal, are cancelled or omitted, and the numerators 1, 7, and the denominators 5 and 8, remain to be multiplied together.

2. Reduce $\frac{2}{3}$ of $\frac{19}{20}$ to a single fraction. Ans. $\frac{38}{60}$ or $\frac{19}{30}$.
3. Reduce $\frac{1}{2}$ of $\frac{2}{3}$ to a single fraction. Ans. $\frac{2}{6}$ or $\frac{1}{3}$.
4. Reduce $\frac{7}{8}$ of $\frac{4}{5}$ of $\frac{9}{10}$ to a single fraction. Ans. $\frac{252}{480}$ or $\frac{21}{40}$.
5. Reduce $\frac{5}{9}$ of $\frac{4}{8}$ of $\frac{3}{4}$ to a single fraction. Ans. $\frac{60}{288}$ or $\frac{5}{24}$.

Case 6.

To reduce a fraction of one denomination to the fraction of another, but greater, retaining the same value.

Rule.

Multiply the denominator of the given fraction by all the denominations between the given one, and that denomination to which it is to be reduced for a new denominator, and place it under the numerator of the given fraction—the result will be the fraction required.

Examples.

1. Reduce $\frac{5}{6}$ of a penny to the fraction of a pound sterling.

$$\overset{d.}{\tfrac{5}{6}}\times\overset{d.}{12}\times\overset{s.}{20}=\tfrac{5}{1440}=\tfrac{1}{288}\text{ part of a £. Ans.}$$

2. Reduce $\frac{1}{10}$ of a cent to the fraction of a dollar.

$$\tfrac{1}{10}\times\overset{c.}{10}\times\overset{di.}{10}=\tfrac{1}{1000}\text{ part of a dollar. Ans.}$$

3. Reduce $\frac{1}{2}$ of a farthing to the fraction of a shilling.

$$\overset{qr.}{\tfrac{1}{2}}\times\overset{qrs.}{4}\times\overset{d.}{12}=\tfrac{1}{96}\text{ part of a shilling. Ans.}$$

4. Reduce $\frac{3}{4}$ of a cent to the fraction of a dollar. Ans. $\frac{3}{400}$\$.
5. Reduce $\frac{4}{5}$ of a pennyweight to the fraction of a lb. Troy. Ans. $\frac{4}{1200}$ or $\frac{1}{300}$pt.

Case 7.

To reduce a fraction of one denomination to the fraction of another, but less, retaining the same value.

Rule.

Multiply the numerator of the given fraction by all the denominations *between* the given one, and that denomination to which it *is to be reduced,* for a new numerator, and place it over the given

denominator—the result will be the fraction required, which must be reduced to its lowest terms.

NOTE.—This case and case the 6th prove each other.

EXAMPLES.

1. Reduce $\frac{1}{288}$ of a £ sterling to the fraction of a penny.

$$\overset{£}{\frac{1}{288}} \times \overset{s.}{20} \times \overset{d.}{12} = \frac{240}{288} = \frac{5}{6} \text{ of a penny. Ans.}$$

2. Reduce $\frac{3}{400}$ of a dollar to the fraction of a cent.

$$\overset{\$}{\frac{3}{400}} \times \overset{c.}{10} \times \overset{di.}{10} = \frac{300}{400} = \frac{3}{4} \text{ of a cent. Ans.}$$

3. Reduce $\frac{1}{96}$ part of a shilling to the fraction of a farthing. Ans. $\frac{1}{2}$qr.
4. Reduce $\frac{1}{1000}$ part of a dollar to the fraction of a cent. Ans. $\frac{1}{10}$c.
5. Reduce $\frac{4}{1200}$ of a lb. Troy to the fraction of a pwt. Ans. $\frac{4}{5}$ of a p.

CASE 8.

To reduce fractions from one denomination to another of the same value, when the numerator of the required fraction is given.

RULE.

Say—As the numerator of the given fraction
Is to its own denominator;
So is the numerator of the required fraction
To its denominator.

EXAMPLES.

1. Reduce $\frac{3}{4}$ to a fraction of the same value, whose numerator shall be 15.

$$\begin{array}{ccccc} n. & & denom. & & n. \\ 3 & : & 4 & :: & 15 \\ & & 15 & & \\ \hline & & 3)60 & & \\ \hline & & 20 & & \end{array}$$

20=new denominator, consequently the required fraction is $\frac{15}{20}$, which is$=\frac{3}{4}$ in its lowest terms.

2. Reduce $\frac{5}{6}$ to a fraction of the same value, whose numerator shall be 25. Ans. $\frac{25}{30}$.

3. Reduce $\frac{7}{8}$ to a fraction of the same value whose numerator shall be 42. Ans. $\frac{42}{48}$.

4. Reduce $\frac{3}{4}$ to a fraction of the same value whose numerator shall be 34. Ans. $\dfrac{34}{45\frac{1}{3}}$.

NOTE. From cases 8 and 9, there arises a new fraction, which may be *properly called* a mixt fraction.

*15

Case 9.

To reduce fractions from one denomination to another of the same value, when the denominator of the required fraction is given.

Rule.

Say—As the denominator of the given fraction
Is to its own numerator;
So is the denominator of the required fraction
To its numerator.

Note—This case and case the 8th prove each other.

Examples.

1. Reduce $\frac{3}{4}$ to a fraction of the same value, whose denominator shall be 20.

denom. nu. denom.
As 4 : 3 :: 20

$$\begin{array}{r} 20 \\ \hline 4)60 \\ \hline \end{array}$$

$$\left.\begin{array}{l} 15 = \text{numerator} \\ \text{—} \\ 20 = \text{denom.} \end{array}\right\} = \tfrac{3}{4}.$$

2. Reduce $\frac{5}{6}$ to a fraction of the same value, whose denominator shall be 30. Ans. $\frac{25}{30}$.

3. Reduce $\frac{7}{8}$ to a fraction of the same value, whose denominator shall be 48. Ans. $\frac{42}{48}$.

4. Reduce $\frac{7}{8}$ to a fraction of the same value, whose denominator shall be 49. Ans. $\dfrac{42\frac{7}{8}}{49}$.

Case 10.

To reduce a mixt fraction to a single one.

Rule 1.

When the numerator has no fractional part joined to it:—Then

1. Multiply the numerator by the denominator of the fractional part of the denominator, for a new numerator.

2. Multiply the denominator of the fraction by the denominator of its fractional part, and to the product add the numerator of the fractional part for a new denominator.

Note.—This part of case the 10th proves the 8th case.

Examples.

1. Reduce $\dfrac{34}{45\frac{1}{3}}$ to a single fraction.

$$\begin{array}{ll} \text{The numerator} & 34\times3 = 102 \\ \text{The denominator} & 45\times3+1 = 136 \end{array} \quad \frac{102}{136} = \tfrac{3}{4} \text{ the lowest terms.}$$

The above question proves the 4th one, in the 8th case.

2. Reduce $\dfrac{73}{131\frac{2}{5}}$ to a single fraction. Ans. $\dfrac{365}{657}$ or $\frac{5}{9}$.

3. *Reduce* $\dfrac{95}{118\frac{9}{12}}$ to a single fraction. Ans. $\dfrac{1140}{1425}$ or $\frac{4}{5}$.

Rule 2.

When a fractional part is joined to the numerator:—Then

1. Multiply the numerator of the fraction by the denominator of its fractional part, and to the product add the numerator of the said fractional part for a new numerator.

2. Multiply the denominator of the fraction, by the denominator of the fractional part of the numerator for a new denominator.

Note.—This part of case the 10th, proves the 9th case.

Examples.

4. Reduce $\frac{42\frac{7}{8}}{49}$ to a single fraction.

The numerator $=42\times8+7=\frac{343}{392}$ or $\frac{7}{8}$. Answer.
The denominator$=49\times8=$

The above example proves the 4th one in the 9th case.

5. Reduce $\frac{34\frac{2}{4}}{46}$ to a single fraction. Ans. $\frac{138}{184}$ or $\frac{3}{4}$.

6. Reduce $\frac{49\frac{7}{9}}{56}$ to a single fraction. Ans. $\frac{448}{504}$ or $\frac{8}{9}$.

Case 11.

To find the value of any fraction in the known parts of the integer, as of coin, weight, measure, &c.

Rule.

Multiply the numerator of the given fraction by the next inferior denomination and divide the product by the denominator; and if there should be a remainder, multiply it by the next inferior denomination and divide the product by the denominator as before, and so on as far as necessary, and the several quotients placed in order will be the answer required.

Examples.

1. Reduce $\frac{2}{3}$ of a pound sterling to its proper value.

£2=the numerator.
20

The denominator=3)40(13*s.* 4*d.*=the answer.
39

1=the remainder.
12

3)12(4 pence.
12

·

2. Reduce $\frac{18}{43}$ of a shilling to its proper value. Ans. 5d. $\frac{1}{43}$.
3. Reduce $\frac{2}{9}$ of a £ to its proper value. Ans. 4s. 5d. 1qr. $\frac{1}{3}$.
4. Reduce $\frac{9}{13}$ of a £ to its proper value. Ans. 13s. 10d. $\frac{2}{13}$.
5. Reduce $\frac{6}{7}$ of 5£ 9s. to its proper value. Ans. 4£ 13s. 5d. $\frac{1}{7}$.
6. Reduce $\frac{5}{8}$ of a dollar to its proper value. Ans. 62½cts.
7. Reduce $\frac{7}{8}$ of a dollar to its proper value. Ans. 87½cts.

Of Weights and Measures.

8. Reduce $\frac{3}{4}$ of a lb. Troy to its proper quantity. Ans. 9oz.
9. Reduce $\frac{2}{13}$ of a ton weight to its proper quantity.
Ans. 3cwt. 0qrs. 8lbs. 9oz. 13drs. $\frac{7}{13}$.
10. Reduce $\frac{5}{9}$ of a lb., Avoirdupois weight, to its proper quantity.
Ans. 8ozs. 14drs. $\frac{2}{9}$.
11. Reduce $\frac{3}{11}$ of a lb., Apothecaries weight, to its proper quantity.
Ans. 3oz. 2drs. 0sc. 10grs. $\frac{10}{11}$.
12. Reduce $\frac{4}{5}$ of an English ell to its proper quantity. Ans. 1yd.
13. Reduce $\frac{3}{8}$ of a yard to its proper quantity. Ans. 1qr. 4½in.
14. Reduce $\frac{2}{7}$ of a quarter of wheat to its proper quantity.
Ans. 2bush. 1pk. 0gal. 1qt. $\frac{1}{7}$.
15. Reduce $\frac{7}{8}$ of a tun of wine to its proper quantity.
Ans. 3hhds. 31½gals.
16. Reduce $\frac{5}{8}$ of an acre to its proper quantity. Ans. 2roo. 20per.
17. Reduce $\frac{6}{7}$ of a mile to its proper quantity.
Ans. 6fur. 34po. 1yd. 1ft. 8in. 2b.c. $\frac{4}{7}$.
18. Reduce $\frac{7}{13}$ of a day to its proper quantity.
Ans. 12hrs. 55m. 23$\frac{1}{13}$sec.
19. Reduce $\frac{5}{8}$ of a degree to its proper quantity. Ans. 37′ 30″.

Case 12.

To reduce any given value or quantity to the fraction of any greater denomination of the same kind.

Rule.

1. Reduce the given quantity to the lowest denomination mentioned for a numerator.

2. Reduce the integer to the same denomination, for a denominator.

3. If there should be a fraction joined to the given quantity, multiply both the above parts by the denominator thereof, and add the numerator to the product of that part, which is to make the numerator of the fraction required.

4. Reduce the new fraction to its lowest terms, and the result will be the answer sought.

Note.—This case and case the 11th prove each other.

EXAMPLES.

1. Reduce 13s. 4d. to the fraction of a pound sterling.

$$\begin{array}{l} \quad\;\; s. \;\; d. \quad d. \\ £ \;\; 13 \;\; 4=\dfrac{160}{240}=\frac{2}{3}£. \text{ Ans.} \\ 1=20 \;\; 0= \end{array}$$

2. Reduce 5d. $\frac{1}{43}$ part of a penny to the fraction of a shilling.

$$\begin{array}{l} \quad\; d. \\ s. \;\; 5\times43+1=216 \\ 1=12\times43 \;\;\; =\overline{516}=\dfrac{18}{43}s. \text{ Ans.} \end{array}$$

3. Reduce 4s. $5\frac{1}{4}$d. $\frac{1}{3}$ part of a qr. to the fraction of a £ sterling.

s. d.	s.
4 $5\frac{1}{4}+\frac{1}{3}$qr.	20
12	12
53	240
4	4
213 qrs.	960 qrs.
3=deno'r.	=3
640=num.	2880=denomina'r.

c. m.=32 $\left\{\dfrac{640}{2880}=\dfrac{2}{9}\right.$£. Ans.

4. What part of 5£ 9s. is 4£ 13s. $5\frac{1}{7}$d.

£ s.	£ s. d.
5 9	4 13 $5+\frac{1}{7}$
20	20
109	93
12	12
1308	1121
7	7
9156 den'r.	7848 numerator.

c. m.=1308 $\left\{\dfrac{7848}{9156}=\dfrac{6}{7}\right.$ of 5£ 9s.

The above examples prove the 1st, 2d, 3d, and 5th in the 11th case.

5. Reduce 13s. 10d. $\frac{2}{13}$ to the fraction of a £ sterling. Ans. $\frac{9}{13}$.
6. What part of 4£ 17s. 3d. is 3£ 17s. $9\frac{1}{2}$d. $\frac{2}{5}$. Ans. $\frac{4}{5}$.
7. What part of a dollar is $31\frac{1}{4}$ cents? Ans. $\frac{5}{16}$.
8. What part of a dollar is $87\frac{1}{2}$ cents? Ans. $\frac{7}{8}$.
9. Reduce 7oz. 10pwts. to the fraction of a lb. Troy. Ans. $\frac{5}{8}$.

ADDITION OF VULGAR FRACTIONS.

CASE 1.

To add fractions that have one common denominator.

RULE.

Add all the numerators together, and set one of the given denominators under the total sum; if the result be an improper fraction, reduce it to a whole or mixt number, by case the 4th of Reduction.

EXAMPLES.

1. Add $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, and $\frac{4}{5}$ together.
$1+2+3+4=\frac{10}{5}=2$. Ans.

2. Add $\frac{2}{6}$, $\frac{3}{6}$, $\frac{4}{6}$, and $\frac{5}{6}$ together.
$2+3+4+5=\frac{14}{6}=2\frac{1}{3}$. Ans.

3. Add $\frac{1}{7}$, $\frac{2}{7}$, $\frac{3}{7}$, $\frac{4}{7}$, $\frac{5}{7}$, and $\frac{6}{7}$ together. Ans. 3.
4. *Add* $\frac{3}{8}$, $\frac{4}{8}$, $\frac{5}{8}$, $\frac{6}{8}$, and $\frac{7}{8}$ together. Ans. $3\frac{1}{8}$.
5. *Add* $\frac{3}{9}$, $\frac{4}{9}$, $\frac{5}{9}$, $\frac{6}{9}$, and $\frac{8}{9}$ together. Ans. $2\frac{8}{9}$.

Case 2.

To add fractions that have different denominators.

Rule.

Reduce the given fractions to others, which shall have one common denominator; then add all the new numerators together, and place the common denominator under their sum. The result will be the answer required.

Examples.

1. Add $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$, and $\frac{11}{12}$ together.

$$\left.\begin{array}{r} 2\times4\times6\times8\times12=4608 \\ 3\times3\times6\times8\times12=5184 \\ 5\times3\times4\times8\times12=5760 \\ 7\times3\times4\times6\times12=6048 \\ 11\times3\times4\times6\times8=6336 \end{array}\right\}\text{Numerators.}$$

Their whole sum $=\dfrac{27936}{6912}=4\frac{288}{6912}$

$3\times4\times6\times8\times12=6912=$ c. de.

or $4\frac{1}{24}$, ans. in the lowest terms.

2. Add $\frac{1}{2}$ and $\frac{2}{3}$ together.

$$\left.\begin{array}{r} 1\times3=3 \\ 2\times2=4 \end{array}\right\}\text{new numerators.}$$

their sum $=\dfrac{7}{6}=1\frac{1}{6}$. Ans.

$2\times3=6=$ common denom.

3. Add $\frac{2}{3}$ and $\frac{3}{4}$ together. Ans. $1\frac{5}{12}$.

4. Add $\frac{1}{2}$, $\frac{2}{3}$, $\frac{5}{6}$, and $\frac{7}{8}$ together. Ans. $2\frac{7}{8}$.

Case 3.

To add mixt numbers whose fractions have a common denominator.

Rule.

Omit the whole numbers, and find the amount of the fractions by case the first, to which add the sum of the whole numbers, and the result will be the answer required.

Examples.

Add $5\frac{6}{12}$, $7\frac{3}{12}$, $8\frac{9}{12}$, $13\frac{3}{12}$, and $6\frac{8}{12}$ into one total sum.

The numerators $=6+3+9+3+8=\dfrac{29}{12}=2\frac{5}{12}$.

Whole numbers $=5+7+8+13+6 = 39$

Ans. $41\frac{5}{12}$.

2. Add $12\frac{5}{20}$, $15\frac{7}{20}$, $18\frac{6}{20}$, and $19\frac{2}{20}$ into one total sum. Ans. 65.
3. Add $2\frac{1}{11}$, $3\frac{2}{11}$, $4\frac{4}{11}$, and $7\frac{9}{11}$ into one sum. Ans. $17\frac{5}{11}$.
4. Add $12\frac{24}{48}$ and $36\frac{40}{48}$ together. Ans. $49\frac{1}{3}$.

Case 4.

To add mixt numbers whose fractions have different denominators.

Rule.

Omit the whole numbers as before, and find the amount of the *fractions by case* the second, to which add the sum of the whole *numbers, and the* result will be the answer required.

EXAMPLES.

1. Add $5\frac{1}{4}$, $6\frac{7}{8}$, and $4\frac{1}{2}$ together.

$1\times8\times2=16$
$7\times4\times2=56$
$1\times4\times8=32$

$4\times8\times2=\frac{104}{64}=1\frac{5}{8}$ of the fract's.
$5+6+4=15$

Ans. $16\frac{5}{8}$.

2. Add $7\frac{1}{2}$, $6\frac{3}{8}$, $10\frac{5}{8}$, and $11\frac{7}{8}$ together. Ans. $36\frac{7}{8}$.

3. Add $12\frac{2}{3}$, $5\frac{4}{9}$, $8\frac{3}{7}$, and $\frac{5}{21}$ together. Ans. $26\frac{7}{9}$.

4. Add $1\frac{2}{3}$, $2\frac{3}{5}$, $7\frac{1}{2}$, and $\frac{4}{9}$ together. Ans. $12\frac{1}{18}$.

5. Add $6\frac{2}{3}$, $\frac{4}{5}$, $1\frac{1}{3}$, and $9\frac{3}{17}$ together. Ans. $17\frac{2}{3}$.

6. Add $1\frac{5}{8}$ and $7\frac{1}{2}$ together. Ans. $9\frac{1}{8}$.

CASE 5.

To add compound fractions, or whole numbers and compound fractions, and mixt numbers.

RULE.

Reduce the compound fractions to single ones, by case the fifth of Reduction; then reduce the single fractions to such as have a common denominator, and add them together as before directed.—When a whole number is given, express it fractionwise by putting 1 for its denominator, then proceed as before.

EXAMPLES.

1. Add $\frac{1}{2}$ of $\frac{7}{8}$ and $\frac{2}{3}$ of $\frac{19}{20}$ together.

$\frac{1}{2}$ of $\frac{7}{8}=\frac{7}{16}$ and $\frac{2}{3}$ of $\frac{19}{20}=\frac{38}{60}$, or $\frac{19}{30}$; now reduce $\frac{7}{16}$ and $\frac{19}{30}$ to a common denominator, and add the results together. Ans. $1\frac{17}{240}$.

2. Add $\frac{1}{3}$ of 95 and $\frac{7}{8}$ of 14 together.

$\frac{1}{3}$ of $\frac{95}{1}=\frac{95}{3}$ and $\frac{7}{8}$ of $\frac{14}{1}=\frac{98}{8}$, or $\frac{49}{4}$; now reduce $\frac{95}{3}$ and $\frac{49}{4}$ to a common denominator, and find the amount. Ans. $43\frac{11}{12}$.

3. Add 6, $\frac{7}{8}$ of $\frac{9}{10}$, $\frac{4}{7}$ of $\frac{1}{2}$, and $7\frac{1}{2}$ together.

$\frac{7}{8}$ of $\frac{9}{10}=\frac{63}{80}$ and $\frac{4}{7}$ of $\frac{1}{2}=\frac{4}{14}$, or $\frac{2}{7}$; now reduce $\frac{63}{80}$, $\frac{2}{7}$, and $\frac{1}{2}$ to a common denominator, and find their amount, to which subjoin the sum of 6 and 7 that were omitted. Ans. $14\frac{321}{560}$.

4. Add $\frac{3}{5}$, $\frac{4}{5}$ of $\frac{1}{3}$ and $\frac{9}{20}$ together. Ans. $1\frac{19}{60}$.

CASE 6.

To add fractions of several denominations.

RULE.

Reduce the given fractions to their proper values or quantities, by case the 11th of Reduction; then add, as in Compound Addition, and to their sum subjoin the sum of the fractional remainders, *(if any,) and the total* will be the answer required.

Examples.

1. Add $\frac{7}{9}$ of a £ sterling, $\frac{3}{10}$ of a shilling, and $\frac{2}{3}$ of a penny together.

$$
\begin{array}{lll}
\overset{\pounds}{1}=\overset{s.}{20} & \overset{s.}{1}=\overset{d.}{12} & \overset{d.}{1}=\overset{qr.}{4} \\
\quad\;\; 7 & \quad\;\; 3 & \quad\;\; 2 \\
9)\overline{140} & 10)\overline{36} & 3)\overline{8} \\
\quad 15\;\overset{d.}{6}\;\overset{qrs.}{2}+\frac{6}{9} & \quad 3\;\overset{qrs.}{2}+\frac{4}{10} & \quad 2\frac{2}{3} \\
\qquad\quad 3\;\;2+\frac{4}{10} & 6\times10\times\;3=180 & \\
\qquad\qquad\;\; 2+\frac{2}{3} & 4\times\;9\times\;3=108 & \\
\overline{15\;\;10\;\;2} & 2\times\;9\times10=180 & \\
\qquad\qquad 1\frac{11}{15} & \qquad\qquad\;\overline{468} & \\
\overline{15\;\;10\;\;3\frac{11}{15}} & 9\times10\times3=\overline{270}=\overset{qr.}{1}\frac{11}{15} &
\end{array}
$$

2. Add $\frac{1}{9}$th of a pound sterling and $\frac{2}{3}$ of a penny together.
Ans. 2s. 3d. $\frac{1}{4}$qr. $\frac{2}{3}$.

3. Add $\frac{1}{2}$ of a lb. Troy to $\frac{7}{12}$ of an oz. Ans. 6oz. 11pwts. 16grs.

4. Add $\frac{4}{7}$ of a ton to $\frac{2}{10}$ of a cwt.
Ans. 12cwt. 1qr. 8lbs. 12oz. 12drs. $\frac{4}{5}$.

5. Add $\frac{5}{8}$ of a lb. of medicine to $\frac{2}{3}$ of an oz. Ans. 7oz. $9\frac{1}{3}$drs.

6. Add $\frac{3}{4}$ of a mile to $\frac{7}{10}$ of a furlong. Ans. 6fur. 28po.

7. Add $\frac{1}{2}$ of a yard, $\frac{2}{3}$ of a foot, and $\frac{3}{4}$ of an inch together.
Ans. 2ft. 2in. 9″.

8. Add $\frac{4}{7}$ of a yard, $\frac{3}{4}$ of a foot, and $\frac{7}{8}$ of a mile together.
Ans. 1540yds. 2ft. 9in.

9. Add $\frac{3}{8}$ of a hhd. of wine and $\frac{4}{9}$ of a gallon together.
Ans. 24gals. $2\frac{1}{3}$pts.

10. Add $\frac{1}{3}$ of a day, $\frac{1}{2}$ of an hour, and $\frac{3}{4}$ of a minute together.
Ans. 8hrs. 30min. 45sec.

11. Add $\frac{1}{2}$ of a sign, $\frac{1}{4}$ of a degree, and $\frac{2}{3}$ of a minute together.
Ans 6si. 7° 30′ 40″.

12. A merchant owned $\frac{3}{8}$ of a ship, valued at $3600.80 cents, and bought $\frac{5}{10}$ of another person's share. What part of the ship belongs to him, and what is it worth? Ans. He owns eleven-sixteenths, which is worth $2475.55cts.

Case 7.

To add mixt fractions, &c.

Rule.

Reduce the mixt fractions to single ones, by case the 10th of Reduction; then reduce the single fractions to such as have a common denominator, and add them together as before directed, and *their sum will be* the answer.

EXAMPLES.

1. Add $\frac{7}{10\frac{1}{2}}$, $\frac{8\frac{1}{4}}{11}$, $\frac{13}{15\frac{1}{6}}$, and $\frac{7\frac{1}{2}}{12}$ together.

1st. $\frac{7}{10\frac{1}{2}}=\frac{2}{3}$, $\frac{8\frac{1}{4}}{11}=\frac{3}{4}$, $\frac{13}{15\frac{1}{6}}=\frac{6}{7}$ and $\frac{7\frac{1}{2}}{12}=\frac{5}{8}$, by case 10th of Reduction.

2d. Reduce the single fractions, viz: $\frac{2}{3}$, $\frac{3}{4}$, $\frac{6}{7}$, and $\frac{5}{8}$, to such as have a common denominator, and find their amount. Ans. $2\frac{151}{168}$.

2. Add $\frac{34}{45\frac{1}{3}}$ and $\frac{42\frac{7}{8}}{49}$ together. Ans. $1\frac{5}{8}$.

3. Add $\frac{73}{131\frac{2}{5}}$ and $\frac{95}{118\frac{9}{12}}$ together. Ans. $1\frac{16}{45}$.

SUBTRACTION OF VULGAR FRACTIONS.

CASE 1.

To find the difference between any two simple fractions that have a common denominator.

RULE.

Subtract the less numerator from the greater one, and set the denominator under the remainder.

EXAMPLES.

From	$\frac{5}{8}$	$\frac{9}{10}$	$\frac{11}{12}$	$\frac{19}{20}$	$\frac{32}{48}$	$\frac{105}{209}$	$\frac{7501}{8000}$	$\frac{11}{30}$	$\frac{101}{270}$	$\frac{99}{100}$	$\frac{14}{15}$.
Take	$\frac{3}{8}$	$\frac{7}{10}$	$\frac{8}{12}$	$\frac{12}{20}$	$\frac{27}{48}$	$\frac{94}{209}$	$\frac{6992}{8000}$	$\frac{4}{30}$	$\frac{99}{270}$	$\frac{78}{100}$	$\frac{7}{15}$.
Answers	$\frac{2}{8}$										
Proof	$\frac{5}{8}$										

CASE 2.

To subtract a fraction or mixt number from a whole number.

RULE.

Subtract the numerator of the given fraction from its denominator and set down the remainder, under which place the denominator, then carry 1 to be subtracted from the whole number.

EXAMPLES.

From	6	1	10	9	1000	7	11	12
Take	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{10}$	$1\frac{1}{2}$	$999\frac{1}{9}$	$\frac{3}{4}$	$5\frac{5}{8}$	8[illegible]
Answers	$5\frac{7}{8}$	$\frac{5}{8}$						
Proof	6	1						

CASE 3.

To find the difference between any two fractions which have *different denominators.*

Rule.

Reduce the given fractions to such as have a common denominator; then subtract the less numerator from the greater one, and place the common denominator under the difference; the result will be the answer, which must be reduced to its lowest terms.

Examples.

1. From $\frac{5}{8}$ take $\frac{1}{2}$.

$5\times2=10$
$1\times8=\ 8$

$$\frac{2}{8\times2=16}=\frac{1}{8}.\ \text{Ans.}$$

2. From $\frac{11}{12}$ take $\frac{3}{4}$. Ans. $\frac{1}{6}$.
3. From $\frac{5}{6}$ take $\frac{4}{5}$. Ans. $\frac{1}{30}$.
4. From $\frac{8}{15}$ take $\frac{9}{20}$. Ans. $\frac{1}{12}$.
5. From $\frac{7}{12}$ take $\frac{1}{4}$. Ans. $\frac{1}{3}$.
6. From $\frac{111}{112}$ take $\frac{3}{4}$. Ans. $\frac{27}{112}$.
7. From $\frac{97}{100}$ take $\frac{3}{7}$. Ans. $\frac{379}{700}$.

Case 4.

To subtract a fraction or mixt number from a mixt number.

Rule.

Reduce the mixt numbers to improper fractions, then reduce the improper fractions to such as have a common denominator, and proceed as in the last case.

Examples.

1. From $12\frac{5}{12}$ take $6\frac{1}{2}$.

First. $12\frac{5}{12}=\frac{149}{12}$ and $6\frac{1}{2}=\frac{13}{2}$

and $\left\{\begin{array}{l}149\times\ 2=298\\ 13\times12=156\end{array}\right\}$ numerators.

$$\frac{142}{12\times2=24}=5\frac{11}{12}.\ \text{Ans.}$$

2. From $13\frac{1}{9}$ take $8\frac{14}{27}$. Ans. $4\frac{16}{27}$.
3. From $6\frac{1}{8}$ take $\frac{2}{8}$. Ans. $5\frac{7}{8}$.
4. From $10\frac{3}{10}$ take $1\frac{7}{12}$. Ans. $8\frac{43}{60}$.
5. From $19\frac{5}{11}$ take $\frac{7}{15}$. Ans. $18\frac{163}{165}$.
6. From $24\frac{1}{4}$ take $11\frac{3}{4}$. Ans. $12\frac{1}{2}$.

Case 5.

To subtract part of a whole number, or compound fraction, from part of another whole number, or compound fraction.

Rule.

1. Reduce the whole numbers to improper fractions by putting 1 for the denominator of each whole number.

2. Reduce all the compound fractions to single ones, then reduce all the single fractions to such as have a common denominator, and proceed as in the third case.

Examples.

1. Subtract $\frac{3}{4}$ of 21 from $\frac{1}{3}$ of 76.

1st. $\frac{3}{4}$ of $\frac{21}{1}=\frac{63}{4}$, and $\frac{1}{3}$ of $\frac{76}{1}=\frac{76}{3}$

and $\left\{\begin{array}{l}76\times4=304\\ 63\times3=189\end{array}\right\}$ numerators.

$$\frac{115}{4\times3=12}=9\frac{7}{12}.\ \text{Ans.}$$

2. From $\frac{2}{3}$ of 19 take $\frac{5}{6}$ of 11. Ans. $3\frac{1}{2}$.
3. From $\frac{109}{110}$ take $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$. Ans. $\frac{163}{220}$.
4. From $14\frac{1}{4}$ take $\frac{2}{3}$ of 19. Ans. $1\frac{7}{12}$.

Case 6.

To find the difference between two fractions of different denominations.

Rule.

Reduce the given fractions to their proper values or quantities by the 11th case in Reduction, and proceed as in Compound Subtraction.

Examples.

1. From $\frac{7}{9}$ of a pound sterling take $\frac{3}{10}$ of a shilling.

$\frac{7}{9}$ of a £=15s. 6d. 2qrs.+$\frac{6}{9}$, or $\frac{2}{3}$ and $\frac{3}{10}$ of 1s.=3d. 2qrs.+$\frac{4}{10}$ or $\frac{2}{5}$.

Before the subtraction can be performed, the fractional remainders $\frac{2}{3}$ and $\frac{2}{5}$ must be reduced to such fractions as have a common denominator, to be used instead of the fractional remainders.

Wherefore $\left\{\begin{matrix} 2\times5=10 \\ 2\times3=\ \ 6 \end{matrix}\right\}$ New numerators,

and $3\times5=15$, the c. d.; consequently, the new fractions are $\frac{10}{15}$ and $\frac{6}{15}$, which must be used instead of $\frac{2}{3}$ and $\frac{2}{5}$.

	s.	d.	qrs.
From	15	6	$2+\frac{10}{15}$
Take	0	3	$2+\frac{6}{15}$
Ans.	15	3	$0+\frac{4}{15}$.

2. From $\frac{3}{4}$ of a £ take $\frac{3}{4}$ of 1 shilling. Ans. 14s. 3d.
3. From $\frac{3}{5}$ of an oz. Troy take $\frac{7}{8}$ of a pwt. Ans. 11pwts. 3grs.
4. From $\frac{1}{2}$ of a cwt. take $\frac{7}{12}$ of a lb. Ans. 1qr. 27lbs. 6oz. 10drs. $\frac{2}{3}$.
5. From 1 E. ell take $\frac{7}{10}$ of a quarter. Ans. 1yd. 1na. $\frac{1}{5}$.
6. From 7 weeks take $9\frac{7}{10}$ days. Ans. 5w. 4da. 7hrs. 12min.

Case 7.

To subtract one mixt fraction from another, &c.

Rule.

Reduce the mixt fractions to single ones, then reduce the single fractions to such as have a common denominator, and in all other respects proceed as before directed.

Examples.

1. From $\frac{8\frac{1}{4}}{11}$ take $\frac{12}{18\frac{3}{4}}$

Now $\frac{8\frac{1}{4}}{11}=\frac{3}{4}$ and $\frac{12}{18\frac{3}{4}}=\frac{16}{25}$

Then $\left\{\begin{matrix} 3\times25=75 \\ 16\times\ \ 4=64 \end{matrix}\right.$

$\overline{\ \ 11\ \ }$

And $4\times25=100$ Ans.

2. From $\frac{34\frac{1}{2}}{46}$ take $\frac{6\frac{3}{4}}{27}$ Ans. $\frac{1}{2}$.

3. From 20 take $\frac{42\frac{7}{8}}{49}$ Ans. $19\frac{1}{8}$.

4. From $\frac{95}{118\frac{3}{12}}$ take $\frac{73}{131\frac{2}{5}}$ Ans. $\frac{11}{45}$.

5. From 9 take $\frac{7}{10\frac{1}{2}}$ Ans. $8\frac{1}{3}$.

MULTIPLICATION OF VULGAR FRACTIONS.

RULE.

1. Reduce mixt numbers to improper fractions, and compound or mixt fractions to single ones.

2. Multiply all the numerators together for a new numerator, and all the denominators for a new denominator; then form the fraction and reduce it to its lowest terms—the result will be the answer required.

NOTE.—When any number, either whole or mixed, is multiplied by a fraction, the product is always less than the multiplicand in the same proportion as the multiplying fraction is less than a unit, or 1.

EXAMPLES.

1. Multiply $\frac{3}{7}$ by $\frac{5}{8}$. Ans. $\frac{15}{56}$.
2. Multiply $\frac{1}{3}$ of $\frac{3}{4}$ by $\frac{7}{10}$ of $\frac{11}{12}$. Ans. $\frac{77}{450}$.
3. Multiply $7\frac{1}{4}$ by $8\frac{1}{2}$. Ans. $61\frac{5}{8}$.
4. Multiply $4\frac{1}{2}$ by $\frac{1}{8}$. Ans. $\frac{9}{16}$.
5. Multiply $12\frac{3}{5}$ by $\frac{1}{3}$ of 7. Ans. $29\frac{2}{5}$.
6. Multiply $1\frac{1}{2}$ by $1\frac{1}{2}$. Ans. $2\frac{1}{4}$.
7. Multiply $7\frac{1}{2}$ by $9\frac{1}{4}$. Ans. $69\frac{3}{8}$.
8. Multiply $\frac{2}{9}$ of $\frac{3}{4}$ by $\frac{5}{8}$ of $7\frac{1}{2}$. Ans. $\frac{25}{32}$.
9. Multiply $\frac{5}{9}$ of $\frac{1}{5}$ by $1\frac{4}{5}$ of 5. Ans. 1.
10. Multiply $3\frac{1}{4}$ by $\frac{2}{3}$ of 7. Ans. $15\frac{1}{6}$.
11. Multiply $\frac{34\frac{1}{2}}{46}$ by $\frac{7}{10\frac{1}{2}}$. Ans. $\frac{1}{2}$.
12. Multiply $\frac{18\frac{3}{4}}{25}$ by $\frac{65}{97\frac{1}{2}}$ of 16. Ans. 8.
13. Multiply $3\frac{1}{4}$ by $\frac{2}{3}$ of 5, and that product by $\frac{3}{4}$ of $\frac{3}{5}$. Ans. $4\frac{7}{8}$.
14. Multiply $2\frac{1}{3}$ by $1\frac{1}{7}$, and that product by $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$. Ans. $\frac{2}{3}$.

DIVISION OF VULGAR FRACTIONS.

RULE.

Prepare the given fractions, if necessary, by the rules in reduction; then, invert the divisor and multiply all the upper terms together for a new numerator, and all the lower ones for a new denominator; next, form the new fraction and reduce it to its lowest terms, and the result will be the answer required.

NOTE.—1. When the dividend is greater than the divisor the quotient will be greater than the dividend, but when the dividend is less than the divisor, then the quotient will be less than the dividend, and in the same proportion, as a unit is greater or less than the *dividing fraction*.

2. *Multiplication* and Division prove each other reciprocally.

Examples.

1. Divide $\frac{17}{21}$ by $\frac{3}{5}$.
Now $\frac{3}{5}$ inverted make $\frac{5}{3}$ and
$\frac{5}{3}\times\frac{17}{21}=\frac{85}{63}=1\frac{22}{63}$ Ans.
2. Divide $\frac{4}{7}$ by $\frac{2}{3}$. Ans $\frac{6}{7}$.
3. Divide $9\frac{1}{6}$ by $\frac{1}{2}$ of 7. Ans. $2\frac{13}{21}$.
4. Divide $\frac{13}{19}$ by $\frac{3}{4}$. Ans. $\frac{52}{57}$.
5. Divide $\frac{15}{56}$ by $\frac{5}{8}$. Ans. $\frac{3}{7}$.
6. Divide $61\frac{5}{8}$ by $8\frac{1}{2}$. Ans. $7\frac{1}{4}$.
7. Divide $61\frac{5}{8}$ by $7\frac{1}{4}$. Ans $8\frac{1}{2}$.
8. Divide $\frac{1}{2}$ of 7 by $\frac{3}{6}$. Ans. 7.
9. Divide 4 by $\frac{7}{8}$. Ans. $4\frac{4}{7}$.
10. Divide $\frac{7}{8}$ by 4. Ans. $\frac{7}{32}$.
11. Divide $1\frac{1}{2}$ by $4\frac{8}{10}$. Ans. $\frac{5}{16}$.
12. Divide 99 by 108. Ans. $\frac{22}{108}$.
13. Divide $\frac{1}{2}$ of $\frac{2}{3}$ by $\frac{2}{3}$ of $\frac{3}{4}$. Ans. $\frac{2}{3}$.
14. Divide $\frac{1}{5}$ of 19 by $\frac{2}{3}$ of $\frac{3}{4}$. Ans. $7\frac{3}{5}$.
15. Divide $\frac{2}{3}$ of $\frac{3}{4}$ by $\frac{1}{2}$ of $\frac{2}{3}$. Ans. $1\frac{1}{2}$.
16. Divide $68\frac{4}{5}$ by $5\frac{3}{8}$. Ans. $12\frac{4}{5}$.

THE SINGLE RULE OF THREE DIRECT, IN VULGAR FRACTIONS.

Rule.

Prepare the given fractions, if necessary, by the rules in reduction; then, state the question as in whole numbers, with the first term of the stating inverted as in division; next, multiply all the terms together, and the product will be the fractional answer, which must be reduced to its proper value or quantity by the 11th case of Reduction.

Examples.

1. If $\frac{3}{5}$ of a yard cost $\frac{7}{12}$£ what will $\frac{6}{15}$ of an English ell cost?

1st. $\frac{3\times4=12}{5\times5=25}$ (yds. qrs.) of an E. E. 2d. As $\overset{E.E.}{\frac{25}{12}} : \overset{£}{\frac{7}{12}} :: \overset{E.E.}{\frac{6}{15}} \cdots \frac{1050}{2160}=\frac{35}{72}$£

And $\frac{35}{72}$£$=$9s. 8d. 2qrs.$+\frac{2}{3}$qr. Ans.

2. If $\frac{6}{15}$ of an English ell cost $\frac{35}{72}$£, what will $\frac{3}{5}$ of a yd. cost? Ans. 11s. 6d.
3. If $1\frac{1}{4}$yds. of linen cost \$$1\frac{1}{2}$, how much will $16\frac{1}{4}$yds. come to? Ans. \$19.50cts.
4. If $\frac{1}{4}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of 2lbs. cost $37\frac{1}{2}$cts., what will $96\frac{7}{8}$lbs. cost? Ans. \$$145.31\frac{1}{4}$cts.
5. If $\frac{1}{3}$ of a lb. less by $\frac{1}{6}$, cost $18\frac{3}{4}$cts., what will 14lbs. less by $\frac{1}{3}$ of 2lbs. come to? Ans. \$15.30cts.
6. If 2 ounces of silver cost \$$2\frac{1}{4}$, what will $\frac{3}{4}$ of a lb. cost? Ans. \$$10.12\frac{1}{2}$cts.
7. If $\frac{3}{4}$ of $\frac{5}{8}$ of a ship cost \$10000, what is the remainder worth? Ans. \$$11333.33\frac{1}{3}$cts.
8. A merchant owning $\frac{4}{5}$ of a vessel, sold $\frac{3}{8}$ of his share for \$957, what was the vessel worth at that rate? Ans. \$$1794.37\frac{1}{2}$cts.
9. If $3\frac{1}{2}$ times $3\frac{1}{2}$lbs. of cinnamon cost \$$7\frac{1}{2}$, what is the value of $\frac{1}{2}$ of $\frac{1}{3}$ of $12\frac{1}{4}$lbs. at the same rate? Ans. \$1.25cts.

10. If $1\frac{1}{3}$ bushels of apples cost $39\frac{3}{5}$cts., what will $3\frac{2}{5}$ bushels cost? Ans. \$$1.01\frac{3}{25}$.

11. How much will $4\frac{3}{5}$lbs. of cheese cost, at $12\frac{1}{5}$ per lb.? Ans. $55\frac{4}{5}$.

12. If $\frac{3}{5}$ of an English ell cost $\frac{2}{3}$ of \$2.28cts., what will 7 ells of the same come to? Ans. \$$17.73\frac{1}{3}$cts.

13. If a yard of broad cloth cost $15\frac{5}{8}$s., what will 4 pieces, each containing $27\frac{3}{8}$yds., come to? Ans. 85£ 10s. 11d. $\frac{1}{4}$qr.

14. A merchant bought $4\frac{3}{4}$ pieces of lutestring, each containing $22\frac{3}{8}$yds., at 8s. 9d. per yd.; how much did the whole amount to? Ans. 46£ 9s. 11d. $2\frac{1}{8}$qrs.

15. If a wedge of gold, weighing $17\frac{3}{7}$lbs. be worth $679\frac{5}{7}$£, what is the value of $\frac{3}{16}$ of a grain? Ans. 2d.

16. A merchant owning $\frac{3}{4}$ of a ship, sold $\frac{1}{4}$ of his interest therein for 250£; what was the whole ship worth at that rate? Ans. 1333£ 6s. 8d.

17. If $\frac{3}{16}$ of a house be valued at $273\frac{1}{8}$£, what are $\frac{5}{32}$ of the same worth? Ans. 227£ 12s. 1d.

18. If $\frac{5}{8}$ of a gallon of wine cost $\frac{5}{8}$£; what will $\frac{5}{9}$ of a tun come to? Ans. 140£.

19. If 1cwt. cost $1\frac{1}{2}$£, what will $3\frac{1}{3}$lbs. come to? Ans. $10\frac{1}{2}$d. $\frac{6}{7}$.

20. If $\frac{3}{5}$ of a lb. of tea cost $\frac{7}{15}$£, what will $\frac{3}{14}$ cost? Ans. 3s. 4d.

21. If a staff $5\frac{2}{3}$ feet long cast a shadow of 6 feet, how high is that steeple whose shadow is 153 feet? Ans. $144\frac{1}{2}$ft.

22. A grocer bought $5\frac{3}{8}$cwt. of sugar at $6\frac{3}{4}$d. per lb. which he bartered for tea at $8\frac{5}{8}$s. per lb.; how much tea did he receive for the sugar? Ans. $43\frac{1}{69}$lbs.

23. How much tobacco can I buy for 4s. $9\frac{3}{5}$d. at the rate of $7\frac{1}{5}$d. per lb.? Ans. 8lbs.

THE SINGLE RULE OF THREE INVERSE, IN VULGAR FRACTIONS.

Rule.

Prepare the given fractions, if necessary, as before directed; then state the question as in whole numbers, with the last term of the stating inverted—next, multiply all the terms together, and the product will be the fractional answer, with which proceed as in direct proportion.

Examples.

1. How many yards of shalloon, that is $\frac{3}{4}$ of a yard wide, will line $9\frac{1}{2}$ yards of cloth that is $2\frac{1}{2}$ yards wide?

1st. $2\frac{1}{2}=\frac{5}{2}$, and $9\frac{1}{2}=\frac{19}{2}$ 2d. As $\overset{yd.}{\frac{5}{2}} : \overset{yd.}{\frac{19}{2}} :: \overset{yd.}{\frac{4}{3}} .. \overset{yd.}{\frac{380}{12}}=\overset{yds.}{31\frac{2}{3}}$. Ans.

2. How much in length, that is $7\frac{1}{2}$ inches wide, will make a superficial foot? As $\frac{12}{1}$in. : $\frac{12}{1}$in. :: $\frac{2}{15}$in. .. $\frac{288}{15}=19\frac{1}{5}$in. Ans.

3. If $3\frac{1}{4}$yds. of cloth, that is $1\frac{1}{8}$yd. wide, be sufficient to make a cloak, how much green Persian, which is but $\frac{3}{4}$ of a yard wide, will be required to line it? Ans. 4yds. $3\frac{1}{2}$qrs.

4. What quantity of shalloon, that is $\frac{3}{4}$ of a yard wide, will be required to line $7\frac{1}{2}$yds. of cloth that is $1\frac{1}{2}$yds. wide? Ans. 15yds.

5. If 16 men finish a piece of work in $28\frac{1}{8}$ days, in what time will 12 men complete the same? Ans. 37d. 9h. 20'.

6. How much cloth, that is $\frac{3}{4}$ of a yard wide, must be bartered for $20\frac{5}{8}$yds. of the same quality, which is $1\frac{1}{4}$yd. wide? Ans. $34\frac{3}{8}$yds.

7. If, when the price of wheat is $6\frac{1}{4}$s. a bushel, the penny loaf weighs 9oz., how much must it weigh when the wheat sells at $4\frac{1}{2}$s. a bushel? Ans. $12\frac{1}{2}$oz.

8. How many pieces of merchandise, at $20\frac{1}{8}$s. a piece, must be given for 240 pieces at $12\frac{1}{2}$s. a piece? Ans. $149\frac{11}{161}$ pieces.

9. How many yards of matting, that is $1\frac{1}{2}$ft. wide, will be necessary to cover a floor which is 18ft. wide and 30ft. long? Ans. 120y.

10. If I lend my friend \$3150 for $6\frac{2}{3}$ months, what sum must he lend me for $3\frac{1}{2}$ years to requite my kindness. Ans. \$500.

THE DOUBLE RULE OF THREE DIRECT, IN VULGAR FRACTIONS.

Rule.

Prepare the given fractions, if necessary, by Reduction, and work with two single statings in direct proportion, remembering to make the answer of the first stating the middle term of the second: or, prepare the fractions as before, and state the question according to the rule given in whole numbers, with the two first terms of the stating inverted; then multiply all the terms together, and their product will be the fractional answer, which must be reduced to its proper value or quantity.

Examples.

1. If £$13\frac{1}{3}$ principal gain £$1\frac{1}{12}$ interest in $\frac{3}{4}$ of a year, what will £50 principal gain in $\frac{5}{12}$ of a year, and at what rate per cent. per annum?

£$13\frac{1}{3} = \frac{40}{3}$£ the principal.

£$1\frac{1}{12} = \frac{13}{12}$£ the interest.

$$\begin{array}{l} \quad\;\; \text{£ } p.\ yr.\ \ \text{£ } int.\quad \text{£ } pr.\ yr.\qquad\quad \text{£} \\ \text{As } \frac{3}{40}\times\frac{4}{3} : \frac{13}{12} :: \frac{50}{1}\times\frac{5}{12} \;..\; \frac{39000}{17280} = 2\text{£ 5s. 1d. } 2\tfrac{2}{3}\text{qrs. interest. Ans.} \end{array}$$

Now to find the rate per cent. per annum.

$$\begin{array}{l} \quad\;\; \text{£ } p.\ yr.\ \ \text{£ } int.\quad \text{£ } pr.\ yr.\qquad\quad \text{£} \\ \text{As } \frac{1}{50}\times\frac{12}{5} : \frac{3900}{1728} :: \frac{100}{1}\times\frac{1}{1} \;..\; \frac{4680|000}{432|000} = 10\text{£ 16s. 8d. the rate p. c.} \end{array}$$

2. Three sailors having been abroad $9\frac{1}{4}$ months, received £40 ...

I demand how much 100 sailors must have for $28\frac{3}{7}$ months, at that rate?

$$\text{As } \overset{sa.}{\frac{1}{3}} \times \overset{mo.}{\frac{4}{37}} : \overset{£}{\frac{603}{15}} :: \overset{sa.}{\frac{100}{1}} \times \overset{mo.}{\frac{199}{7}} \;..\; \overset{£}{\frac{47998800}{11655}} = 4118£\ 6s.\ 0d.\ \tfrac{1}{4}qr.\ \tfrac{29}{259}.\ \text{Ans.}$$

3. If $\frac{3}{4}$ of a yard of cloth, that is $\frac{7}{8}$ of a yard wide, cost $\frac{2}{5}$£, what is the value of $\frac{5}{8}$ of a yard which is $1\frac{3}{4}$ of a yard wide, the cloth being of the same quality?

$$\text{As } \overset{y.l.}{\tfrac{4}{3}} \times \overset{y.w.}{\tfrac{8}{7}} : \overset{£}{\tfrac{2}{5}} :: \overset{y.l.}{\tfrac{5}{8}} \times \overset{y.w.}{\tfrac{7}{4}} \;..\; \overset{£}{\tfrac{2240}{3360}} = 13s.\ 4d.\ \text{Ans.}$$

4. A man and his wife, having wrought 1 day, earned $4\frac{5}{8}$s.; I demand how much they must have for $10\frac{1}{2}$days, when their two sons helped them?

$$\text{As } \overset{per.}{\frac{1}{2}} \times \overset{d.}{\frac{1}{1}} : \overset{s.}{\frac{37}{8}} :: \overset{per.}{\frac{4}{1}} \times \overset{d.}{\frac{21}{2}} \;..\; \overset{s.}{\frac{3108}{32}} = 4£\ 17s.\ 1\tfrac{1}{2}d.\ \text{Ans.}$$

5. If 9 students spend £$10\frac{7}{8}$ in 18 days, what sum will 20 students spend in 30 days? Ans. 39£ 18s. 4d. $\frac{20}{81}$.

6. If 3 men, in $19\frac{1}{2}$ days, earn £$8\frac{9}{10}$, how much must 20 men have for $100\frac{1}{4}$ days' work, at the same rate? Ans. 305£ 0s. 8d. $\frac{8}{35}$.

7. If 5 persons drink $7\frac{4}{5}$ gallons of beer in a week, how many gallons will be drank in $22\frac{1}{2}$ weeks, when 3 persons more join them? Ans. $208\frac{1}{8}$ gallons.

8. If 50£ principal, in $\frac{5}{12}$ of a year, gain 2£ 5s. 1d. $2\frac{2}{3}$qrs., how much will $13\frac{1}{3}$£ principal gain in $\frac{3}{4}$ of a year? Ans. 1£ 1s. 8d.

9. If $2\frac{1}{4}$ yards of cloth, $1\frac{3}{4}$ yards wide, cost $\$3\frac{3}{5}$, what is the value of $38\frac{1}{4}$ yards that is 2 yards wide? Ans. \$255.

10. If the expenditures of two brothers, for tuition fees, &c., during $\frac{3}{4}$ of a year, amount to $\$56\frac{1}{4}$, how much specie will liquidate the expenses of 3 students $5\frac{1}{3}$ years, at the same rate? Ans. \$600.

11. If 12 persons use $1\frac{1}{3}$lb. of tea in $\frac{1}{12}$ of a year, how much should a family of 8 persons provide for $\frac{1}{2}$ year? Ans. $4\frac{1}{2}$lbs.

12. If the carriage of $12\frac{1}{2}$cwt. 150 miles amount to $\$18\frac{3}{4}$, how much will I have to pay for the carriage of $6\frac{1}{4}$ cwt. 240 miles, at the same rate? Ans. \$15.

THE DOUBLE RULE OF THREE INVERSE, IN VULGAR FRACTIONS.

Rule.

Prepare the given fractions, if necessary, by Reduction; then state the question according to the rule given in whole numbers, with the third and fourth terms of the stating inverted; next, multiply all the terms together, and their product will be the fractional answer, which must be reduced to its proper value or quantity.—Or, work by two single statings, one of which will be direct and the other inverse proportion.

EXAMPLES.

1. If 50£ principal, in $\frac{5}{12}$ of a year, gain 2£ 5s. 1d. $2\frac{2}{3}$qrs. interest, in what time will $13\frac{1}{3}$£ gain 1£ 1s. 8d. interest?

First. 2£ 5s. 1d. $2\frac{2}{3}$qrs.$=\frac{6500}{3}$ of a farthing.
Second. $13\frac{1}{3}$£$=\frac{40}{3}$ of a pound sterling.
Third. 1£ 1s. 8d.$=\frac{1040}{1}$ of a farthing.

$$\text{Fourth. As } \overset{yr.}{\frac{5}{12}}\times\overset{\pounds\ p.}{\frac{50}{1}} : \overset{qr.\ in.}{\frac{3}{6500}}\times\overset{\pounds\ p.}{\frac{3}{40}} :: \overset{qr.\ in.}{\frac{1040}{1}} \;..\; \overset{yr.}{\frac{234|000}{312|000}}=\frac{3}{4} \text{ of a year or 9m. Ans.}$$

2. What principal will be required to gain $\$289\frac{3}{4}$ interest in $4\frac{3}{4}$ years, at $\$7\frac{5}{8}$ per cent. per annum?

$$\text{As } \overset{\$\ p.}{\frac{100}{1}}\times\overset{yr.}{\frac{1}{1}} : \overset{\$\ in.}{\frac{8}{61}}\times\overset{yr.}{\frac{4}{19}} :: \overset{\$\ in.}{\frac{1159}{4}} \;..\; \$800. \text{ Ans.}$$

3. If the carriage of 60cwt. 20 miles cost $\$14\frac{1}{2}$, what weight can I have carried 30 miles for $5\frac{7}{16}$ dollars. Ans. 15cwt.

4. Seven men, with their wives, upon examining into their expenditures for 20 weeks past, found that they had expended $40\frac{4}{5}$£; in what time will 50 people expend $21\frac{6}{7}$£ in the same proportion?
Ans. 3 weeks.

5. A man with his family, which in all were 5 persons, usually drank $7\frac{4}{5}$ gallons of cider in one week; in what time will $280\frac{4}{5}$ gallons be consumed in the same family, when 3 persons more are added to it? Ans. 22 weeks.

6. If a man travel $266\frac{2}{3}$ miles in $12\frac{1}{2}$ days, when they are $12\frac{4}{5}$ hours long, how many days of $9\frac{3}{8}$ hours in length will he require to travel a journey of $734\frac{3}{8}$ miles? Ans. 47 days.

DECIMAL FRACTIONS.

Decimal Fractions, like whole numbers and vulgar fractions, may be reduced from one denomination to another, without altering their intrinsic value.

REDUCTION OF DECIMAL FRACTIONS.

CASE 1.

To reduce a vulgar fraction to its equivalent decimal value.

RULE.

Make a point on the right hand of the given numerator, and annex a sufficient number of ciphers to it; then divide by the denominator of the given fraction, and the quotient will be the decimal required.

Examples.

1. Reduce $\frac{3}{16}$ to its equivalent decimal.

16)3.0000(.1875. Ans.
16
—
140
128
—
120
112
—
80
80
—

2. Reduce $\frac{4}{7}$ to its equivalent decimal.

7)4.0000
———
Ans. .5555+5rem.

In the above example we obtain a continual repetition of the figures 5, &c. without any possibility of coming to an end. And here, it may not be improper to observe, that three decimal places are sufficient in all common business; but when a greater degree of exactness is necessary in the calculation, the decimals may be continued as far as the nature of the case may require.

3. Reduce $\frac{74}{99}$ to a decimal.

99)74.00000(.74747. Ans.
693
—
470
396
—
740
693
—
470
396
—
740
693
—
47 rem.

4. Reduce $\frac{1}{3}$ to a decimal. Ans. .3333+1 rem.

5. Reduce $\frac{7}{28}$ to a decimal. Ans. .17857+.

6. Reduce $\frac{5}{26}$ to a decimal. Ans. .1923+.

7. Reduce $\frac{1}{8}$ to a decimal. Ans. .125.

8. Reduce $\frac{1}{4}$ to a decimal. Ans. .25.

9. Reduce $\frac{1}{2}$ to a decimal. Ans. .5.

10. Reduce $\frac{3}{4}$ to a decimal. Ans. .75.

11. Reduce $\frac{2}{5}$ to a decimal. Ans. .4.

12. If the student will memorize the several respective decimals, answering to $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{4}{5}$, $\frac{1}{8}$, $\frac{3}{8}$, $\frac{5}{8}$, and $\frac{7}{8}$, he will be amply remunerated for his labor, in the facility with which he will be enabled to make decimal calculations.

Case 2.

To reduce any sum or quantity of different denominations to its equivalent decimal value.

Rule.

1. Divide the given sum or quantity in its lowest denomination

mentioned, by the proposed integer reduced into the same denomination, and the quotient will be the answer; or

2. Write the given numbers in a perpendicular column, proceeding orderly from the least to the greatest denomination given. Then divide each of them by such a number as will reduce it to the next superior denomination, and annex the quotient as decimal parts on the right hand of the succeeding number, &c.—the last quotient will be the answer as before.

EXAMPLES.

1. Reduce 17s. 4½d. to the equivalent decimal of a £.

```
 4 | 2.0
12 | 4.5..
20 | 17.375..
     ------
```

Ans. .86875£. Ans. by the 2d. rule, it being the shortest.

2. Reduce 19s. 11d. ¾qrs. to the equivalent decimal of a £.

```
 4 | 3.00
12 | 11.750
20 | 19.979+2 rem.
     ----------
```

.99895£. Ans.

3. Reduce 1oz. 10pwts. 12grs. to the decimal of a lb. Troy wt.

```
lb. oz.    oz. pwts. grs.
1=12     |  1  10  12
   20    | 20
  ---    | --
  240    | 30
   24    | 24
  ---    | --
  960    | 132
  480    | 60
  ---    | --
 5760)     732.0(.127. Ans.
           5760  by the first
           ----       rule.
           15600
           11520
           -----
            40800
            40320
            -----
             480 rem.
```

The same by the second rule.

```
24 | 12.0
20 | 10.500
12 |  1.525000
      --------
```

Ans. .127083+Ans.

4. Reduce 18 grains to the decimal of a lb. Troy. Ans. .003125.

5. Reduce 20 grains to the decimal of a lb. Troy. Ans. .0415.

6. Reduce 14lbs. 8ozs. to the decimal of an cwt. Ans. .1294625.

7. Reduce 6ozs. 4drs. 1sc. 10 grs. to the decimal of a lb. Apothecaries' weight. Ans. .546875.

8. Reduce 2qrs. 2 nails to the decimal of a yard. Ans. .625.

9. Reduce 3 pecks to the decimal of a barrel. Ans. .15.

10. Reduce 1 gallon to the decimal of a bushel. Ans. .125.

11. Reduce 6 gallons 2 quarts 1 pint of ale to the decimal of a barrel. Ans. .20703125.

12. Reduce 7 gallons 3 quarts 1 pint of wine to the decimal of a hogshead. Ans. .125.

13. *Reduce 440 yards* to the decimal of a mile. Ans. .25.

14. Reduce 2 roods 10 perches to the decimal of an acre. Ans. .5625.

15. Reduce 30min. 45sec. to the decimal of an hour. Ans. .5125.

16. Reduce 15min. 15sec. to the decimal of a degree. Ans. .2625.

Case 3.

To find the value of any given decimal in the known terms of the integer.

Rule.

1. Multiply the given decimal by the next inferior denomination, and point off as many places at the right hand of the product as there are places in the given decimal.

2. Multiply the figures on the right hand of the decimal point by the next less denomination, and point off as many places in the product as there are figures in the multiplicand.

3. Proceed in the same manner through all the denominations of the given integer, and the several parts standing on the left hand of the decimal points will form the true value of the given decimal.

Note.—This case and case the 2nd mutually prove each other.

Examples.

1. Reduce .86875 decimals of a £ to their proper value. Ans. 17s. $4\frac{1}{2}$d.

```
 .86875£
     20
 -------
17.37500
     12
 -------
 4.500
 4
 -----
```

2. Reduce .99895 decimals of a £ to their proper value. Ans. 19s. 11d. $\frac{3}{4}$qrs.

```
 99895
    20
 -------
19.97900
    12+2 rem.
 -------
11.750
    4
 -----
 3.00
```

3. What is the value of .127083+4 decimals of a lb. Troy wt.? Ans. 1oz. 10pwt. 12grs.

4. What is the value of .003125 decimals of a lb. Troy? Ans. 18gr.

5. What is the value of .0415 decimals of a lb. Troy? Ans. 20gr.

6. Reduce .1294625 decimals of an cwt. to their proper value? Ans. 14lbs. 8ozs.

7. What is the value of .546875 decimals of a lb. Apothecaries' weight? Ans. 6ozs. 4drs. 1sc. 10grs.

8. What is the value of .625 decimals of a yd.? Ans. 2qrs. 2nails.

9. What is the value of .15 of a barrel of corn? Ans. 3 pecks.

10. What is the value of .125 decimals of a bushel. Ans. 1 gallon.

11. What is the value of .20703125 decimals of a barrel of ale?
Ans. 6gals. 2qts. 1pt.

12. What is the value of .125 decimals of a hhd. of wine?
Ans. 7gals. 3qts. 1pt.

13. What is the value of .25 decimals of a mile? Ans. 440yds.

14. Reduce .5625 decimals of an acre to their proper value.
Ans. 2roo. 10per.

15. Reduce .5125 decimals of an hour to their proper value.
Ans. 30min. 45sec.

16. What is the value of .2625 decimals of a degree?
Ans. 15min. 15sec.

Case 4.

To reduce any number of shillings, pence, and farthings, to the decimal of a pound sterling by *inspection.*

Rule.*

1. Set down the greatest even number of the given shillings for the first decimal figure, to which add .05, or .050 decimals, if the shillings be odd, and the result will be the decimal value of the shillings. 2. Set the farthings contained in the given pence and farthings, (increased by 1 when they exceed 12, and by 2 when they exceed 36,) under the 2d and 3d places of the above decimal and add them to it, and that sum will be the correct answer, within $\frac{1}{2}$ of $\frac{1}{1000}$ part of a pound.

Examples.

1. Reduce 17s. $4\frac{1}{2}$d. to the equivalent decimal of a £ by inspection.

.8=$\frac{1}{2}$ the greatest even number of s.
.050=the value of the odd shilling.
18=the quarters contained in $4\frac{1}{2}$d.
Add 1 because the qrs. exceed 12.

Ans. .869 which is too much by .00025£.

2. Reduce 19s. $11\frac{3}{4}$d. to the equivalent decimal of a £ by inspection.

.9=$\frac{1}{2}$ of 18 shillings.
.050=for the odd shilling.
47=qrs. in $11\frac{3}{4}$d.
Add 2 because the qrs. exceed 36.

Ans. .999 which is too much by .00005£.

*1. The reason of this rule is plain—because 2s. is one-tenth part of 1£, and the decimal expression of one-tenth is .1=one-tenth; therefore, as 2s. is .1 of a £, 1s. must be one-twentieth=.05, consequently the $\frac{1}{2}$ of any even number of shillings will be their equivalent decimal value, but when they are odd the decimal of 1s. must be added to complete the answer.

2. If 1000qrs., instead of 960, were equal to £1, each quarter would be one-thousandth part of 1£=.001£, but 960qrs. increased by their 24th part=1000qrs.; consequently, any number of quarters, increased by their 24th part, will result in so many decimals of a £, but *for the sake of brevity,* we add 1 when they exceed 12, and 2 when they exceed 36.

Case 5.

To find the value of any decimal of a pound sterling by *inspection*.

Rule.

Double the first figure after the decimal point, for shillings, and add 1 to the result, if the second figure be 5 or more; if the second figure be 5 or more, deduct 5 from it and call the remainder so many farthings, from which take 1 if they exceed 12, and 2 if they are above 36, and change the residue to pence, &c. When there are only two figures in the given decimal, annex a cipher to it, and proceed as above directed, and you will obtain the answer required.

Examples.

1. Reduce .869 decimals of a £ to their proper value.

s.
16=the double of .8
Add 1 for 5 in the second place, which must be taken from 6, and there remains 19qrs., from which take 1, and the residue is 18=6½d.
6½d.

Ans. 17 6½d. which proves the 1st query.

2. Reduce .999 decimals of a £ to their proper value.

	.9	99	
	2	5	
	18	49	remainder.
Add	1	2	
		4)47	residue.
Ans.	19	11¾	

3. What is the value of .375£? Ans. 7s. 6d.
4. What is the value of .425£? Ans. 8s. 6d.
5. What is the value of .125£? Ans. 2s. 6d.
6. What is the value of .188£? Ans. 3s. 9d.
7. What is the value of .75£? Ans. 15s.

THE SINGLE RULE OF THREE DIRECT IN DECIMALS.

General Rule.

State the questions by the rule given in whole numbers, and if the first and third terms are of different denominations, reduce them to the same; then multiply the second and third terms together, and divide the product by the first term—the quotient thence arising will be the answer in decimals, and of like name with the middle term.

Note.—Be very careful in pointing off the decimals according to the rules given in Multiplication and Division of Decimals.

Examples.

1. If 1.6 of a cwt. of sugar cost 3£ 12.76s. what will 3 hhds. each weighing 11cwt. 3qrs. 10.12lbs. come to at the same rate?

```
Cwt.    £   s.        cwt. qrs.  lbs.
1.6 : 3   12.76 :: 11   3   10.12=wt. of 1 hhd.
  4   20                          3
----  -----         ---------------
 6.4  72.76          35   2    2.36=wt. of 3 hhds.
 28                   4
----                ---
 512                142
128                  28
----                ----
179.2               1138
                    284
                    ------
                    3978.36
                      72.76
                    -------
                    2387016
                   2784852
                   795672
                  2784852          2,0
                  ---------        ---
          179.2)289465.4736(161,5.320
                1792        Ans. 80£ 15s. 3d. ¾qrs.+.36 rem.
                ----
                11026
                10752
                -----
                  2745
                  1792
                  ----
                   9534
                   8960
                   ----
                    5747
                    5376
                    ----
                     3713
                     3584
                     ----
                     1296 rem.
```

2. What will 4.7 tons of iron amount to, when I pay at the rate of $4.972 *per cwt.?* Ans. $467.36cts. 8ms.

3. What will 986 feet of plank cost at $1.4375 per hundred feet? Ans. $14.17cts. 3¾ mills.

4. If .0625 of a cargo of tea cost $100, how much will the whole cargo amount to? Ans. $1600.

5. If a man expend .125 of his estate in 2.5 years time, how long will the whole last him? Ans. 20 years.

6. If 673 bushels of wheat cost $769.239, what is one bushel of it worth? Ans. $1.14cts. 3 mills.

7. When iron is sold at $5.04 by the cwt. what is one pound of it worth? Ans. 4cts. 5 mills.

8. What will 4 hhds. of rum (containing 79.5, 84, 101.5 and 112 gallons respectively) cost at $1.125 per gallon? Ans. $424.12½cts.

9. If 16.75cwt. of Crawley steel cost $360.125ms. what must I pay for 1 quarter? Ans. $5.37cts. 5ms.

10. A bankrupt stands indebted to sundry creditors in the sum of $7066.25cts. and his effects are found to be worth only $4416.40cts. 625 decimals, which he gives up to his creditors; how much do they receive on the dollar? Ans. 6cts. 2½ms.

11. A grocer bought 3 pipes of wine (containing 120.5, 124, and 126.75 gallons, respectively,) at 93cts. 7½ms. per gallon; what sum did they amount to? Ans. $348.04cts. 6m.+.875 decimals.

12. A grocer bought 5.8 tuns of oil for $963.66875, but by misfortune it chanced to leak out 59.9 gallons; how must he sell the rest by the gallon to be no loser? Ans. 68¾cts. per gallon.

13. A brewer made a quantity of beer, which cost him $300, and afterwards sold it out at $5.76cts. per barrel, by which he gained $135.84cts. in the whole; I demand the quantity that was brewed? Ans. 75 barrels, 24 gallons.

14. If a man's yearly income be $1682.70cts. how much may he expend, one day with another, to lay up $678.95 cents at the termination of the year? Ans. $2.75cts.

15. A merchant bought 3cwt. 1.5qr. of cloves, at the rate of 47 cents by the lb., and sold the whole quantity for $220.91cts.; what did he gain or lose by the bargain? Ans. He gained $43.25cts.

SINGLE RULE OF THREE INVERSE, IN DECIMALS.

Rule.

State the questions by the rule given in whole numbers, and if the first and third terms are of different denominations, reduce them to the same—then, multiply the first and second terms together, and *divide the product* by the third; the quotient thence arising will be *the answer in* decimals, and of like name with the middle term.

EXAMPLES.

1. What quantity of shalloon that is .75 of a yard wide, will line 7.5yds. of cloth that is 1.5 yards wide?

yds. w. yds. long. yds. w.
As 1.5 : 7.5 :: .75
1.5

375
75

.75)11.25(15yds. Ans.
75

375
375

2. How many yards of carpeting 2ft. 6in. wide, will cover a floor that is 27ft. long and 20ft. wide?

ft. w. ft. long ft. w.
20 : 27 :: 2.5
20

2.5)540.0

3)216 feet.

Ans. 72 yards.

3. How much in length that is $4\frac{1}{2}$ inches wide will make one superficial foot? Ans. 32in. long.

4. How many yards of cloth .75yd. wide, are equal in quantity to 30yds. that is $1\frac{1}{4}$yds. wide? Ans. 50yds.

5. How many yards of canvass, 1 English ell wide, will line 20 yards of say, that is .75 of a yard wide? Ans. 12yds.

6. How many yards of flannel .75 of a yard wide will line a cloak that has in it $5\frac{3}{4}$yds. of cloth $1\frac{5}{8}$ yds. wide? Ans. 12yds. 2qrs. 3na.+

7. How many yards of paper 1.25yds. wide will be required to hang round a room, which is 20yds. in circumference and 4yds. in height? Ans. 64yds.

8. If, when wheat is 60cts. a bushel, the cent loaf weigh 5 pounds, what must it weigh when wheat is but 40cts. a bushel? Ans. 12oz.

DOUBLE RULE OF THREE DIRECT, IN DECIMALS.

The questions are stated and the operations performed according to the directions given in whole numbers—due regard being had to pointing off the decimals.

EXAMPLES.

1. If 9 students expend $36.25 in 18 days, what sum will 20 students expend in 30 days? Ans. $134.25, 9m.+.

2. If a man and his wife earn $2.50cts. in one day, how much ought they to have for 10.5 days work, when their two sons helped them? Ans. $52.50cts.

3. If 5 men drink 7.8 gallons of perry in a week, how much will suffice 8 men 22.5 weeks? Ans. 280.8 gals.

4. If 3 men earn $29.25cts. in 19.5 days, how much ought 24 men to have for 100.25 days? Ans. $1203.

5. If 12 oxen graze 16.25 acres in 20 days, how many acres of pasture will serve 24 oxen 100 days? Ans. 162.5 acres.

6. Three sailors having been abroad 9.25 months, received $134; therefore, how much ought 100 sailors to have for their service 28¾ months? Ans. $13810.45cts.+.

7. If a family of 12 people use 1.125lbs. of tea in 1 month, how much will serve 8 people 6 months? Ans. 4lbs. 8ozs.

8. If 2.25yds. of cloth, 1¾yd. wide, cost $3.60 cents, what will 38¼yds. that is 2yds. wide, amount to? Ans. $255.

9. If $100 principal gain $7.62cts. 5m. in one year, how much will $800 principal gain in 4.75 years? Ans. $289.75c.

DOUBLE RULE OF THREE INVERSE, IN DECIMALS.

The questions are stated and the operations performed according to the directions given in whole numbers, due regard being had to pointing off the decimals.

Examples.

1. If $100 principal gain $3.50cts. interest in one year, what principal will gain $38.50cts. in 1.25 of a year? Ans. $880.

2. What principal will be required to gain $289.75cts. interest in 4.75 years, at $7.62cts. 5m. per cent. per annum? Ans. $800.

3. If $100 principal gain $7.62½cts. in 1 year, in what time will $800 principal gain $289.75cts. interest? Ans. 4yrs. 9mo.

4. In what time will $880 principal gain $38.50 interest, at $3. 50cts. per cent. per annum? Ans. 1yr. 3mo.

5. If 5 men can mow 52.2 acres of meadow in 6 days, how many men will be required to mow 417.6 acres of meadow in 12 days? Ans. 20 men.

6. A cellar 22.5 feet long, 17.3 feet wide, and 10.25 feet deep, being dug in 2.5 days by 6 men, working 12.3 hours in each day, how many days of 8.2 hours each will 9 men require to excavate another cellar which measures 45ft. long, 34.6ft. wide, and 12.3ft. deep?

	feet.		feet.	
	22.5	long.	45	long.
	17.3	wide.	34.6	wide.
	389.25		1557.0	
	10.25	deep.	12.3	deep.
	3989.8125		19151.10	
	8.2	hours.	12.3	hours.

d. m.		men.	
As 2.5×6 :	32716.46250×9	::	235558.530
6	9		15.0
15.0	294448.16250		)3533377.95000(12 days. Ans.
			3533377 95000

EXCHANGE.

1. Exchange is the paying or giving of the money, weights, or measures of one place or country for an equal value thereof in the money, weights, or measures of another place or country.

2. Real money is a piece of metal coined by the authority of the state or country, and is current at a certain price, by virtue of the said authority, or of its own intrinsic value.

3. Imaginary money is a denomination used to express a sum of money, of which there is no real species extant.

4. Agio is the difference between bank money and current money.

5. Par signifies the equality of money in value.

6. The course of exchange is the current or running price of exchange, and is frequently above or below par, varying according to the occurrences of trade or demand for money.

7. Current money is that which passes from hand to hand in the receiving and paying of such sums of money as are due from one man to another in the transaction of commerce.

8. Usance is a certain time allowed for the payment of bills of exchange, but variable according to the law and custom of the place where the bill is made, compared with that on which it is drawn.

9. Days of grace.—It is customary, in some places, to allow three days to the time mentioned in the bill, which are called days of grace, on the last of which the bill must be demanded, and if it is not paid must be immediately protested.

I. THE U. STATES WITH ENGLAND AND SCOTLAND.

The denominations of English or Sterling Money are—

4 farthings	make	1 penny.
12 pence	make	1 shilling.
20 shillings	make	1 pound sterling.
21 shillings	make	1 guinea sterling.

NOTE.—The federal dollar of the United States is 4s. 6d. sterling, and 1£ sterling is equal to $4.44cts. 4m. in the federal currency of the United States.

1. *To change Sterling Money to Federal Currency.*

RULE 1.—If the given sum be pounds only, annex three ciphers to it; then multiply by 4 and divide the product by 9, and the quotient will be the answer in cents.

2. If there be shillings, &c. given with the pounds, annex their equivalent decimal value (found by inspection) to the pounds, and proceed according to the first rule, the result will be the answer required in cents, &c.

EXAMPLES.

1. Change 9£ sterling to federal currency.

£9.000
4

9)36.000

4000cts.=$40 Ans.

2. Change 389£ 17s. 6d. sterling to federal money.

£389.875decimals.
4

9)1559.500

Ans. $1732.77,7$\frac{7}{9}$

3. Change 72£ sterling into federal currency. Ans. $320.
4. Change 756£ 19s. 6d. sterling into fed. currency. Ans. 3364.33\frac{1}{3}$.

2. To change Federal Currency to Sterling Money.

RULE.—Multiply the federal currency in cents by 9, and divide the product by 4; then point off three figures in the quotient for decimals, and reduce them to their proper value (by inspection); the result will be the answer in sterling money.

NOTE.—If mills should occur in the given sum, then point off four figures in the quotient, and proceed as above directed.

EXAMPLES.

1. Change $40 United States currency to sterling money.

$40.00
9

4)36000

9.000=9£ Ans.

2. Change $1732.77cts. 7$\frac{7}{9}$m. U. S. currency to sterling money.

1732.777\frac{7}{9}$
9

4)15595000

Ans. 389.8750=389£ 17s. 6d.

3. Change $320 U. S. currency to sterling money. Ans. 72£.
4. Change 3364.33\frac{1}{3}$cts. into sterling money. Ans. 756£ 19s. 6d.

APPLICATION.

N. B.—When the rate of exchange is above or below the rate given in the note under the table, the operation must be performed by the Rule of Three or by Practice.

EXAMPLES.

5. A merchant in Virginia bought goods in Liverpool amounting to 943£ 17s. sterling; how much federal money will discharge the debt, exchange being at $3.80cts. per pound sterling?

As 1£ : $3.80cts. :: 943£ 17s. .. $3586.63cts. Ans.

6. A merchant in London bought goods in Virginia amounting to $3586.63cts.; how much sterling money will discharge the debt, exchange being at $3.80cts. per pound sterling?

As $3.80cts. : 1£ :: $3586.63cts. .. 943£ 17s. Ans.

7. A Virginia merchant has remitted \$8837.96cts. $8\frac{3}{4}$m. to his correspondent in London; what is the amount thereof in sterling money, exchange being at \$4.75cts. per £? Ans. 1860£ 12s. 6d.

8. A merchant in Richmond received an invoice of goods from Liverpool amounting to 196£ 14s. 6d. sterling, which he sold immediately at 25 per cent. advance; what sum did he receive in federal money? Ans. \$1092.91$\frac{2}{3}$cts.

II. WITH IRELAND.

The denominations of money in Ireland are the same as in England, but different in value.

The par of exchange between England and Ireland is $8\frac{1}{3}$ per cent.; that is, 100£ sterling is equal to 108£ 6s. 8d. Irish money. 1£ sterling=1£ 1s. 8d. Irish, and 1 guinea sterling=1£ 2s. 9d. in Ireland. 1£ of Irish money=\$4.10cts. federal money, and \$1 federal money=4s. $10\frac{1}{2}$d. in Ireland.

1. *To change Irish Money to Federal Currency.*

RULE.—Reduce the given sum to half-pence, then annex two ciphers to it, and divide by 117, (the half-pence in a dollar Irish currency,) and the quotient will be the answer in cents.

AN EXAMPLE.

Change 278£ 15s. 9d. Irish to federal money.

```
  £   s. d.
 278  15 9
  20
 ----
 5575
   12
 -----
 66909
     2
 ----------
117)133818.00($1143.74cts. Ans.
 ----------
     42 rem.
```

2. *To change Federal Money to Irish Currency.*

RULE.—Multiply the given sum in cents by 117, and divide the product by 100; the quotient will be the answer in half-pence.

AN EXAMPLE.

Change \$1143.74cts.+42 rem. to Irish currency. Ans. 278£ 15s. 9d.

APPLICATION.

1. A merchant in Dublin drew a bill of exchange on a merchant in Philadelphia for 400£ 17s. $9\frac{1}{2}$d. Irish currency; how much federal money will discharge the bill? Ans. \$1644.67cts. 5m.+25 rem.

2. A merchant of Dublin purchased flour in Baltimore, amounting to \$1644.67cts. 5ms.+25 rem.; how much Irish currency will discharge the debt? Ans. 400£ 17s. $9\frac{1}{2}$d.

3. How much federal money will pay for 50 pieces of Irish linen, each containing 20 English ells, at 2s. $4\frac{1}{2}$d. Irish per yd.?

Ans. $608.97cts. +

4. A New York merchant shipped 400 barrels of flour at $9.75 cents per barrel, to his agent in Belfast; how much Irish currency may he expect in return, when the charges for shipping, &c. cost him $100? Ans. £926 5s.

III. WITH FRANCE.

The denominations of money in France are—

12 deniers	make	1 sol or sous=$9\frac{1}{4}$ mills.
20 sols	make	1 liver-turnois=$18\frac{1}{2}$cts.
3 livers	make	1 ecu of exchange=$55\frac{1}{2}$cts.
6 livers	make	1 crown=111cts.
24 livers	make	1 louis-d'or=$4.44cts.

Note.—The present money of account in France, is francs and centimes, or hundredths—80 francs make 81 livers, consequently, 1 franc=18cts. 7 mills, $\frac{5}{16}$m. or 18cts. 73125 decimals of a cent in federal money.

1. *To change Livers Turnois to Francs, &c.*

Rule.—Multiply the livers turnois by 80 and divide the product by 81, the quotient will be francs—multiply the remainder by 100, continue the division, and the result will be centimes.

Examples.

1. Change 5469 livers turnois to francs, &c.

```
       5469
         80
       ------ francs. centimes.
 81)437520(5401.48+
    405      Answer.
    ---
     325
     324
     ---
       120
        81
       ---
        39.00
        324
        ----
          660
          648
          ---
           12 rem.
```

2. Change 3802 livers 19 sols to francs, &c.

```
     liv.
   3802.95 decimals.
          80
   ---------
81)304236.00(3756 francs. Ans.
   243
   ---
    612
    567
    ---
     453
     405
     ---
      486
      486
      ---
```

3. *Change* 10125 livers turnois to francs, &c. Ans. 1000 francs.

4. *Change* 2437 livers 17 sols 6 deniers to francs, &c.

Ans. 2407frs. 77cts.+63.

2. *To change Francs, &c. to Livers Turnois, &c.*

Rule.—Multiply the francs, &c. by 81, adding in the remainder, and divide the product by 80—the quotient will be the answer in livers turnois, &c.

Examples.

1. Change 2407 francs, 77 centimes, to livers turnois, &c.

```
      2407.77+63
            81
      ------
      240840
    1926216
  ----------
8,0)19503,0.00
```

Ans. 2437 liv. 17 sols. 6 deniers.

2. Change 5401 francs 48 cen. to livers turnois, &c. Ans. 5469 livers.

3. Change 3756 francs to livers turnois. Ans. 3802 livers 19 sols.

4. Change 10000 francs to livers turnois. Ans. 10125 liv.

3. *To change French Money to Federal Currency.*

Rule.—Multiply the French money by the corresponding federal value of one integer found in the table, and the product will be the answer in federal money.

Examples.

1. Change 56 louis d'ors to federal money. Ans. $248.64cts.
2. Change 476 ecues of exchange to federal money. Ans. $264.18c.
3. Change 762 crowns to federal money. Ans. $845.82cts.
4. Change 8744 sols to federal money. Ans. $80.88cts. 2ms.

4. *To change Federal Currency to French Money.*

Rule.—Divide the given sum in mills by 185, (the mills in 1 liver) and the quotient will be the answer in livers turnois; which may be changed to any other denomination required.

Examples.

1. Change $248.64cts. to louis d'ors. Ans. 56 louis d'ors.
2. Change $264.18cts to ecues of exchange. Ans. 476 ecues.
3. Change $845.82cts. to crowns. Ans. 762 Fr. cro.
4. Change $80.88cts. 2ms. to sols. Ans. 8744 sols.

IV. WITH SPAIN.

There are two kinds of money in Spain, namely:

1. The money of Velon, denominated *current* dollars.
2. The money of plate, denominated *hard*, or *plate* dollars.

34 maravedies	make	1 rial velon=5cts.
2 rials velon	make	1 rial of plate=10cts.
8 rials of plate	make	1 piaster, or pezo=80cts.
10 rials of plate	make	1 hard dollar=100cts.
11 rials of plate	make	1 crown=110cts.
36 rials of plate	make	1 Spanish pistole=360cts.

1. *To change Spanish Money, to Federal Currency.*

RULE.—Multiply the Spanish money by the corresponding value of the integer in federal money, and the product will be the answer required.

EXAMPLES.

1. Change 172 pistoles to federal money. Ans. \$619.20cts.
2. Change 4764 rials velon to federal money. Ans. \$238.20cts.
3. Change 7462 rials of plate to federal money. Ans. \$746.20cts.
4. Change 698 piasters to federal money. Ans. \$558.40cts.

2. *To change Federal Currency to Spanish Money.*

RULE.—Divide the federal sum in cents by the corresponding federal value of any Spanish coin that may be required, and the quotient will be the answer.

EXAMPLES.

1. Change \$619.20cts. to Spanish pistoles. Ans. 172.
2. Change \$238.20cts. to rials velon. Ans. 4764.
3. Change \$746.20cts. to rials of plate. Ans. 7462.
4. Change \$558.40cts. to piasters. Ans. 698.

V. WITH BARCELONA, SARAGOSSA, &c.

The denominations of money Barcelona, Saragossa, &c. are—

17 maravedies	make	1 soldo	= 06¼cts.
2 soldoes	make	1 rial of old plate	= 12½cts.
8 rials old plate	make	1 hard dollar	=100cts.
10 rials old plate	make	1 ducat	=125cts.
30 rials old plate	make	1 pistole	=360cts.

1. *To change the Currencies of Barcelona, &c. to Federal Money.*

RULE.—Multiply the Barcelona, &c. currency by the corresponding federal value of one piece of the given coin, and the product will be the answer in federal money.

EXAMPLES.

1. Change 946 pistoles to federal money. Ans. \$3405.60cts.
2. Change 1872 ducats to federal money. Ans. \$2340.00cts.
3. Change 8746 rials to federal money. Ans. \$1093.25cts.
4. Change 9996 soldoes to federal money. Ans. \$624.75cts.

2. *To change Federal Money to the Currencies of Barcelona, &c.*

RULE.—Divide the federal sum by the corresponding federal value of the required coin, and the quotient will be the answer.

EXAMPLES.

1. *Change* \$3405.60cts. to pistoles. Ans. 946.
2. *Change* \$2340.00cts. to Spanish ducats. Ans. 1872.

3. Change $1093.25cts. to rials of old plate. Ans. 8746.
4. Change $624.75cts. to soldoes. Ans. 9996.

VI. WITH PORTUGAL.

The merchants in Portugal keep their accounts in milrees and rees, allowing 1000 rees to make one milree, which is equal to 5s. 7½d. sterling, and $1.25cts. in the United States. There are various sorts of money in circulation among traders, &c., which may be seen in the following table:

20 rees	make	1 vintin=2½cts.
5 vintins	make	1 testoon=12½cts.
4 testoons	make	1 crusado=50cts.
10 testoons	make	1 milree=125cts.

1. *To change the Currency of Portugal to Federal Money.*

RULE.—Reduce the Portugal money to rees, then divide by 8, and the quotient will be the answer in cents; or, increase the rees in the given sum by their fourth part, and the result will be the answer in mills.

EXAMPLES.

1. Change 579 milrees 740 rees to federal money.

First method.	Second method.
8)579.740 rees.	4)579.740 rees.
	144.935
$724.67½ Ans.	
	Ans. 724.67,5 as before.

2. Change 156 testoons 4 vintins to federal money. Ans. $19.60c.
3. Change 894 milrees 672 rees to federal money. Ans. $1118.34c.
4. Change 1755 crusadoes to federal money. Ans. $877.50c.

2. *To change Federal Money to Portugal Currency.*

RULE.—Multiply the given sum in cents by 8, and the product will be rees, from which point off three figures for rees, and those on the left hand of the point will be milrees; or you may divide the given sum in cents, &c. by the federal value of any coin that may be required, and the quotient will be the answer sought.

EXAMPLES.

1. Change $724.67cts. 5ms. to milrees, &c. Ans. 579m. 740r.
2. Change $19.60cts. to testoons, &c. Ans. 156t. 4v.
3. Change $1118.34cts. to milrees, &c. Ans. 894m. 672r.
4. Change $877.50cts. to crusadoes. Ans. 1755cru.

VII. WITH JAMAICA AND BERMUDAS.

The merchants in Jamaica, &c. keep their accounts in pounds, shillings, &c. The Spanish dollar passes for 6s. 8d.—three of them *are equal to 20s.=1£* Jamaica currency.

1. *To change Jamaica, &c. Currency to Federal Money.*

RULE.—Annex two ciphers to the pence in the given sum, then divide by 80, (the pence in 1$ Jamaica currency,) and the quotient will be the answer in cents.

EXAMPLES.

1. Change 45£ Jamaica currency to federal money. Ans. $135.
2. Change 54£ 12s. 6d. Jamaica currency to federal money. Ans. $163.87½cts.
3. How much federal money must be remitted to Kingston, in Jamaica, to pay for 8519lbs. of sugar, at 3£ 10s. per 100lbs.? Ans. $894.49½cts.
4. A Virginia merchant bought 9103lbs. of sugar, in Jamaica, at 75s. per 100lbs.; how much federal money will discharge the debt? Ans. $1024.08⅗cts.

2. *To change Federal Money to Jamaica, &c. Currency.*

RULE.—Divide the given sum by 3—the quotient will be pounds and decimal parts; or deduct one-fifth part of the cents in the given sum, and the remainder will be pence.

EXAMPLES.

1. Change $135 into Jamaica currency. Ans. 45£.
2. Change $163.87½ into Jamaica currency. Ans. 54£ 12s. 6d.
3. If 8519lbs. of sugar cost $894.49½cts., what are 100lbs. of it worth in Jamaica currency? Ans. 3£ 10s.
4. A merchant in Jamaica sold 9103lbs. of sugar to a Virginia merchant for $1024.08⅗cts.; how much did it cost by the hundred in Jamaica currency? Ans. 3£ 15s.

VIII. WITH BARBADOES.

The denominations of money in Barbadoes are the same as in Jamaica, but different in value. The Spanish dollar passes for 6s. 3d.—$3⅕=20s.=1£ Barbadoes currency.

1. *To change Barbadoes Currency to Federal Money.*

RULE.—Increase the pence in the given sum by one-third part, and the result will be the answer in cents.

EXAMPLES.

1. Change 49£ 11s. 9d. Barbadoes currency to federal money. Ans. $158.68cts.
2. Change 164£ 16s. 8d. Barbadoes currency to federal money. Ans. $527.46⅔cts.

2. *To change Federal Money to Barbadoes Currency.*

RULE.—Subtract one-fourth part from the cents in the given sum, and the remainder will be the answer in pence.

EXAMPLES.

1. Change $527.46⅔cts. to Barbadoes currency.

```
   $  cts.
 4)527.46⅔
   131.86⅔
 ---------
12)395 60 . pence.
 ---------
 2,0)329,6 8d.
 ---------
```

Ans. 164£ 16s. 8d.

2. Change $158.68cts. to Barbadoes currency. Ans. 49£ 11s. 9d.

3. Change $469.60cts. to Barbadoes currency. Ans. 146£ 15s.

IX. WITH MARTINICO, TOBAGO, AND ST. CHRISTOPHER'S, &c.

The above islands are inhabited by French and English promiscuously. The French keep their accounts in livers, sols, and deniers—the English in pounds, shillings, and pence.

NOTE.—The Spanish dollar passes for 8¼ livers in some places, and in others for 9 livers. A current dollar is 8s. 3d., and a round dollar passes for 9 shillings.

1. *To change the currency of any place where the dollar passes for 8s. 3d. to Federal Money.*

RULE.—Annex two ciphers to the pence in the given sum, then divide by 99, (the pence in 8s. 3d.,) and the quotient will be the answer in cents; or, increase the pence by one 99th part for the answer in cents.

EXAMPLES.

1. Change 25£ 3s. 3d. Martinico currency to federal money.
Ans. $61.

2. Change 397£ 13s. Tobago currency to federal money.
Ans. $964.

3. How much federal money must be remitted to a merchant in St. Christopher's island, to pay for a quantity of groceries, charged in the invoice at 81£ 9s. 4½d. of that currency? Ans. $197.50cts.

4. A grocer of Norfolk, in Virginia, purchased a quantity of salt in Turk's island, amounting by the invoice to 103£ 16s. 11¼d; how much federal money will discharge the debt? Ans. $251.75cts.

2. *To change Federal Money to the currency of any place where the dollar passes for 8s. 3d.*

RULE.—Multiply the cents in the given sum by 99, then divide the product by 100, and the quotient will be the answer in pence; or, subtract 100th part from the cents, and the remainder will be the answer as before.

Examples.

1. Change $61 federal money to Martinico cur. Ans. 25£ 3s. 3d.
2. Change $964 federal money to Tobago cur. Ans. 379£ 13s.
3. Change $197.50cts. to pounds, shillings, &c. Ans. 81£ 9s. 4½d.
4. Change $251.75c. to pounds, shillings, &c. Ans. 103£ 16s. 11¼d.

8. To change livers to dollars of 8¼ *livers each.*

Rule.—Multiply the livers by 4, and divide the product by 33; or, divide the given number of livers by 8¼, decimally, and the quotient by either operation will be dollars.

Examples.

1. Change 10692 livers to Spanish dollars. Ans. $1296.
2. Change 15477 livers to Spanish dollars. Ans. $1876.
3. Change 3762 livers to Spanish dollars. Ans. $456.
4. Change 6253 livers 10 sols to Spanish dollars. Ans. $758.

X. WITH HAMBURG.

The denominations of money in Hamburg are—

12	deniers	make	1 stiver=2$\frac{1}{12}$ cents.
16	stivers	make	1 mark=33⅓ cents.
3	marks	make	1 rix-dollar=100 cents.
4	marks	make	1 silver ducatoon=133⅓ cents.
7½	marks	make	1 pound Flemish=250 cents.
Also, 12	pence Flem.	make	1 shilling Flemish=12½ cents.
20	shillings	make	1 pound Flemish.

Note.—The current money of Hamburg is inferior to bank money, (commonly called banco,) the agio or rate per cent. being variable.

1. To change Hamburg Currency into Bank Money.

Rule.—As 100, with the agio added, is to 100; so is the given sum in current money to the bank money required.

Examples.

1. Change 560 marks 8 stivers, current money, into bank money, the agio being 18 marks per cent.

$$\begin{array}{l} \textit{m.c.} \\ 100 \\ \quad 18 \text{ agio.} \\ \hline \end{array}$$

$$\text{As } 118 : \overset{\textit{m. banco.}}{100} :: \overset{\textit{m. c.}}{560}\ \overset{\textit{sti.}}{8} \therefore \overset{\textit{m. ban.}}{475} \text{ the answer.}$$

2. Change 2366 marks, current money, into bank money, the agio being 20 marks per cent. Ans. 1971mks. 10⅔sti. banco.

2. To change Bank Money into Current Money.

Rule.—As 100 marks banco is to 100 marks current, with the *agio added,* so is the given sum in bank money to the currency *required.*

EXAMPLES.

1. Change 475 marks banco into current money, the agio being 18 marks per cent.

marks cur.
100
18 agio.

m. banco. m. ban. m. sti. currency.
As 100 : 118 :: 475 .. 560 8 the answer.

2. Change 1971 marks 10⅔ stivers, banco, into current money, the agio being 20 per cent. Ans. 2366 marks current money.

3. *To change Bank Money to Federal Money.*

RULE.—Divide the given sum in marks by 3, and the quotient will be dollars, &c. If stivers be given with the marks, add their equivalent federal value to the above quotient, and that sum will be the answer required; or, reduce the given sum to stivers and multiply them by $2\frac{1}{12}$—the product thence arising will be the answer in cents.

EXAMPLES.

1. Change 4967 marks 8 stivers, bank money, into federal money.

m. banco.
3)4967
1655.66⅔c.
8 sti.=16⅔
Ans. $1655.83⅓cts.

m. sti.
4967 8
16
79480
$2\frac{1}{12}$
$1655.83⅓cts. as before.

2. Change 12843 marks banco into federal money. Ans. $4281.

3. Change 18640 currency into bank money, and then into federal, the agio being 25 per cent. Ans. 14912 m. b. and $4970.66⅔.

4. *To change Federal Money into Marks Banco, &c.*

RULE.—Multiply the given sum in cents by 3, then point off two figures at the right hand of the product, and you will have the answer in marks and decimal parts.

EXAMPLES.

1. Change $1655.83⅓ cents to marks banco, &c. of Hamburg.

$ cts.
1655.83⅓
3
Ans. 4967.50=4967 marks and 8 stivers, banco.

2. Change $970.66⅔ cents to marks banco, and then to current money of Hamburg, the agio being 25 per cent.

Ans. 14912 marks banco, and 18640 marks current money.

XI. WITH HOLLAND.

The denominations of money in Holland are—

8 pennings	make	1 groat=1cent.
16 pennings	make	1 stiver=2cts.
20 stivers	make	1 guilder, or florin=40cts.
6 guilders	make	1 pound Flemish=240cts.
2½ guilders	make	1 rix-dollar=100cts.
5 guilders	make	1 ducat=200cts.

To change current money into bank money, and the contrary—work by the rules given under exchange with Hamburg.

Examples.

1. Change 794 guilders 15 stivers, current money, into bank florins, the agio being 4⅜ per ct.

Ans. 761flor. 8st. 11pen.+

guil. g. st. pen.
4⅜=4 7 8
100

104 7 8 : 100 flor. :: 794 g. 15 st.
20 20

2087 15895
16 16

33400 254320
100

334,00)254320,00(761 guil.

146 rem.
20

334)2920(8sti.

248 rem.
16

334)3968(11pe.

294 rem.

2. Change 761 florins 8 stivers 11 pennings, bank money, into current, agio at 4⅜ per cent.

g. g. st. p.
4⅜=4 7 8
100

g. 100 : 104 7 8 :: g. st. p. 761 8 11
20 20 20

2000 2087 15228
16 16 16

32000 33400 243659
33400

8138210600
Add 294 rem.

32,000)8138240,000

16)254320pen.

2,0)1589,5sti.

Ans. 794g. 15sti.

3. Change 823 guilders 9⅕ stivers, current money, into bank, agio at 4½ per cent. Ans. 788 guilders banco.

4. Change 788 guilders, bank money, into current, the agio being *4½ per cent.* Ans. 823guilders 9¼ stivers.

XII. WITH ITALY, &c.

The money of Italy is very different in value—the bankers in Genoa keep their accounts in pezzoes, piasters or dollars, soldi, and denari.

12 denari	make	1 soldi=5cts.
20 soldi	make	1 pezzo=100cts.

The above value is the money of exchange; but, in general, accounts are kept in lire, soldi, and denari. The liver of Genoa is equal to only one-fifth of the exchange money; and the money of Leghorn is equal to one-third of the same.

12 denari	make	1 soldi=1ct.
20 soldi	make	1 lire, or liver=20cts.
30 soldi	make	1 testoon=30cts.
6 testoons	make	1 geroni=180cts.

The bankers and merchants in Venice keep their accounts in ducats, sols, and deniers.

12 deniers	make	1 sol=4cts 6½ mills. +
20 sols	make	1 ducat=93cts. +

The money of Venice is of three kinds, namely: bank, banco-current, and picoli. The bank money is 20 per cent. better than banco-current, and banco-current 20 per cent. better than picoli money. The money of exchange is only imaginary, 100 ducats of which is equal to 120 ducats current—the difference is called agio.

1. *To change Italian Money to Federal Currency.*

RULE.—Multiply the given number of coins by the corresponding federal value of one piece, and the product will be the answer in federal money.

EXAMPLES.

1. Change 589 pezzoes 17 soldi 6 denari to federal money.
Ans. \$589.87½cts.
2. Change 75 geroni 4 testoons 24 soldi to federal money.
Ans. \$136.44cts.
3. Change 758 Venice ducats to federal money. Ans. \$704.94c.

2. *To change Federal Currency to Italian Money.*

RULE.—Divide the federal sum by the corresponding federal value of one of the greatest coin required, then divide the remainder (if any) by the federal value of one of the next inferior coin, and so on—the several quotients will compose the answer required.

EXAMPLES.

1. Change \$589.87cts. 5m. to pezzoes, soldi, &c.
Ans. 589pez. 17sol. 6 denari.
2. Change \$136.44cts. to geroni, &c. Ans. 75ger. 4 tes. 24sol.
3. *Change \$704.94*cts. to Venice ducats. Ans. 758 ducats.

ARBITRATION OF EXCHANGES.

This Rule is used by merchants to determine which is the best way of remitting money from one country to another.

RULE.

The operation is performed by conjoined proportion, or by the Single Rule of Three Direct.

EXAMPLES.

1. Suppose a merchant in Richmond has \$3530 at Amsterdam, which he can have remitted by way of Lisbon at 840 rees per dollar; thence to Richmond at \$1.30cts. per milree. Or by way of Nantz at 5$\frac{2}{5}$ livers per dollar; thence to Richmond at \$1,10cts. per crown of 6 livers. It is required to arbitrate these exchanges, that is, to choose the most advantageous route.

\$1.00 at Amsterdam=840 rees at Lisbon.
1000 rees at Lisbon =1.30cts. in Richmond.
\$3530 in Amsterdam.

$$\frac{\$3530\times\$1.30\text{cts.}\times840\text{ rees}}{1000\times1.00}=\frac{3854760.00}{1000.00}=\$3854.76\text{cts. Ans.}$$

by the way of Lisbon.

Again—\$1 at Amsterdam =5$\frac{2}{5}$ livers at Nantz.
6 livers at Nantz=110cts at Richmond.
\$3530 in Amsterdam.

$$\frac{\$3530\times1.10\text{cts.}\times5.4\text{liv.}}{6\times1}=\frac{20968.200}{6}=\$3494.70.\text{ Ans. by Nantz.}$$

\$3854.76cts.—\$3494.70=\$360.06cts. in favor of remitting by the way of Lisbon.

Performed by the Single Rule of Three.

1st. As \$1 : 840 rees :: \$3530 .. 2965200 rees.
2d. As 1000 rees : \$1.30cts. :: 2965200 rees .. \$3854.76cts. Ans. as before, by the way of Lisbon.

Again—As \$1 : 5.4liv. :: \$3530 .. 190620liv.
2d. As 6liv. : \$1.10 :: 190620 .. \$3494.70cts. Ans. as before by the way of Nantz.

The first method is preferable on account of its brevity.

2. A merchant in London has 500 piasters in Leghorn, for which he can draw directly at 52d. sterling per piaster; but choosing to try a circular route, he sent them to Venice, at 95 piasters for 100 ducats banco; thence to Cadiz, at 350 maravidies per ducat banco; thence to Lisbon, at 630 rees per piaster of 272 maravidies; thence to Amsterdam, at 48d. Flemish for 400 rees; thence to Paris, at 54d. Flemish per crown; thence to London, at 30d. sterling per crown. What is the arbitrated price between London and Leghorn, *per piaster*, and what is gained or lost by this circular remittance without reckoning expenses?

95 piasters	=	100 ducats banco.
1 ducat banco	=	350 maravidies.
272 maravidies	=	630 rees at Lisbon.
400 rees at Lisbon	=	48 pence Flemish.
54d. Flemish	=	1 crown at Paris.
1 crown at Paris	=	30d. sterling
		500 piasters

$$\frac{500\times30\times48\times630\times350\times100}{95\times400\times272\times95}=\frac{15876000000|000}{558144|000}=\overset{£.}{118}\ \overset{s.}{10}\ \overset{d.}{4\tfrac{1}{4}}+$$

As 500pia. : 118£ 10s. 4¼d. :: 1pia. .. 56⅞d.+ arbitrated price. Ans.

The amount received by the circular route 118£ 10s. 4¼d.+
500 piasters at 52d. each = 108 6 8

The merchant gained 10 3 8¼ Ans.

3. Amsterdam changes with London at 34s. 3d. Flem. per pound sterling, and with Lisbon at 52d. Flemish for 400 rees; how then ought the exchange to go between London and Lisbon?

Ans. 75⅜d.+per milree.

4. A merchant in Richmond has 225£ sterling in London, which he can draw for at 54d. sterling per dollar; but choosing to try a circular route, he orders it to be sent to Dublin, at 100£ sterling for 109£ Irish; thence to Hamburg, at 12½ marks banco, for 1£ Irish; thence to Amsterdam, at 33 florins for 40 marks banco; thence to Copenhagen, at 5 rials for 2 rix dollars of Denmark; thence to Bremen, at 3 marks for 1 rix dollar of Denmark; thence to Russia, at 5 marks for two rubles; thence to Bordeaux, at 5 francs per ruble; thence to Cadiz at 18 rials of plate for 10 francs; thence to Lisbon, at 1250 rials of plate, for 100 milrees; thence to Leghorn, at 750 soldi for 88 milrees; thence to Smyrna, at 2 soldi per piaster; thence to Jamaica, at 24d. Jamaica currency, for 1 piaster; and thence to Richmond in Virginia, at 80d. Jamaica currency, for $1 United States currency. What did he gain or lose by the above circuitous remittance? Ans. The merchant gained $117.42cts.+

100£ sterling	=	100£ Irish.
1£ Irish	=	12½ marks at Hamburg.
40 marks in Hamburg	=	33 florins at Amsterdam.
5 florins at Amsterdam	=	2 rix dollars of Den.
1 rix dollar of Denmark	=	3 marks in Bremen.
5 marks in Bremen	=	2 rubles in Russia.
1 ruble in Russia	=	5 francs in Bordeaux.
10 francs in Bordeaux	=	18 rials of plate in Cadiz.
1250 rials of plate in Cadiz	=	100 milrees in Lisbon.
88 m. r. in Lisbon	=	750 soldi in Leghorn.
2 soldi in Leghorn	=	1 piaster in Smyrna.
1 piaster in Smyrna	=	24 in Jamaica.
80 pence in Jamaica	=	1$ in Richmond, Va.
		225£ sterling.

INVOLUTION,

OR THE RAISING OF POWERS.

A Power is the product arising from multiplying any given number into itself continually, a certain number of times: Thus—

2=the root, or first power of 2.
2×2=4, the second power, or square of 2.
2×2×2=8, the third power, or cube of 2.
2×2×2×2=16, the 4th power, or biquadrate of 2.

The number denoting the power is called the index or the exponent of that power. If two or more powers are multiplied together, their product is that power whose index is equal to the sum of the exponents of the factors: thus—16 is the fourth power or biquadrate of 2, and 16×16=256, the 8th power of 2, or square biquadrate, &c.

A TABLE OF THE FIRST NINE POWERS.

Roots, or first powers.	Squares, or second pow.	Cubes, or third powers.	Biquadrates, or fourth powers.	Sursolids, or fifth powers.	Square Cubes, or sixth powers.	Second Sursolids, or seventh powers.	Biquadrates squared, or eighth powers.	Cubes cubed, or ninth powers.
1	1	1	1	1	1	1	1	1
2	4	8	16	32	64	128	256	512
3	9	27	81	243	729	2187	6561	19683
4	16	64	256	1024	4096	16384	65536	262144
5	25	125	625	3125	15625	78125	390625	1953125
6	36	216	1296	7776	46656	279936	1679616	10077696
7	49	343	2401	16807	117649	823543	5764801	40353607
8	64	512	4096	32768	262144	2097152	16777216	134217728
9	81	729	6561	59049	531441	4782969	43046721	387420489

EXAMPLES.

1. What is the 2d power, or square, of 36? Ans. 1296.
2. What is the 3d power, or cube, of 4.39? Ans. 84.604519.
3. What is the 4th power, or biquadrate, of 96? Ans. 84934656.
4. What is the 5th power, or sursolid, of .029? Ans. .000000020511149.
5. What is the 6th power, or square cube of 5.03? Ans. 16196.005304479729.
6. What is the 7th power, or second sursolid, of .029? Ans. 0000000000017249876309.
7. What is the 8th power, or square biquadrate, of 9.6? Ans. 72138957.89838336.

EVOLUTION,

OR THE EXTRACTION OF ROOTS.

The root of any number or power being multiplied into itself a certain number of times, will produce that power: thus—2 is the square root of 4, because $2\times2=4$; 3 is the cube root of 27, because $3\times3\times3=27$; and 4 is the biquadrate root of 256, because $4\times4\times4\times4=256$.

OF THE SQUARE ROOT.

The extraction of the Square Root is the finding of such a number, as being multiplied into itself, will produce the given number.

RULE.

1. Distinguish the given number into periods of two figures each, beginning at the unit's place or decimal point, and when the decimal does not consist of an even number of places, annex a cipher to it, and equal to the periods of the whole numbers and decimals respectively will be the places of each in the root.

2. Place the greatest square number contained in the first or left hand period under it, and set the root thereof on the right hand of the given number, like a quotient in division.

3. Subtract the said square number from the period above it, and to the remainder annex the next period in the given number to form a resolvend or dividual.

4. Place the double of the root, already found, on the left hand of the resolvend, for a divisor.

5. Seek how often the said divisor is contained in the resolvend, (omitting the unit's figure,) and set the result in the root, and on the right hand of the divisor.

6. *Multiply the divisor with the figure annexed to it by the last figure in the root, and subtract the product from the resolvend.*

7. Annex the third period in the given number to the remainder for a new resolvend.

8. Double all the figures in the root, now found, for a new divisor, and set it on the left hand of the new resolvend; then find the next figure of the root, as before directed, and continue the operation in the same manner till you have brought down all the periods in the given number.

NOTE.—The operation may be continued to any degree of exactness by annexing pairs of ciphers to the remainder.—See the 4th example, and examine it attentively.

PROOF.

Square the root and add the remainder (if any) to the product; the result will be equal to the given number, if the operation is right.

EXAMPLES.

1. What is the square root of 1296? Ans. 36.

```
   1296(36=the root.
   9
 66)396=resolvend.
    396
    ...
```

2. What is the square root of

```
133225(365 Answer.
9
66)432=resolvend.
   396
725)3625=new resolvend.
    3625
    ....
```

3. What is the square root of

```
5499025(2345 Answer.
4
43)149=1st resolvend.
   129
464)2090=2d resolvend.
    1856
4685)23425=3d resolvend.
     23425
     .....
```

4. What is the square root of 2.2710957?

```
2.27109570(1.507015 Ans.
1
25)127
   125
300)210
    000
3007)21095
     21049
30140)4670
      0000
301401)4670,00
       3014 01
3014025)16559900
        15070125
         1489775 remainder.

        1.507015
        1.507015
         7535075
        1507015
      10549105
     7535075
    1507015
  2.271094210225=product.
         1489775=rem.
  2.271095700000 proof.
```

5. What is the square root of 30138.696025? Ans. 173.605.
6. What is the square root of 1444? Ans. 38.
7. What is the square root of 219961? Ans. 469.
8. What is the square root of 151834.9156? Ans. 389.66.
9. What is the square root of 10? Ans. 3.162277+.
10. What is the square root of .0003272481? Ans. .01809.

N. B.—In the 4th example the number of decimal places is odd, wherefore I annex one cipher on the right hand to make it even; then I put a dot over the whole number 2, it being the unit's place; next over 7, 0, 5, and 0. When decimals are given, the periods must be pointed off both ways from the unit's place or decimal point.

OF THE SQUARE ROOT OF VULGAR FRACTIONS.

Rule.

1. Reduce the given fraction to its lowest terms.
2. Extract the square root of the numerator for a new numerator.
3. Extract the square root of the denominator for a new denominator.

Examples.

1. What is the square root of $\frac{9}{16}$? Ans. $\frac{3}{4}$.
2. What is the square root of $\frac{25}{36}$? Ans. $\frac{5}{6}$.
3. What is the square root of $\frac{49}{64}$? Ans. $\frac{7}{8}$.
4. What is the square root of $\frac{64}{81}$? Ans. $\frac{8}{9}$.
5. What is the square root of $\frac{36}{49}$? Ans. $\frac{6}{7}$.
6. What is the square root of $\frac{3044}{6849}$? Ans. $\frac{4}{9}$.
7. What is the square root of $\frac{2016}{6561}$? Ans. $\frac{2}{3}$.
8. What is the square root of $\frac{2704}{4225}$? Ans. $\frac{4}{5}$.

OF SURDS.

When the given fraction is a surd, that is, such a number whose square root cannot be exactly found.

Rule.

Reduce it to a decimal, and extract the square root thereof.

Examples.

1. What is the square root of $\frac{1}{2}$? Ans. .707+.
2. What is the square root of $\frac{1}{3}$? Ans. .5773+.
3. What is the square root of $\frac{4}{9}$? Ans. .666+.
4. What is the square root of $\frac{6}{7}$? Ans. .9258+.

OF MIXED NUMBERS.

Rule.

1. Reduce the fractional part of the mixt number to its lowest terms, and then *reduce* the mixed number to an improper fraction.

EXAMPLES.

1. Change $61 federal money to Martinico cur. Ans. 25£ 3s. 3d.
2. Change $964 federal money to Tobago cur. Ans. 379£ 13s.
3. Change $197.50cts. to pounds, shillings, &c. Ans. 81£ 9s. 4½d.
4. Change $251.75c. to pounds, shillings, &c. Ans. 103£ 16s. 11¼d.

3. *To change livers to dollars of* 8¼ *livers each.*

RULE.—Multiply the livers by 4, and divide the product by 33; or, divide the given number of livers by 8¼, decimally, and the quotient by either operation will be dollars.

EXAMPLES.

1. Change 10692 livers to Spanish dollars. Ans. $1296.
2. Change 15477 livers to Spanish dollars. Ans. $1876.
3. Change 3762 livers to Spanish dollars. Ans. $456.
4. Change 6253 livers 10 sols to Spanish dollars. Ans. $758.

X. WITH HAMBURG.

The denominations of money in Hamburg are—

12	deniers	make	1 stiver=2$\frac{1}{12}$ cents.
16	stivers	make	1 mark=33⅓ cents.
3	marks	make	1 rix-dollar=100 cents.
4	marks	make	1 silver ducatoon=133⅓ cents.
7½	marks	make	1 pound Flemish=250 cents.
Also, 12	pence Flem.	make	1 shilling Flemish=12½ cents.
20	shillings	make	1 pound Flemish.

NOTE.—The current money of Hamburg is inferior to bank money, (commonly called banco,) the agio or rate per cent. being variable.

1. *To change Hamburg Currency into Bank Money.*

RULE.—As 100, with the agio added, is to 100, so is the given sum in current money to the bank money required.

EXAMPLES.

1. Change 560 marks 8 stivers, current money, into bank money, the agio being 18 marks per cent.

```
   m.c.
   100
    18 agio.
   ———     m. banco.   m. c. sti.   m. ban.
As 118  :  100    ::   560 8   ..   475 the answer.
```

2. Change 2366 marks, current money, into bank money, the agio being 20 marks per cent. Ans. 1971mks. 10⅖sti. banco.

2. *To change Bank Money into Current Money.*

RULE.—As 100 marks banco is to 100 marks current, with the *agio added, so is* the given sum in bank money to the currency *required.*

2. An acre of land contains 160 square perches, therefore please to tell me how many perches in length will make one side of a square acre? Ans. 12.649 perches.+

PROBLEM 3.

When any number of men are to be placed in rank and file, so that the number in rank may be 2, 3, 4, or 5 times, &c. more than in the file.

RULE.

Divide the given number of men by the proportion between the rank and file, and the square root of the quotient will be the number in file; then multiply the number in file by the same proportion, and the product will be the number to be placed in the rank.

EXAMPLES.

1. Let 10952 men be formed in such a manner that the number in rank may be double the file. Ans. 74 in file and 148 in rank.

2. Let 8192 men be placed in such a manner that the number in rank may be 8 times the file. Ans. 32 in file and 256 in rank.

PROBLEM 4.

The diameter of one circle being given, to find the diameter of another, which shall be 2, 3, or 4 times, &c. greater or less than the given one.

RULE.

1. Square the diameter of the given circle, and if the required one be greater, multiply the said square by the given proportion; then extract the square root of the product, and that will be the diameter of the required circle.

2. If the required circle be less than the given one, divide the said square by the given proportion, and the square root of the quotient will be the diameter of the required circle.

EXAMPLES.

1. If the diameter of a given circle be 4 inches, what is the diameter of another one 3 times as large? Ans. 6.928+ inches.

2. If the diameter of a given circle be 20, what is the diameter of another 4 times less than the given one? Ans. 10.

PROBLEM 5.

The area of a circle being given to find the diameter.

RULE.

Divide the area of the given circle by .7854, and the square root of the quotient will be the diameter required.

NOTE.—.7854 is the area of a circle whose diameter is unity, or 1.

EXAMPLES.

1. A horse in the midst of a meadow suppose,
Made fast to a stake by a line from his nose;
How long must this line be, that, feeding around,
Permits him to graze just an acre of ground?

perches.
4)160=the area of an acre.
30¼

4800
40

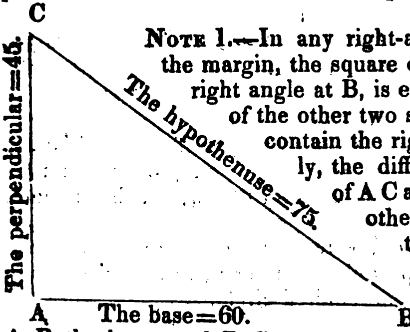
4840=square yards in 1 acre.
9 feet=1 square yard.

.7854)43560.0000(55462.18ft.=the square of the diameter of 1 acre.

3828 remainder.

$\sqrt{55462.18}$=235.5 ft.+=the diameter of an acre, and 117.75ft.= the half diameter; therefore, the true length of the line required is 117 feet 9 inches.

2. The superficial content of a circular deer park is 49 acres 14 perches, the diameter of which is required. Ans. 100 perches.



NOTE 1.—In any right-angled triangle, as A B C in the margin, the square of the side A C, opposite the right angle at B, is equal to the sum of the squares of the other two sides, A B and B C, which contain the right angle at B; consequently, the difference between the square of A C and the square of either of the other sides will be the square of the remaining side. Hence the 6th, 7th, & 8th problems.

NOTE 2.—The side A C is called the hypothenuse, A B the base, and B C the perpendicular.

PROBLEM 6.

Given the base and perpendicular, to find the hypothenuse.

RULE.

Extract the square root of the sum of the squares of the base and perpendicular, and that will be the length of the hypothenuse required.

AN EXAMPLE.

When the base A B is 60 and the perpendicular B C 45 equal parts, what is the length of the hypothenuse A C?

$60 \times 60 = 3600$
$45 \times 45 = 2025$

$\sqrt{5625} = 75$ Answer.

PROBLEM 7.

Given the base and hypothenuse, to find the perpendicular.

RULE.

Extract the square root of the difference of the squares of the base and hypothenuse, and that will be the height of the perpendicular required.

AN EXAMPLE.

When the base A B is 60, and the hypothenuse A C 75 equal parts, what is the height of the perpendicular B C?

$$75 \times 75 = 5625$$
$$60 \times 60 = 3600.$$

$$\sqrt[2]{2025} = 45. \text{ Ans.}$$

PROBLEM 8.

Given the hypothenuse and perpendicular, to find the base.

RULE.

Extract the square root of the difference of the squares of the hypothenuse and perpendicular, and that will be the length of the base required.

AN EXAMPLE.

When the hypothenuse A C is 75, and the perpendicular B C 45 equal parts, what is the length of the base A B?

$$75 \times 75 = 5625$$
$$45 \times 45 = 2025$$

$$\sqrt[2]{3600} = 60 \text{ Ans.}$$

PROBLEM 9.

Given the height of the roof and the width of the house, to find the length of the rafter.

RULE.

Extract the square root of the sum of the squares of the height of the roof and half the width of the house, and that will be the length of the rafter.

AN EXAMPLE.

Admit a house to be 32 feet wide, and the height of the roof 12 feet, what is the length of the rafters?

$$16 \times 16 = 256$$
$$12 \times 12 = 144$$

$$\sqrt[2]{400} = 20\text{ft. Ans.}$$

PROBLEM 10.

Given the length of a rafter and the width of the house, to find the *height of the roof* or length of the king post.

Rule.

Subtract the square of half the width of the house from the square of the rafter; then extract the square root of the remainder, and that will be the height of the roof.

An Example.

If a house be 24 feet wide, and the rafters 18 feet long, what is the height of the roof?

$18\times18=324$ the square of the rafter.
$12\times12=144$ the square of half the width.

$\sqrt{180}=13$ft. 4 tenths. Ans.

Practical Questions.

1. How many superficial feet are contained on a floor that is 75 feet square? Ans. 5625.

2. What is the difference between 9 feet square and 9 square feet? Ans. 72 square feet.

3. A certain square pavement contains 197136 square stones, all of the same size; how many are contained in one of its sides? Ans. 444.

4. Being about to plant out 5292 trees, equally distant from each other, in rows; the length of the grove is to be 3 times the breadth; how many trees must there be in each row?
Ans. 42 in the short rows, and 126 in the long ones.

5. A ladder 40 feet long will exactly reach from the top of a fort to the opposite side of a ditch that is 24 feet wide; what is the height of the wall? Ans. 32 feet.

6. The wall of a town that is 18 feet high, is surrounded by a moat 20 yards wide; required the length of a ladder that will reach from the outside of the moat to the top of the wall? Ans. 62.6ft.+

7. Suppose a ladder 60 feet long, be so planted as to reach a window 37 feet from the ground on one side of a street, and without moving it at the foot, will reach another window 23 feet high on the other side; please to tell me the breadth of the street?
Ans. 102.649 feet.+

8. If a field be 800 yards long, and 600 yards wide, what is the distance between two opposite corners? Ans. 1000 yards.

9. There is a square field containing 10 acres; what is the distance from the centre to each corner? Ans. 28.28 rods+.

10. The height of a tree growing in the centre of a circular island (44 feet in diameter) is 75 feet, and a line stretched from the top of it, over to the hither edge of the water is 256 feet long; what is the breadth of the stream, supposing the land to be level on each side of the water? Ans. 222.767 feet+.

11. A castle wall there was, whose top was found
To be one hundred feet above the ground;
Against the wall a ladder stood upright,
Of the same length the castle was in height.
A waggish youth the ladder's foot did slide
Exactly ten feet from the lower side;
Now I would know how far the top did fall,
By pulling out the ladder from the wall? Ans. 6in.+

12. As I was walking out one day,
Which happened on the first of May,
As luck would have it, I did spy
A may-pole raised up on high,
The which at first me much surpris'd,
Not being before hand advertis'd
Of such a strange uncommon sight;
I said I would not stir that night,
Nor rest content, until I'd found
It's height exact from off the ground.
But when these words I just had spoke,
A blast of wind the may-pole broke,
Whose broken piece I found to be
Exact in length, yards sixty-three,
Which by its fall broke up a hole,
Twice fifteen yards from off the pole;
But this being all that I could do,
The may-pole now being broke in two
Unequal parts, to aid a friend,
Ye youths, pray then an answer send.
Ans. 118.3985yds.=118yds. 1ft. 2in.+.346.

OF THE CUBE ROOT.

The cube of any number is the product arising from that number being multiplied into itself three times.

The extraction of Cube Root is the finding of a number which being multiplied into itself three times will produce the given number.

RULE.

1. Distinguish the given number into periods of three figures each, beginning at the unit's place or decimal point, and when the decimal does not consist of a complete period, or periods, annex a cipher or ciphers to make it so; and the figures in the root will be as many as the periods of the given cube, in whole numbers and decimals respectively.

2. Find the greatest root contained in the left hand period, and place it *on the right* hand of the given number, then subtract its cube

from the said period, and to the remainder annex the next period for a resolvend.

3. Take the triple square of the ascertained root, for a defective divisor.

4. Try how often the said defective divisor is contained in the resolvend (omitting the two right hand figures)—place the result of this trial in the root, and its square on the right hand of the said defective divisor, supplying the place of tens with a cipher, if the square be less than ten.

5. Complete the divisor by adding thereto the product of the last figure in the root, multiplied by the rest and by 30.

6. Multiply, subtract, and bring down the next period for a new resolvend.

7. Find a new defective divisor, by adding the last complete divisor, the number which completed it, and twice the square of the last figure in the root, together.

8. Complete the new defective divisor as before, and proceed on as above directed, till all the periods are used.

NOTE.—The operation may be continued to any degree of exactness by annexing triplets of ciphers to the last remainder, and so on.

PROOF.

Cube the whole root, that is, multiply it into itself three times, and add the last remainder (if any) to the last product, and the total sum will be equal to the given number, if the work is right.

EXAMPLES.

1. What is the cube root of 240061.988375?

10804	
360	240061.988375(62.15 = cube root required.
	216
Complete divisor=11164	
8	24061=the 1st resolvend.
1153201	22328
62×1×30= 1860	
	1733988=the 2d resolvend.
Complete divisor=1155061	1155061
Twice the square of 1= 2	
	578927375=3d resolvend.
115692325	578927375
621×5×30= 93150	
Complete divisor=115785475	000000000

1st. I put a point over the unit's figure 1, and then proceed both ways to distinguish the given number into periods of three figures each, according to the rule. 2d. The greatest root contained in the left hand period 240 is 6, which I place on the right hand of the

given number, and the cube of 6 is 216, which I subtract from the said period, and the remainder is 24, to which I annex the second period 061 to form a resolvend. 3d. The triple square of $6=108$, which I set on the left hand of the given number for the defective divisor. 4th. I try how often the defective divisor 108 is contained in the resolvend 24061 (omitting the two right hand figures 61) and the result is 2, which I place after 6 in the root, and the square of $2=4$, which is less than 10, therefore I supply the place of tens with a cipher thus, 04, and set it on the right hand of the defective divisor 108, and the result is 10804. 5th. Now 2 is the last figure in the root, therefore $2\times6\times30=360$, which I add to 10804, and the sum is $11164=$ the complete divisor. 6th. I multiply the complete divisor by 2 and subtract the product from the resolvend 24061, and the remainder is 1733, to which I annex the next period 988, for a new resolvend. 7th. Twice the square of 2 (the last figure in the root as yet)$=8$, which I set under the last complete divisor, and say $8+11164+360=11532$ the second defective divisor, with which I proceed as before, and so on till the work is finished.

2. What is the cube root of 22069810125? Ans. 2805.
3. What is the cube root of 673373097125? Ans. 8765.
4. What is the cube root of 262144? Ans. 64.
5. What is the cube root of 34965783? Ans. 327.
6. What is the cube root of 268336125? Ans. 645.
7. What is the cube root of 84.604519? Ans. 4.39.
8. What is the cube root of 138.188413? Ans. 5.17.
9. What is the cube root of 5232228.323420125? Ans. 173.605.
10. What is the cube root of .001906624? Ans. .124.

OF THE CUBE ROOT OF VULGAR FRACTIONS.

Rule.

1. Reduce the given fraction to its lowest terms (if necessary) then extract the cube root of the numerator for a new numerator, and of the denominator for a new denominator.

2. If the given fraction be a surd, then reduce it to its equivalent decimal value, and extract the cube root thereof.

Examples.

1. What is the cube root of $\frac{27}{64}$? Ans. $\frac{3}{4}$.
2. What is the cube root of $\frac{256}{500}$? Ans. $\frac{4}{5}$.
3. What is the cube root of $\frac{1000}{1728}$? Ans. $\frac{5}{6}$.
4. What is the cube root of $\frac{648}{1029}$? Ans. $\frac{6}{7}$.
5. What is the cube root of $\frac{2744}{4096}$? Ans. $\frac{7}{8}$.
6. What is the cube root of $\frac{6144}{8748}$? Ans. $\frac{8}{9}$.

OF SURDS.

7. What is the cube root of $\frac{4}{9}$? Ans. .763.+
8. *What is the* cube root of $\frac{6}{7}$? Ans. .949.+
9. *What is the* cube root of $\frac{1}{2}$? Ans. .793.+

7. Annex the third period in the given number to the remainder for a new resolvend.

8. Double all the figures in the root, now found, for a new divisor, and set it on the left hand of the new resolvend; then find the next figure of the root, as before directed, and continue the operation in the same manner till you have brought down all the periods in the given number.

NOTE.—The operation may be continued to any degree of exactness by annexing pairs of ciphers to the remainder.—See the 4th example, and examine it attentively.

PROOF.

Square the root and add the remainder (if any) to the product; the result will be equal to the given number, if the operation is right.

EXAMPLES.

1. What is the square root of 1296? Ans. 36.

```
    1296(36=the root.
     9
   ---
 66)396=resolvend.
    396
    ---
    ...
```

2. What is the square root of

```
  133225(365 Answer.
  9
  --
66)432=resolvend.
   396
   ----
725)3625=new resolvend.
    3625
    ----
    ....
```

3. What is the square root of

```
  5499025(2345 Answer.
  4
  --
43)149=1st resolvend.
   129
   ----
464)2090=2d resolvend.
    1856
    -----
4685)23425=3d resolvend.
     23425
     -----
     .....
```

4. What is the square root of 2.2710957?

```
         2.27109570(1.507015 Ans.
         1
        ---
      25)127
         125
        -----
     300)210
         000
        -----
    3007)21095
         21049
        ------
   30140)4670
         0000
        --------
  301401)4670,00
         3014 01
        ---------
 3014025)16559900
         15070125
        ---------
          1489775 remainder.

         1.507015
         1.507015
        ---------
          7535075
         1507015
       10549105
      7535075
     1507015
    -------------------
    2.271094210225=product.
          1489775=rem.
    -------------------
    2.271095700000 proof.
```

5. What is the square root of 30138.696025? Ans. 173.605.
6. What is the square root of 1444? Ans. 38.
7. What is the square root of 219961? Ans. 469.
8. What is the square root of 151834.9156? Ans. 389.66.
9. What is the square root of 10? Ans. 3.162277+.
10. What is the square root of .0003272481? Ans. .01809.

N. B.—In the 4th example the number of decimal places is odd, wherefore I annex one cipher on the right hand to make it even; then I put a dot over the whole number 2, it being the unit's place; next over 7, 0, 5, and 0. When decimals are given, the periods must be pointed off both ways from the unit's place or decimal point.

OF THE SQUARE ROOT OF VULGAR FRACTIONS.

Rule.

1. Reduce the given fraction to its lowest terms.
2. Extract the square root of the numerator for a new numerator.
3. Extract the square root of the denominator for a new denominator.

Examples.

1. What is the square root of $\frac{9}{16}$? Ans. $\frac{3}{4}$.
2. What is the square root of $\frac{25}{36}$? Ans. $\frac{5}{6}$.
3. What is the square root of $\frac{49}{64}$? Ans. $\frac{7}{8}$.
4. What is the square root of $\frac{64}{81}$? Ans. $\frac{8}{9}$.
5. What is the square root of $\frac{36}{49}$? Ans. $\frac{6}{7}$.
6. What is the square root of $\frac{3024}{4725}$? Ans. $\frac{4}{5}$.
7. What is the square root of $\frac{2016}{4536}$? Ans. $\frac{2}{3}$.
8. What is the square root of $\frac{2704}{4225}$? Ans. $\frac{4}{5}$.

OF SURDS.

When the given fraction is a surd, that is, such a number whose square root cannot be exactly found.

Rule.

Reduce it to a decimal, and extract the square root thereof.

Examples.

1. What is the square root of $\frac{1}{2}$? Ans. .707+.
2. What is the square root of $\frac{1}{3}$? Ans. .5773+.
3. What is the square root of $\frac{4}{9}$? Ans. .666+.
4. What is the square root of $\frac{6}{7}$? Ans. .9258+.

OF MIXED NUMBERS.

Rule.

1. Reduce the fractional part of the mixt number to its lowest terms, and then reduce the mixed number to an improper fraction.

EXAMPLES.

1. A stone of a cubical form contains 474552 solid inches; what is the superficial content of one of its sides? Ans. 6084 inches.

2. If a stone of a cubical form contain 21952 solid feet, what is the superficial content of one of its sides? Ans. 784 feet.

OF THE BIQUADRATE ROOT.

A biquadrate is the product or power arising from the involution of any number into itself four times.

The extraction of the biquadrate root is the finding of a number which being involved into itself four times, will produce the given number.

RULE.

Extract the square root of the given number, then extract the square root of that square root, and it will be the biquadrate root required.

EXAMPLES.

1. What is the biquadrate root of 5308416?

```
 5308416(2304=the first        2304(48=the biquadrate
 4            square root.     16         root required.
 --                            --
43)130                        88)704
   129                           704
   ---                           ---
4604)18416
     18416
     -----
```

2. Required the biquadrate root of 48382841521. Ans. 469.

3. Required the biquadrate root of 72138957.89838336. Ans. 92.16.

4. Required the biquadrate root of 908340998.087350800625. Ans. 173.605.

OF THE SURSOLID ROOT.

Any number being involved into itself five times produces a sursolid.

The extraction of the sursolid root is the finding of a number, which being involved into itself 5 times, will produce the given number.

RULE.

1. Point off the given number into periods of five figures each.

2. Find the greatest root contained in the left hand period, by trial, or in the table of powers, and subtract its fifth power therefrom.

3. *To the remainder annex the first figure in the next period for a dividend.*

2. An acre of land contains 160 square perches, therefore please to tell me how many perches in length will make one side of a square acre? Ans. 12.649 perches.+

PROBLEM 3.

When any number of men are to be placed in rank and file, so that the number in rank may be 2, 3, 4, or 5 times, &c. more than in the file.

RULE.

Divide the given number of men by the proportion between the rank and file, and the square root of the quotient will be the number in file; then multiply the number in file by the same proportion, and the product will be the number to be placed in the rank.

EXAMPLES.

1. Let 10952 men be formed in such a manner that the number in rank may be double the file. Ans. 74 in file and 148 in rank.

2. Let 8192 men be placed in such a manner that the number in rank may be 8 times the file. Ans. 32 in file and 256 in rank.

PROBLEM 4.

The diameter of one circle being given, to find the diameter of another, which shall be 2, 3, or 4 times, &c. greater or less than the given one.

RULE.

1. Square the diameter of the given circle, and if the required one be greater, multiply the said square by the given proportion; then extract the square root of the product, and that will be the diameter of the required circle.

2. If the required circle be less than the given one, divide the said square by the given proportion, and the square root of the quotient will be the diameter of the required circle.

EXAMPLES.

1. If the diameter of a given circle be 4 inches, what is the diameter of another one 3 times as large? Ans. 6.928+ inches.

2. If the diameter of a given circle be 20, what is the diameter of another 4 times less than the given one? Ans. 10.

PROBLEM 5.

The area of a circle being given to find the diameter.

RULE.

Divide the area of the given circle by .7854, and the square root of the quotient will be the diameter required.

NOTE.—.7854 is the area of a circle whose diameter is unity, or 1.

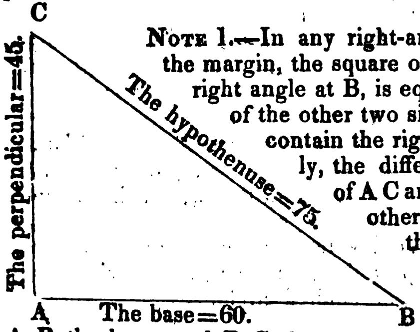
EXAMPLES.

1. A horse in the midst of a meadow suppose,
Made fast to a stake by a line from his nose;
How long must this line be, that, feeding around,
Permits him to graze just an acre of ground?

perches.
4)160=the area of an acre.
  30¼
———
 4800
   40
———
 4840=square yards in 1 acre.
   9 feet=1 square yard.
———
.7854)43560.0000(55462.18ft.=the square of the diameter of 1 acre.
———
3828 remainder.

$\sqrt{55462.18}$=235.5 ft.+=the diameter of an acre, and 117.75ft.= the half diameter; therefore, the true length of the line required is 117 feet 9 inches.

2. The superficial content of a circular deer park is 49 acres 14 perches, the diameter of which is required. Ans. 100 perches.



NOTE 1.—In any right-angled triangle, as A B C in the margin, the square of the side A C, opposite the right angle at B, is equal to the sum of the squares of the other two sides, A B and B C, which contain the right angle at B; consequently, the difference between the square of A C and the square of either of the other sides will be the square of the remaining side. Hence the 6th, 7th, & 8th problems.

NOTE 2.—The side A C is called the hypothenuse, A B the base, and B C the perpendicular.

PROBLEM 6.

Given the base and perpendicular, to find the hypothenuse.

RULE.

Extract the square root of the sum of the squares of the base and perpendicular, and that will be the length of the hypothenuse required.

AN EXAMPLE.

When the base A B is 60 and the perpendicular B C 45 equal parts, what is the length of the hypothenuse A C?

$$60\times60=3600$$
$$45\times45=2025$$
———
$$\sqrt{5625}=75 \text{ Answer.}$$

PROBLEM 7.

Given the base and hypothenuse, to find the perpendicular.

RULE.

Extract the square root of the difference of the squares of the base and hypothenuse, and that will be the height of the perpendicular required.

AN EXAMPLE.

When the base A B is 60, and the hypothenuse A C 75 equal parts, what is the height of the perpendicular B C ?

$$75\times75=5625$$
$$60\times60=3600$$

$$\sqrt[2]{2025}=45 \text{ Ans.}$$

PROBLEM 8.

Given the hypothenuse and perpendicular, to find the base.

RULE.

Extract the square root of the difference of the squares of the hypothenuse and perpendicular, and that will be the length of the base required.

AN EXAMPLE.

When the hypothenuse A C is 75, and the perpendicular B C 45 equal parts, what is the length of the base A B ?

$$75\times75=5625$$
$$45\times45=2025$$

$$\sqrt[2]{3600}=60 \text{ Ans.}$$

PROBLEM 9.

Given the height of the roof and the width of the house, to find the length of the rafter.

RULE.

Extract the square root of the sum of the squares of the height of the roof and half the width of the house, and that will be the length of the rafter.

AN EXAMPLE.

Admit a house to be 32 feet wide, and the height of the roof 12 feet, what is the length of the rafters ?

$$16\times16=256$$
$$12\times12=144$$

$$\sqrt[2]{400}=20\text{ft. Ans.}$$

PROBLEM 10.

Given the length of a rafter and the width of the house, to find the *height of the roof* or length of the king post.

Rule.

Subtract the square of half the width of the house from the square of the rafter; then extract the square root of the remainder, and that will be the height of the roof.

An Example.

If a house be 24 feet wide, and the rafters 18 feet long, what is the height of the roof?

$18\times18=324$ the square of the rafter.
$12\times12=144$ the square of half the width.

$\sqrt{180}=13$ft. 4 tenths. Ans.

Practical Questions.

1. How many superficial feet are contained on a floor that is 75 feet square? Ans. 5625.

2. What is the difference between 9 feet square and 9 square feet? Ans. 72 square feet.

3. A certain square pavement contains 197136 square stones, all of the same size; how many are contained in one of its sides? Ans. 444.

4. Being about to plant out 5292 trees, equally distant from each other, in rows; the length of the grove is to be 3 times the breadth; how many trees must there be in each row? Ans. 42 in the short rows, and 126 in the long ones.

5. A ladder 40 feet long will exactly reach from the top of a fort to the opposite side of a ditch that is 24 feet wide; what is the height of the wall? Ans. 32 feet.

6. The wall of a town that is 18 feet high, is surrounded by a moat 20 yards wide; required the length of a ladder that will reach from the outside of the moat to the top of the wall? Ans. 62.6ft.+

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11. A castle wall there was, whose top was found
To be one hundred feet above the ground;
Against the wall a ladder stood upright,
Of the same length the castle was in height.
A waggish youth the ladder's foot did slide
Exactly ten feet from the lower side;
Now I would know how far the top did fall,
By pulling out the ladder from the wall? Ans. 6in.+

12. As I was walking out one day,
Which happened on the first of May,
As luck would have it, I did spy
A may-pole raised up on high,
The which at first me much surpris'd,
Not being before hand advertis'd
Of such a strange uncommon sight;
I said I would not stir that night,
Nor rest content, until I'd found
It's height exact from off the ground.
But when these words I just had spoke,
A blast of wind the may-pole broke,
Whose broken piece I found to be
Exact in length, yards sixty-three,
Which by its fall broke up a hole,
Twice fifteen yards from off the pole;
But this being all that I could do,
The may-pole now being broke in two
Unequal parts, to aid a friend,
Ye youths, pray then an answer send.
Ans. 118.3985yds.=118yds. 1ft. 2in.+.346.

OF THE CUBE ROOT.

The cube of any number is the product arising from that number being multiplied into itself three times.

The extraction of Cube Root is the finding of a number which being multiplied into itself three times will produce the given number.

Rule.

1. Distinguish the given number into periods of three figures each, beginning at the unit's place or decimal point, and when the decimal does not consist of a complete period, or periods, annex a cipher or ciphers to make it so; and the figures in the root will be as many as the periods of the given cube, in whole numbers and decimals respectively.

2. Find the greatest root contained in the left hand period, and place *it on the right* hand of the given number, then subtract its cube

INVOLUTION,
OR THE RAISING OF POWERS.

A Power is the product arising from multiplying any given number into itself continually, a certain number of times: Thus—

2=the root, or first power of 2.
2×2=4, the second power, or square of 2.
2×2×2=8, the third power, or cube of 2.
2×2×2×2=16, the 4th power, or biquadrate of 2.

The number denoting the power is called the index or the exponent of that power. If two or more powers are multiplied together, their product is that power whose index is equal to the sum of the exponents of the factors: thus—16 is the fourth power or biquadrate of 2, and 16×16=256, the 8th power of 2, or square biquadrate, &c.

A TABLE OF THE FIRST NINE POWERS.

Roots, or first powers.	Squares, or second pow.	Cubes, or third powers.	Biquadrates, or fourth powers.	Sursolids, or fifth powers.	Square Cubes, or sixth powers.	Second Sursolids, or seventh powers.	Biquadrates squared, or eighth powers.	Cubes cubed, or ninth powers.
1	1	1	1	1	1	1	1	1
2	4	8	16	32	64	128	256	512
3	9	27	81	243	729	2187	6561	19683
4	16	64	256	1024	4096	16384	65536	262144
5	25	125	625	3125	15625	78125	390625	1953125
6	36	216	1296	7776	46656	279936	1679616	10077696
7	49	343	2401	16807	117649	823543	5764801	40353607
8	64	512	4096	32768	262144	2097152	16777216	134217728
9	81	729	6561	59049	531441	4782969	43046721	387420489

EXAMPLES.

1. What is the 2d power, or square, of 36? Ans. 1296.
2. What is the 3d power, or cube, of 4.39? Ans. 84.604519.
3. What is the 4th power, or biquadrate, of 96? Ans. 84934656.
4. What is the 5th power, or sursolid, of .029? Ans. .000000020511149.
5. What is the 6th power, or square cube of 5.03? Ans. 16196.005304479729.
6. What is the 7th power, or second sursolid, of .029? Ans. 000000000017249876309.
7. What is the 8th power, or square biquadrate, of 9.6? Ans. 72138957.89838336.

EVOLUTION,
OR THE EXTRACTION OF ROOTS.

The root of any number or power being multiplied into itself a certain number of times, will produce that power: thus—2 is the square root of 4, because $2\times2=4$; 3 is the cube root of 27, because $3\times3\times3=27$; and 4 is the biquadrate root of 256, because $4\times4\times4\times4=256$.

OF THE SQUARE ROOT.

The extraction of the Square Root is the finding of such a number, as being multiplied into itself, will produce the given number.

RULE.

1. Distinguish the given number into periods of two figures each, beginning at the unit's place or decimal point, and when the decimal does not consist of an even number of places, annex a cipher to it, and equal to the periods of the whole numbers and decimals respectively will be the places of each in the root.

2. Place the greatest square number contained in the first or left hand period under it, and set the root thereof on the right hand of the given number, like a quotient in division.

3. Subtract the said square number from the period above it, and to the remainder annex the next period in the given number to form a resolvend or dividual.

4. Place the double of the root, already found, on the left hand of the resolvend, for a divisor.

5. Seek how often the said divisor is contained in the resolvend, (omitting the unit's figure,) and set the result in the root, and on the right hand of the divisor.

6. *Multiply the divisor* with the figure annexed to it by the last *figure in the root,* and subtract the product from the resolvend.

7. Annex the third period in the given number to the remainder for a new resolvend.

8. Double all the figures in the root, now found, for a new divisor, and set it on the left hand of the new resolvend; then find the next figure of the root, as before directed, and continue the operation in the same manner till you have brought down all the periods in the given number.

NOTE.—The operation may be continued to any degree of exactness by annexing pairs of ciphers to the remainder.—See the 4th example, and examine it attentively.

PROOF.

Square the root and add the remainder (if any) to the product; the result will be equal to the given number, if the operation is right.

EXAMPLES.

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   1296(36=the root.
   9
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```
 133225(365 Answer.
 9
 ---
66)432=resolvend.
   396
   ----
725)3625=new resolvend.
    3625
    ----
    ....
```

3. What is the square root of

```
 5499025(2345 Answer.
 4
 --
43)149=1st resolvend.
   129
   ----
464)2090=2d resolvend.
    1856
    -----
4685)23425=3d resolvend.
     23425
     -----
     .....
```

4. What is the square root of 2.2710957?

```
        2.27109570(1.507015 Ans.
        1
        ---
     25)127
        125
        ----
    300)210
        000
        -----
   3007)21095
        21049
        ------
  30140)4670
        0000
        ---------
 301401)4670,00
        3014 01
        ---------
3014025)16559900
        15070125
        --------
         1489775 remainder.

         1.507015
         1.507015
         --------
         7535075
        1507015
      10549105
     7535075
    1507015
   ---------------
   2.271094210225=product.
          1489775=rem.
   ---------------
   2.271095700000 proof.
```

5. What is the square root of 30138.696025? Ans. 173.605.
6. What is the square root of 1444? Ans. 38.
7. What is the square root of 219961? Ans. 469.
8. What is the square root of 151834.9156? Ans. 389.66.
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N. B.—In the 4th example the number of decimal places is odd, wherefore I annex one cipher on the right hand to make it even; then I put a dot over the whole number 2, it being the unit's place; next over 7, 0, 5, and 0. When decimals are given, the periods must be pointed off both ways from the unit's place or decimal point.

OF THE SQUARE ROOT OF VULGAR FRACTIONS.

RULE.

1. Reduce the given fraction to its lowest terms.
2. Extract the square root of the numerator for a new numerator.
3. Extract the square root of the denominator for a new denominator.

EXAMPLES.

1. What is the square root of $\frac{9}{16}$? Ans. $\frac{3}{4}$.
2. What is the square root of $\frac{25}{36}$? Ans. $\frac{5}{6}$.
3. What is the square root of $\frac{49}{64}$? Ans. $\frac{7}{8}$.
4. What is the square root of $\frac{64}{81}$? Ans. $\frac{8}{9}$.
5. What is the square root of $\frac{36}{49}$? Ans. $\frac{6}{7}$.
6. What is the square root of $\frac{3044}{6849}$? Ans. $\frac{4}{5}$.
7. What is the square root of $\frac{2916}{6561}$? Ans. $\frac{2}{3}$.
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OF SURDS.

When the given fraction is a surd, that is, such a number whose square root cannot be exactly found.

RULE.

Reduce it to a decimal, and extract the square root thereof.

EXAMPLES.

1. What is the square root of $\frac{1}{2}$? Ans. .707+.
2. What is the square root of $\frac{1}{3}$? Ans. .5773+.
3. What is the square root of $\frac{4}{9}$? Ans. .666+.
4. What is the square root of $\frac{6}{7}$? Ans. .9258+.

OF MIXED NUMBERS.

RULE.

1. Reduce the fractional part of the mixt number to its lowest terms, and then reduce the mixed number to an improper fraction.

EXAMPLES.

1. A stone of a cubical form contains 474552 solid inches; what is the superficial content of one of its sides? Ans. 6084 inches.

2. If a stone of a cubical form contain 21952 solid feet, what is the superficial content of one of its sides? Ans. 784 feet.

OF THE BIQUADRATE ROOT.

A biquadrate is the product or power arising from the involution of any number into itself four times.

The extraction of the biquadrate root is the finding of a number which being involved into itself four times, will produce the given number.

RULE.

Extract the square root of the given number, then extract the square root of that square root, and it will be the biquadrate root required.

EXAMPLES.

1. What is the biquadrate root of 5308416?

```
      5308416(2304=the first
      4            square root.
      ---
   43)130
      129
     -----
 4604)18416
      18416
      -----
```

```
    2304(48=the biquadrate
    16          root required.
    ----
 88)704
    704
    ---
```

2. Required the biquadrate root of 48382841521. Ans. 469.
3. Required the biquadrate root of 72138957.89838336. Ans. 92.16.
4. Required the biquadrate root of 908340998.0873350800625. Ans. 173.605.

OF THE SURSOLID ROOT.

Any number being involved into itself five times produces a sursolid.

The extraction of the sursolid root is the finding of a number, which being involved into itself 5 times, will produce the given number.

RULE.

1. Point off the given number into periods of five figures each.
2. Find the greatest root contained in the left hand period, by trial, or in the table of powers, and subtract its fifth power therefrom.
3. *To the remainder annex the first figure in the next period for a dividend.*

2. An acre of land contains 160 square perches; therefore please to tell me how many perches in length will make one side of a square acre? Ans. 12.649 perches.+

PROBLEM 3.

When any number of men are to be placed in rank and file, so that the number in rank may be 2, 3, 4, or 5 times, &c. more than in the file.

RULE.

Divide the given number of men by the proportion between the rank and file, and the square root of the quotient will be the number in file; then multiply the number in file by the same proportion, and the product will be the number to be placed in the rank.

EXAMPLES.

1. Let 10952 men be formed in such a manner that the number in rank may be double the file. Ans. 74 in file and 148 in rank.

2. Let 8192 men be placed in such a manner that the number in rank may be 8 times the file. Ans. 32 in file and 256 in rank.

PROBLEM 4.

The diameter of one circle being given, to find the diameter of another, which shall be 2, 3, or 4 times, &c. greater or less than the given one.

RULE.

1. Square the diameter of the given circle, and if the required one be greater, multiply the said square by the given proportion; then extract the square root of the product, and that will be the diameter of the required circle.

2. If the required circle be less than the given one, divide the said square by the given proportion, and the square root of the quotient will be the diameter of the required circle.

EXAMPLES.

1. If the diameter of a given circle be 4 inches, what is the diameter of another one 3 times as large? Ans. 6.928+ inches.

2. If the diameter of a given circle be 20, what is the diameter of another 4 times less than the given one? Ans. 10.

PROBLEM 5.

The area of a circle being given to find the diameter.

RULE.

Divide the area of the given circle by .7854, and the square root of the quotient will be the diameter required.

NOTE.—.7854 is the area of a circle whose diameter is unity, or 1.

EXAMPLES.

1. A horse in the midst of a meadow suppose,
Made fast to a stake by a line from his nose;
How long must this line be, that, feeding around,
Permits him to graze just an acre of ground?

2. Extract the roots of the numerator and denominator for a ne numerator and denominator; then form the new fraction, and r duce it to its proper terms.

EXAMPLES.

1. What is the square root of $51\frac{21}{25}$? Ans. $7\frac{1}{5}$.
2. What is the square root of $37\frac{36}{49}$? Ans. $6\frac{1}{7}$.
3. What is the square root of $17\frac{80}{125}$? Ans. $4\frac{1}{5}$.
4. What is the square root of $4\frac{264}{512}$? Ans. $2\frac{1}{8}$.

WHEN THE FRACTIONAL PARTS ARE SURDS.

RULE.

Annex the equivalent decimal value of the fractional part to the whole number, and then extract the root decimally.

EXAMPLES.

1. What is the square root of $76\frac{14}{17}$? Ans. 8.7649+.
2. What is the square root of $7\frac{9}{11}$? Ans. 2.7961+.
3. What is the square root of $6\frac{2}{3}$? Ans. 2.5819+.
4. What is the square root of $85\frac{14}{15}$? Ans. 9.27+.

APPLICATION.

PROBLEM 1.

To find a mean proportional between two given numbers.

RULE.

Multiply the two given numbers together, and the square root of the product will be the mean proportional required.

EXAMPLES.

1. What is the mean proportional between 24 and 96?

$$24\times96=2304(48 \text{ Ans.}$$

```
16
---
88)704
   704
```

2. What is the mean proportional betwen 18 and 72? Ans. 36.

PROBLEM 2.

To find the side of a square that shall be equal in area to any given superficies whatever.

RULE.

Extract the square root of any given superficies, and that will be the side of a square that is equal in

E

1. If the superficial con *is the length of one sid given circle?*

Examples.

1. Required the square biquadrate-root of 28179280429056?

$\sqrt[2]{28179280429056}=5308416$ and $\sqrt[2]{5308416}=2304$=the biquadrate-root, and $\sqrt[2]{2304}=48$, the square biquadrate-root required.

2. Required the square biquadrate-root of 7213895789838336? Ans. 96.

3. Required the square biquadrate-root of 472769874482845188096? Ans. 384.

4. Required the square biquadrate-root of .000000000000500246412961? Ans. .029.

OF THE CUBED CUBE-ROOT.

Any number involved into itself nine times, produces a cubed cube, or ninth power.

The extraction of the cubed cube-root is the finding of a number, which, being involved into itself nine times, will produce the given number.

Rule.

Extract the cube root of the cube root of the given number, and the result will be the cubed cube-root required.

Examples.

1. Required the cubed cube-root of 1352605460594688? Ans. 48.

2. Required the cubed cube-root of 692533995824480256? Ans. 96.

3. Required the cubed cube-root of 181543631801412552228864? Ans. 384.

4. Required the cubed cube-root of .0000000000000104507145975869? Ans. .029.

OF SIMPLE INTEREST BY DECIMALS.

NOTE.—P=any principle.
T=the given time.
R=the ratio.
A=the amount.

The ratio signifies the simple interest of one dollar, or one pound, for one year, at any proposed rate of interest per cent., and is found by the following proportion:

As \$100 : 6 :: \$1
1

100)6.00(.06=the ratio at \$6 per cent.
6.00

000

A TABLE OF RATIOS.

Rate per cent.	*Ratio.*	*Rate per cent.*	*Ratio.*	*Rate per cent.*	*Ratio.*
2	.02	6	.06	9½	.095
2½	.025	6½	.065	10	.10
3	.03	7	.07	10½	.105
3½	.035	7½	.075	11	.11
4	.04	8	.08	11½	.115
4½	.045	8½	.085	12	.12
5	.05	9	.09	12½	.125
5½	.055				

Case 1.

When P, T, and R are given, to find A.

Rule.—PTR+P=A.

Note.—Any number of letters joined together, like the letters in a word, denote the continual multiplication of the terms represented by those letters. The number represented by a single letter must be added or subtracted, according to the sign prefixed to it.

Examples.

1. What sum will 567£ 10s. amount to in 9 years, at 6£ per cent. per annum?

£567.5=p.
9yrs.=t.

5107.5=pt.
.06=r.

306.450=ptr.
567.5=p.

Ans. 873.950=ptr+p=a.

2. What will $419 amount to in 1 year, at $6 per cent.? Ans. $444.14cts.

3. What will $964.75cts. 6m. amount to in 6 years, at 4½ per cent. per annum? Ans. $1225.24cts.+012.

4. What will $1296.87cts.5m. amount to in 4 years, at $7½ per cent. per annum? Ans. $1685.93cts. 7½ms.

Case 2.

When the given time does not consist of whole years.

Rule.

Find the equivalent decimal answering to the odd time, (in the *following table,*) to which join the whole years, (if any,) and then *proceed by the first* case.

2. An acre of land contains 160 square perches, therefore please to tell me how many perches in length will make one side of a square acre? Ans. 12.649 perches.+

Problem 3.

When any number of men are to be placed in rank and file, so that the number in rank may be 2, 3, 4, or 5 times, &c. more than in the file.

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1. If the diameter of a given circle be 4 inches, what is the diameter of another one 3 times as large? Ans. 6.928+ inches.

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Made fast to a stake by a line from his nose;
How long must this line be, that, feeding around,
Permits him to graze just an acre of ground?

perches.
4)160=the area of an acre.
30¼

4800
40

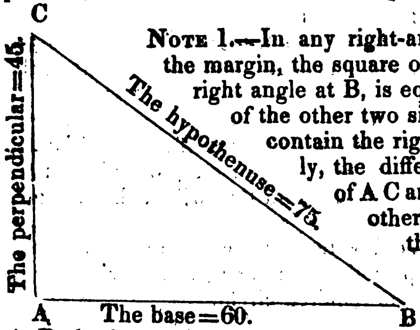
4840=square yards in 1 acre.
9 feet=1 square yard.

.7854)43560.0000(55462.18ft.=the square of the diameter of 1 acre.

3828 remainder.

$\sqrt{55462.18}$=235.5 ft.+=the diameter of an acre, and 117.75ft.= the half diameter; therefore, the true length of the line required is 117 feet 9 inches.

2. The superficial content of a circular deer park is 49 acres 14 perches, the diameter of which is required. Ans. 100 perches.



Note 1.—In any right-angled triangle, as A B C in the margin, the square of the side A C, opposite the right angle at B, is equal to the sum of the squares of the other two sides, A B and B C, which contain the right angle at B; consequently, the difference between the square of A C and the square of either of the other sides will be the square of the remaining side. Hence the 6th, 7th, & 8th problems.

Note 2.—The side A C is called the hypothenuse, A B the base, and B C the perpendicular.

Problem 6.

Given the base and perpendicular, to find the hypothenuse.

Rule.

Extract the square root of the sum of the squares of the base and perpendicular, and that will be the length of the hypothenuse required.

An Example.

When the base A B is 60 and the perpendicular B C 45 equal parts, what is the length of the hypothenuse A C?

$60 \times 60 = 3600$
$45 \times 45 = 2025$

$\sqrt{5625} = 75$ Answer.

Problem 7.

Given the base and hypothenuse, to find the perpendicular.

Rule.

Extract the square root of the difference of the squares of the base and hypothenuse, and that will be the height of the perpendicular required.

An Example.

When the base A B is 60, and the hypothenuse A C 75 equal parts, what is the height of the perpendicular B C?

$$75 \times 75 = 5625$$
$$60 \times 60 = 3600$$

$$\sqrt[2]{2025} = 45 \text{ Ans.}$$

Problem 8.

Given the hypothenuse and perpendicular, to find the base.

Rule.

Extract the square root of the difference of the squares of the hypothenuse and perpendicular, and that will be the length of the base required.

An Example.

When the hypothenuse A C is 75, and the perpendicular B C 45 equal parts, what is the length of the base A B?

$$75 \times 75 = 5625$$
$$45 \times 45 = 2025$$

$$\sqrt[2]{3600} = 60 \text{ Ans.}$$

Problem 9.

Given the height of the roof and the width of the house, to find the length of the rafter.

Rule.

Extract the square root of the sum of the squares of the height of the roof and half the width of the house, and that will be the length of the rafter.

An Example.

Admit a house to be 32 feet wide, and the height of the roof 12 feet, what is the length of the rafters?

$$16 \times 16 = 256$$
$$12 \times 12 = 144$$

$$\sqrt[2]{400} = 20\text{ft. Ans.}$$

Problem 10.

Given the length of a rafter and the width of the house, to find the height of the roof or length of the king post.

7. Annex the third period in the given number to the remainder for a new resolvend.

8. Double all the figures in the root, now found, for a new divisor, and set it on the left hand of the new resolvend; then find the next figure of the root, as before directed, and continue the operation in the same manner till you have brought down all the periods in the given number.

NOTE.—The operation may be continued to any degree of exactness by annexing pairs of ciphers to the remainder.—See the 4th example, and examine it attentively.

PROOF.

Square the root and add the remainder (if any) to the product; the result will be equal to the given number, if the operation is right.

EXAMPLES.

1. What is the square root of 1296? Ans. 36.

```
   1296(36=the root.
    9
  ----
 66)396=resolvend.
    396
    ---
    ...
```

2. What is the square root of

```
  133225(365 Answer.
  9
  ----
66)432=resolvend.
   396
   ----
725)3625=new resolvend.
    3625
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```
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   4
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     -----
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      23425
      -----
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```

4. What is the square root of 2.2710957?

```
        2.27109570(1.507015 Ans.
        1
        ---
     25)127
        125
        ---
    300)210
        000
        -----
   3007)21095
        21049
        -----
  30140)4670
        0000
        -------
 301401)4670,00
        3014 01
        --------
3014025)16559900
        15070125
        --------
         1489775 remainder.

         1.507015
         1.507015
         --------
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         1507015
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5. What is the square root of $\frac{36}{49}$? Ans. $\frac{6}{7}$.
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OF MIXED NUMBERS.

Rule.

1. Reduce the fractional part of the mixt number to its lowest terms, and then reduce the mixed number to an improper fraction.

annum will $728.85cts. payable $1\frac{1}{2}$ years hence, produce $678 in cash? Ans. $5 per cent.

6. At what rate per cent. per annum will $840, payable 4 years hence, produce $600 present money? Ans. $10 per cent.

ANNUITIES OR PENSIONS IN ARREARS, AT SIMPLE INTEREST.

Annuities or Pensions are said to be in arrears when they are payable yearly, half yearly, or quarterly; and remain unpaid for any number of payments.

NOTE.—U represents the annuity or pension, &c., R, T, and A as before.

CASE 1.

When U, R, and T are given, to find A.

$$\text{RULE.}— \frac{tut - tu}{2} \times r + tu = a.$$

EXAMPLES.

1. If an annuity of $70 be forborne 5 years, what will it amount to in that time, at $5 per cent. per annum?

$$\begin{array}{r} 5\text{ yrs.}=t. \\ 70=u. \\ \hline 350=tu. \\ 5=t. \\ \hline 1750=tut. \\ 350=tu. \\ \hline 2)1400 \\ \hline 700 \\ .05=r \\ \hline 35.00 \\ \text{Add } 350=tu. \\ \hline \end{array}$$

$385.00=a, or the amt. Ans.

2. If the payment of a pension of $156 be omitted for 7 years; what will it amount to in that time, at $6 per cent. per annum? Ans. $1288.56cts.

3. If a house be let upon a lease for 8 years, at $80 per year, what will it amount to in that time at $4 per cent. per annum.? Ans. $729.60cts.

4. Suppose a salary of 333.33\frac{1}{3}$cts. a year be omitted for 6 years; what will it amount to in that time, at 4\frac{1}{2}$ per cent. per annum? Ans. $2225.

NOTE.—When the annuities, pensions, &c. are to be paid half yearly or quarterly, as most generally they are—then, for half yearly payments, take half of the ratio, half of the annuity or yearly rent, and twice the number of years for R, U, and T. But, for quarterly payments, take one fourth part of the ratio, one fourth part of the annuity or yearly rent, and ··r times the number of years; then proceed as before.

2. An acre of land contains 160 square perches, therefore please to tell me how many perches in length will make one side of a square acre? Ans. 12.649 perches.+

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When any number of men are to be placed in rank and file, so that the number in rank may be 2, 3, 4, or 5 times, &c. more than in the file.

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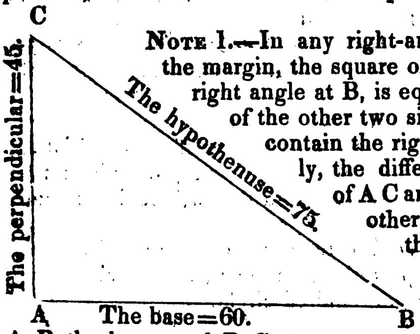
2. The superficial content of a circular deer park is 49 acres 14 perches, the diameter of which is required. Ans. 100 perches.



NOTE 1.—In any right-angled triangle, as A B C in the margin, the square of the side A C, opposite the right angle at B, is equal to the sum of the squares of the other two sides, A B and B C, which contain the right angle at B; consequently, the difference between the square of A C and the square of either of the other sides will be the square of the remaining side. Hence the 6th, 7th, & 8th problems.

NOTE 2.—The side A C is called the hypothenuse, A B the base, and B C the perpendicular.

PROBLEM 6.

Given the base and perpendicular, to find the hypothenuse.

RULE.

Extract the square root of the sum of the squares of the base and perpendicular, and that will be the length of the hypothenuse required.

AN EXAMPLE.

When the base A B is 60 and the perpendicular B C 45 equal parts, what is the length of the hypothenuse A C?

$60 \times 60 = 3600$
$45 \times 45 = 2025$

$\sqrt{5625} = 75$ Answer.

Problem 7.

Given the base and hypothenuse, to find the perpendicular.

Rule.

Extract the square root of the difference of the squares of the base and hypothenuse, and that will be the height of the perpendicular required.

An Example.

When the base A B is 60, and the hypothenuse A C 75 equal parts, what is the height of the perpendicular B C?

$$75\times75=5625$$
$$60\times60=3600$$

$$\sqrt[2]{2025}=45 \text{ Ans.}$$

Problem 8.

Given the hypothenuse and perpendicular, to find the base.

Rule.

Extract the square root of the difference of the squares of the hypothenuse and perpendicular, and that will be the length of the base required.

An Example.

When the hypothenuse A C is 75, and the perpendicular B C 45 equal parts, what is the length of the base A B?

$$75\times75=5625$$
$$45\times45=2025$$

$$\sqrt[2]{3600}=60 \text{ Ans.}$$

Problem 9.

Given the height of the roof and the width of the house, to find the length of the rafter.

Rule.

Extract the square root of the sum of the squares of the height of the roof and half the width of the house, and that will be the length of the rafter.

An Example.

Admit a house to be 32 feet wide, and the height of the roof 12 feet, what is the length of the rafters?

$$16\times16=256$$
$$12\times12=144$$

$$\sqrt{400}=20\text{ft. Ans.}$$

Problem 10.

Given the length of a rafter and the width of the house, to find the height *of the roof or* length of the king post.

Rule.

Subtract the square of half the width of the house from the square of the rafter; then extract the square root of the remainder, and that will be the height of the roof.

An Example.

If a house be 24 feet wide, and the rafters 18 feet long, what is the height of the roof?

$18\times 18=324$ the square of the rafter.
$12\times 12=144$ the square of half the width.

$\sqrt{180}=13$ft. 4 tenths. Ans.

Practical Questions.

1. How many superficial feet are contained on a floor that is 75 feet square? Ans. 5625.
2. What is the difference between 9 feet square and 9 square feet? Ans. 72 square feet.
3. A certain square pavement contains 197136 square stones, all of the same size; how many are contained in one of its sides? Ans. 444.
4. Being about to plant out 5292 trees, equally distant from each other, in rows; the length of the grove is to be 3 times the breadth; how many trees must there be in each row? Ans. 42 in the short rows, and 126 in the long ones.
5. A ladder 40 feet long will exactly reach from the top of a fort to the opposite side of a ditch that is 24 feet wide; what is the height of the wall? Ans. 32 feet.
6. The wall of a town that is 18 feet high, is surrounded by a moat 20 yards wide; required the length of a ladder that will reach from the outside of the moat to the top of the wall? Ans. 62.6ft.+
7. Suppose a ladder 60 feet long, be so planted as to reach a window 37 feet from the ground on one side of a street, and without moving it at the foot, will reach another window 23 feet high on the other side; please to tell me the breadth of the street? Ans. 102.649 feet.+
8. If a field be 800 yards long, and 600 yards wide, what is the distance between two opposite corners? Ans. 1000 yards.
9. There is a square field containing 10 acres; what is the distance from the centre to each corner? Ans. 28.28 rods+.
10. The height of a tree growing in the centre of a circular island (44 feet in diameter) is 75 feet, and a line stretched from the top of it, over to the hither edge of the water is 256 feet long; what is the breadth of the stream, supposing the land to be level on each side of the water? Ans. 222.767 feet+.

A TABLE

For the ready finding of the decimal parts of a year equal to any number of months and days, allowing the year to be 12 calendar months of 30 days each.

Days.	Decimal parts.	Days.	Decimal parts.	Months.	Decimal parts.
1	.00277	30	.08333	1	.08333
2	.00555	40	.11111	2	.16666
3	.00833	50	.13888	3	.25000
4	.01111	60	.16666	4	.33333
5	.01388	70	.19444	5	.41666
6	.01666	80	.22222	6	.50000
7	.01944	90	.25000	7	.58333
8	.02222	100	.27777	8	.66666
9	.02500	200	.55555	9	.75000
10	.02777	300	.83333	10	.83333
20	.05555			11	.91666

EXAMPLES.

1. What will $500 amount to in 4 years 11 months and 7 days, at $4 per cent. per annum?

Decimals.

11 months=.91666
7 days =.01944

yrs. 4.93610=t.
.04=r.

.197440=tp.
500=p.

98.7220000=ptr.
500.=p.

Ans. $598.72,2ms.=a.

2. What will $600 amount to in 356 days, at $6 per cent. per annum?

Ans. $635.59cts. 9m.+32.

3. What will $875.25cts. amount to in 400 days, at $8 per cent. per annum?

Ans. $953.04cts. 9m+222.

4. What will $1200 amount to in 389 days, at $5½ per cent. per annum?

Ans. $1271.31cts. 6m.+3.

5. What will $800 amount to in 2 years 5 months and 29 days, at $4½ per cent. per annum?

Ans. $889.89cts. 9m+56.

6. What will $475.87cts. 5m. amount to in 70 days, at $7½ per cent. per annum? Ans. $482.81cts. 4ms.+685125.

CASE 3.

When A, T, and R are given, to find P.

RULE.— $\frac{a}{tr+1}=P.$

from the said period, and to the remainder annex the next period for a resolvend.

3. Take the triple square of the ascertained root, for a defective divisor.

4. Try how often the said defective divisor is contained in the resolvend (omitting the two right hand figures)—place the result of this trial in the root, and its square on the right hand of the said defective divisor, supplying the place of tens with a cipher, if the square be less than ten.

5. Complete the divisor by adding thereto the product of the last figure in the root, multiplied by the rest and by 30.

6. Multiply, subtract, and bring down the next period for a new resolvend.

7. Find a new defective divisor, by adding the last complete divisor, the number which completed it, and twice the square of the last figure in the root, together.

8. Complete the new defective divisor as before, and proceed on as above directed, till all the periods are used.

NOTE.—The operation may be continued to any degree of exactness by annexing triplets of ciphers to the last remainder, and so on.

PROOF.

Cube the whole root, that is, multiply it into itself three times, and add the last remainder (if any) to the last product, and the total sum will be equal to the given number, if the work is right.

EXAMPLES.

1. What is the cube root of 240061.988375?

	10804	
	360	240061.988375(62.15 = cube root required.
		216
Complete divisor=	11164	
	8	24061=the 1st resolvend.
	1153201	22328
62×1×30=	1860	
		1733988=the 2d resolvend.
Complete divisor=	1155061	1155061
Twice the square of 1=	2	
	115692325	578927375=3d resolvend.
621×5×30=	93150	578927375
Complete divisor=	115785475	000000000

1st. I put a point over the unit's figure 1, and then proceed both ways to distinguish the given number into periods of three figures each, according to the rule. 2d. The greatest root contained in the left hand period 240 is 6, which I place on the right hand of the

given number, and the cube of 6 is 216, which I subtract from the said period, and the remainder is 24, to which I annex the second period 061 to form a resolvend. 3d. The triple square of 6=108, which I set on the left hand of the given number for the defective divisor. 4th. I try how often the defective divisor 108 is contained in the resolvend 24061 (omitting the two right hand figures 61) and the result is 2, which I place after 6 in the root, and the square of 2=4, which is less than 10, therefore I supply the place of tens with a cipher thus, 04, and set it on the right hand of the defective divisor 108, and the result is 10804. 5th. Now 2 is the last figure in the root, therefore $2 \times 6 \times 30 = 360$, which I add to 10804, and the sum is 11164=the complete divisor. 6th. I multiply the complete divisor by 2 and subtract the product from the resolvend 24061, and the remainder is 1733, to which I annex the next period 988, for a new resolvend. 7th. Twice the square of 2 (the last figure in the root as yet)=8, which I set under the last complete divisor, and say $8+11164+360=11532$ the second defective divisor, with which I proceed as before, and so on till the work is finished.

2. What is the cube root of 22069810125? Ans. 2805.
3. What is the cube root of 673373097125? Ans. 8765.
4. What is the cube root of 262144? Ans. 64.
5. What is the cube root of 34965783? Ans. 327.
6. What is the cube root of 268336125? Ans. 645.
7. What is the cube root of 84.604519? Ans. 4.39.
8. What is the cube root of 138.188413? Ans. 5.17.
9. What is the cube root of 5232228.323420125? Ans. 173.605.
10. What is the cube root of .001906624? Ans. .124.

OF THE CUBE ROOT OF VULGAR FRACTIONS.

Rule.

1. Reduce the given fraction to its lowest terms (if necessary) then extract the cube root of the numerator for a new numerator, and of the denominator for a new denominator.

2. If the given fraction be a surd, then reduce it to its equivalent decimal value, and extract the cube root thereof.

Examples.

1. What is the cube root of $\frac{27}{64}$? Ans. $\frac{3}{4}$.
2. What is the cube root of $\frac{256}{500}$? Ans. $\frac{4}{5}$.
3. What is the cube root of $\frac{1000}{1728}$? Ans. $\frac{5}{6}$.
4. What is the cube root of $\frac{648}{1029}$? Ans. $\frac{6}{7}$.
5. What is the cube root of $\frac{2744}{4096}$? Ans. $\frac{7}{8}$.
6. What is the cube root of $\frac{6144}{8748}$? Ans. $\frac{8}{9}$.

OF SURDS.

7. What is the cube root of $\frac{4}{9}$? Ans. .763.+
8. What is the cube root of $\frac{6}{7}$? Ans. .949.+
9. What is the cube root of $\frac{1}{2}$? Ans. .793.+

EXAMPLES.

1. I demand what principal will amount to 873£ 19s. in 9 years, at $6 per cent. per an.?

9 years=t.
.06=r.

.54=tr.
add 1.00

£ £
1.54)873.950=a(567.5=P.
000 000

2. I demand what principal will amount to $444.14cts. in 1 year, at $6 per cent. per annum? Ans. $419.

3. I demand what principal will amount to $1225.24c.+012, in 6 years, at $4½ per cent. per annum? Ans. $964.75cts. 6m.

4. I demand what principal will amount to $1685.93cts. 7½m. in 4 years, at $7½ per cent. per annum? Ans. $1296.87cts. 5m.

5. I demand what principal will amount to $598.72cts. 2m. in 4 years 11 months and 7 days, at $4 per cent. per an.? Ans. $500.

6. I demand what principal will amount to $635.59cts. 9m.+32 in 356 days, at $6 per cent. per annum? Ans. $600.

CASE 4.

When A, P, and T are given, to find R.

RULE.— $\frac{a-p}{tp}$=R, or the ratio.*

EXAMPLES.

1. At what rate per cent. will 567£ 10s. amount to 873£ 19s. in 9yrs.?

p=£567.5 | 873.950=a.
t= 9yrs. | 567.5=p.

tp=5107.5)306.450(.06=R, or the ratio.
100
000 000
6.00=$6 per cent.

2. At what rate per cent. will $419 amount to $444.14 cents, in one year? Ans. $6 per cent.

3. At what rate per cent. will $964.75cts. 6m. amount to $1225 24cts.+012rem. in 6 years? Ans. $4½ per cent. per annum.

4. At what rate per cent. will $1296.87cts. 5m. amount to $1685 93cts. 7½m., in 4 years? Ans. $7½ per cent. per annum.

5. At what rate per cent. will $500 amount to $598.7[illegible] in 4 years 11 months and 7 days? Ans. $4 per cent. p[illegible]

6. At what rate per cent. will $600 amount to $635.[illegible] 32 rem. in 356 days? Ans. $6 per[illegible]

CASE 5.

When A, P, and R are given, t[illegible]

RULE.— $\frac{a-p}{rp}$=T, or the[illegible]

EXAMPLES.

1. In what time will 567£ 10s. amount to 873£ 19s., at $6 per cent. per annum?

p=£567.5|873.950=a.
r= .06|567.5=p.

rp=34.050)306.450(9yrs.=t.

000 000

2. In what time will $419 amount to $444.14cts., at $6 per cent.? Ans. 1 year.

3. In what time will $964.75 cents amount to $1225.24cts.+012rem., at $4½ per cent. per annum? Ans. 6 years.

4. In what time will $1296.87 cts. 5m. amount to $1685.93cts. 7½ mills, at $7½ per cent. per annum? Ans. 4 years.

5. In what time will $500 amount to $598.72cts. 2 mills, at $4 per cent. per annum? Ans. 4yrs. 11mo. and 7ds.

OF DISCOUNT BY DECIMALS.

Let A=the amount of the debt, or given sum.
P=the present worth of the given sum.
T=the time before the debt becomes due.
R=the ratio of the rate per cent.

CASE 1.

When A, T, and R are given, to find P.

RULE.— $\frac{a}{tr+1}$=P, or the present worth.

EXAMPLES.

1. I demand the present worth of $100, which is not due till the termination of one year, allowing discount at $6 per cent. per an.?

t=1 year.
r=.06

.06
Add 1. $=a. $ cts. m.
tr+1=1.06)100.00(94.33,9
954 or pre

460
424

36
3

2. I demand the present worth of $1200, due 6 months hence, allowing discount at $6 per cent. per annum? Ans. $1165+50.

3. I demand the present worth of $500, due 2½ years hence, allowing discount at $10 per cent. per annum? Ans. $400.

4. I demand the present worth of $5 due 4½ months hence, ount at $8 per cent. Ans. $5000.

d the present worth ents, due 1½ years ount at $5 Ans.$678.

nt worth e 1 year e, allow- per cent. payment? +1761

Examples.

1. If a house be leased at \$50 a year, payable half yearly, how long may it be possessed by the lessee for \$262.50cts. present money, allowing \$5 per cent. for prompt payment? Ans. 6 years.

2. If a house be leased at \$50 a year, payable quarterly, how long may the lessee keep possession of it, for \$263.94cts. 2m.+8 rem. present money, allowing him \$5 per cent. for prompt payment?

$ cts. m.
263.94,2=p
2

u=12.5)527.884000=2p.

42.23072=2p÷u.
1.....

43.23072=2p÷u+1.

r=.0125)2.0000

160
43.23072

116.76928=x.
116.76928=x.

4)13635.0647517184=xx.

3408.7661879296=xx÷4.
3378.46272=the quotient of 2p÷ru.

√6787.2289079296=82.38464
x÷2 =58.38464

r= .0125
u= 12.5

ru=.15625

263.942+8 rem.
2

527.8848000000

2p÷ru = 3378.46272

Ans. 24 payments=6 years.

OF ANNUITIES, LEASES, &c. IN REVERSION.

Annuities, leases or pensions, &c., are said to be in reversion when they are not to commence till the expiration of a certain period of time.

Case 1.

To find the present worth of an annuity, &c., taken in reversion.

Rule.

1. *Find the* present worth of the yearly sum for the time of its

continuance, at the given rate per cent; to do which, U, T, and R are given, to find P by the following theorem:

$$\text{Viz. } \frac{rtt-rt+2t\times u}{2rt+2}=P.$$

2. Find what sum will amount to the present worth, at the same rate per cent. and for the time before the annuity commences; to do which, change P into A and proceed by the following theorem:

$$\text{Viz. } \frac{a}{tr+1}=P.$$

EXAMPLES.

1. I demand the present worth of a lease of $30 per year, for 3 years, but not to commence till the end of 2 years, allowing $4 per cent. for prompt payment?

.04=r.
3=t=time of continuance.
.12=rt.
3=t.
.36=rtt.
.12=rt
.24=rtt—rt.
6.=2t added.
6.24
30=u.

rt=.12
2
.24
2.
2.24) 187.2000(83.57=p. ($ cts.)
32 rem.

Now I change P, which is=$83.57cts. into A, and proceed by the second theorem.

r=.04
t= 2yrs. before the lease begins.
tr=.08
1
1.08)83.57000(77.37,9=p, or present worth required. ($ cts. m.)
88 rem.

2. I have the promise of a pension of $60 a year for 7 years, but it does not commence till the end of 4 years; what is it worth in ready money, allowing the purchaser $5 per cent. for prompt payment? Ans. $298.14+2 rem.

3. Suppose I have rented a farm for 10 years, at $100 a year, but am not to get possession till the end of 3 years, what sum must I pay in hand, when the renter allows me $6 per cent. for prompt payment? Ans. $672.66,9m.

4. A benevolent gentleman settled a pension of $120 a year for 12 years, on an unbeneficed minister, to commence at the termination of 5 years, but he being in want of money to purchase a library, disposes of the pension at $7½ per cent. discount; I demand the present worth? Ans. $778.56cts.

CASE 2.

To find the yearly income of a pension, &c. taken in reversion.

Rule.

1. Find the amount of the present worth at the given rate per cent. and for the time before the reversion; to do which, there are given P, T, and R to find A by the following theorem:

$$\text{Viz. } PTR+P=A.$$

2. Find what yearly rent or pension, &c. being put out at interest, will produce the amount of the present worth at the same rate and for the time of its continuance; to do which, change A into P, and proceed by the following theorem:

$$\frac{2p\times\overline{rt+1}}{rtt-rt+2t}=U.$$

Examples.

1. The lease of a house for 3 years, which does not commence till the end of 2 years, is sold for the present worth of $77.37cts. 9m.+68 rem.; what is the yearly rent, allowing the purchaser $4 per cent. for his ready money?

By the first theorem.

Present worth is $77.379c.=p.
Time before reversion=2yrs.=t.

154.758=pt.
.04=r.
6.19032=ptr.
77.379=p.
83.56932
68 rem.
$83.57000=a.

Now, by the second theorem.

r= .04
t= 3yrs=time of continuance.

rt=	.12	$83.57=p.
t=	3	2
rtt=	.36	167.14
rt=	.12	1.12
	.24	187.1968
Add 2t=	6.	32 rem. add.
	6.24	187.2000($30=u.
		1872

2. I have the promise of a pension for 7 years, but it does not commence till the expiration of 4 years, and I have sold my right of it for $298.14cts+2m.; what is the yearly income, allowing the purchaser $5 per cent. for prompt payment? Ans $60.

3. Suppose I have leased a farm for 10 years, but am not to get possession till the end of 3 years; what is the yearly rent, when the lessor receives $672.66cts. 9m.+58 rem. for the present worth, allowing $6 per cent. for prompt payment? Ans. $100.

4. A benevolent gentleman settled a yearly gratuity on an unbeneficed minister for 12 years, to commence at the termination of 5 years; what is the yearly amount, when the gratuity was sold for *$778.56cts.* ready money, allowing the purchaser $7½ per cent. for *prompt payment.* Ans. $120.

OF COMPOUND INTEREST BY DECIMALS.

Let P, T, R, and A represent the principle, time, ratio, and amount, as in Simple Interest, &c.

The ratio in Compound Interest signifies the amount of $1 or 1£ for one year, at any proposed rate of interest per cent., and is found by the subsequent proportion.

As $100 : $106 :: 1 .. $1.06.

A TABLE OF RATIOS,

Or the amount of $1 or 1£ for one year.

Rate per cent.	*Ratio.*	*Rate per cent.*	*Ratio.*	*Rate per cent.*	*Ratio.*
2	1.02	5½	1.055	8	1.08
3	1.03	6	1.06	8½	1.085
3½	1.035	6½	1.065	9	1.09
4	1.04	7	1.07	9½	1.095
4½	1.045	7½	1.075	10	1.1
5	1.05				

Case 1.

When P, T, and R are given, to find A.

Rule.—$P \times r^t = A$.

N. B.—r^t signifies that the ratio is involved up to the number of years indicated by the t, which is placed nearly over the r.

Examples.

1. What will $480 amount to in 3 years, at $5 per cent. per annum?

$1.05cts.=r, or the amount of $1 for 1year.
1.05

1.1025=r^t, or the amount of $1 for 2 years.
1.05

1.157625=r^t, or the amount of $1 for 3 years.
480=p, or the principal.

Ans. $555.660000=a, or the whole amount.

2. What will $750 amount to in 4 years, at $6 per cent. per annum? Ans. $946.85cts. 7m.+.72 rem.

3. What will $1250 amount to in 5 years, at $6 per cent. per annum? Ans. $1672.78cts. 1m.+.972 rem.

What will $480 amount to in 6 years, at $5 per cent. per annum? Ans. $643.24cts. 5m.+.9075 rem.

Case 2.

When A, R, and T are given, to find P.

Rule.— $\frac{a}{r^t}=P.$

Examples.

1. What principal will amount to $555.66cts. in 3 years at $5 per cent. per annum?

$$\overline{1.05}|^{3}=1.157625)555.660000(\$480=P. \text{ Ans.}$$

($ cts.=a.)

2. What principal will amount to $946.85cts. 7m.+72 rem. in 4 years, at $6 per cent. per annum? Ans. $720.

3. What principal will amount to $1672.78cts. 1m.+972 rem. in 5 years, at $6 per cent. per annum? Ans. $1250.

4. What principal will amount to $643.24cts. 5m.+9075 rem. in 6 years, at $5 per cent. per annum? Ans. $480.

Case 3.

When P, R, and A, are given, to find T.

Rule.— $\frac{a}{p}=r^t$ { then involve r, till it is equal to r^t, and the index of the power will be t.

Examples.

1. In what time will $480 amount to $556.66cts. at $5 per cent?

($ cts.=a.)

$$p=\$480)555.660000(1.157625=r^t,$$ which = 3 involutions of r, wherefore the time is 3 years.

2. In what time will $750 amount to $946.85cts. 7m.+72 rem. at $6 per cent. per annum? Ans. 4 years.

3. In what time will $1250 amount to $1672.78cts. 1m.+972 rem. at $6 per cent. per annum? Ans. 5 years.

4. In what time will $480 amount to $643.24cts. 5m.+9075 rem. at $5 per cent per annum? Ans. 6 years.

Case 4.

When P, A, and T are given, to find R.

Rule.— $\frac{a}{p}=r^t$ { then extract that root of r^t which is indicated by the number of years in the question for r.

EXAMPLES.

1. At what rate per cent. will $480 amount to $555.66cts. in 3yrs?

480)555660000(1.157625=$\overset{t}{r}$, and the $\sqrt[3]{1.157625}$=1.05=r. Ans.

2. At what rate per cent. will $750 amount to $946.85cts. 7m. +72 rem. in 4 years? Ans. $6 per cent.

3. At what rate per cent. will $1250 amount to $1672.78cts. 1m. +972 rem. in 5 years? Ans. $6 per cent.

4. At what rate per cent. will $480 amount $643.24cts. 5 m.+ 9075 rem. in 6 years? Ans. $5 per cent.

OF ANNUITIES OR PENSIONS, &c. IN ARREARS AT COMPOUND INTEREST.

NOTE.—U represents the annuity, pension, &c., R, T, and A as heretofore.

CASE 1.

When U, T, and R are given, to find A.

$$\text{RULE.—}\frac{u\times r^{t}-u}{r-1}=A.$$

EXAMPLES.

1. What will an annuity of $80 a year, payable yearly, amount to in 4 years, at $5 per per cent. per annum?

$\overline{1.05}|^{4}$=1.21550625=r^{t},
80=u.

1.05	97.24050000
1	80=u, subtracted.
.05	17.2405

Ans. $344.81cts.=a.

2. What will an annuity of $250 per annum, payable yearly, amount to in 5 years, at $6 p. c.? Ans. $1409.27c. 3m.+.24rem.

3. What will an annuity of $240 a year, payable yearly, amount to in 6 years, at $5 p. c.? Ans. $1632.45c. 9m.+.075 rem.

4. What will an annuity of $480 a year, payable yearly, amount to in 7 years, at $5 p. c.? Ans.$3908.16c. 4m.+.0575rem.

CASE 2.

When R. T, and A are given, to find U.

$$\text{RULE.—}\frac{ra-a}{r^{t}-1}=U.$$

EXAMPLES.

1. What annuity, being forborne 4 years, will amount to $344.81 cents, at $5 per cent. per annum?

$$\$344.81\text{cts.}=a.$$
$$1.05=r.$$

$$\overline{1.05}|^{4}=1.21550625 \quad | \quad 362.0505=ra.$$
$$1 \quad | \quad 344.81=a, \text{ subtracted.}$$
$$.21550625\ )\ 17.24050000(\$80=u.\ \text{Ans.}$$

2. What annuity, being forborne 5 years, will amount to \$1409. 27cts. 3m.+24 rem., at \$6 per cent. per annum? Ans. \$250.

3. What annuity, being forborne 6 years, will amount to \$1632. 45cts. 9m.+075 rem., at \$5 per cent. per annum? Ans. \$240.

4. What annuity, being forborne 7 years, will amount to \$3908. 16cts. 4m.+.0575 rem., at \$5 per cent. per annum? Ans. \$480.

Case 3.

When U, A, and R are given, to find T.

Rule.— $\frac{ar+u-a}{u}=r^{t}$, then involve r, till it is equal to r^{t}, and the index of the power will be $=t$.

Examples.

1. In what time will \$80 annuity per an. amount to \$344. 81cts., allowing \$5 per cent. for the forbearance of payment?

$$\$344.81\text{cts.}=a.$$
$$1.05=r.$$
$$362.0505=ar.$$
$$80.=u, \text{ added.}$$
$$442.0505$$
$$344.81=a, \text{ subtracted.}$$
$$u=80)\ 97.24050000$$
$$1.21550625=r^{t}, \text{ which}$$

is equal to 4 involutions of r, wherefore, the required time is 4 years.

2. In what time will a pension of \$250 a year amount to \$1409. 27cts. 3m.+24 rem., at \$6 per cent.? Ans. 5 years.

3. In what time will the rent of a plantation, at \$240 a year, amount to \$1632.45cts. 9m.+075 rem., at \$5 per cent? Ans. 6 years.

4. In what time will a salary of \$480 a year amount to \$3908. 16cents 4mills+0575? Ans. 7 years.

OF THE PRESENT WORTH OF ANNUITIES, &c. AT COMPOUND INTEREST.

Note.—P represents the present worth, U, T, and R, as before.

Case 1.

When U, T, and R are given, to find P.

RULE.— $\overline{u - \frac{u}{r^t}}\ ;\ \div \overline{r - 1} = P.$

EXAMPLES.

1. What is the present worth of an annuity of $80 a year, to continue 4 years, at $5 per cent. per annum discount?

$$\overline{1.05}|^{4} = 1.21550625)80.00000000000(65.816 = u \div r^{t}. \quad (\$ = u.\ \ \$\ cts.)$$

24065000 rem. rejected.

$$\begin{array}{r|l} r = 1.05 & \$80 = u. \\ 1 & 65.816 = u \div r^{t}, \\ \hline .05 & 14.1840 \end{array}$$

$283.68cts. = p. Ans.

2. What is the present worth of a salary of $250 a year, to continue 5 years, at $6 per cent. per annum discount? Ans. $1053.10cts.

3. What is the present worth of a pension of $240 a year, to continue 6 years, discounting $5 per cent. per annum for prompt payment? Ans. $1218.16cts, 8m.+.

4. A superannuated officer has a pension of $480 a year allowed him for 7 years; what is it worth in present money, allowing the purchaser $5 per cent. discount? Ans. $2777.46cts.+

CASE 2.

When P, T, and R are given, to find U.

RULE.— $\dfrac{r^{t}\, p \times r - 1}{r^{t} - 1} = U.$

EXAMPLES.

1. What annuity, to continue 4 years, may be bought with $283.68cts., allowing $5 per cent. per annum for prompt payment?

$$\overline{1.05}|^{4} = 1.21550625 = r^{t}$$

283.68 = present worth.

$$344.8148130000 = p \times r^{t}.$$

$$.05 = r - 1.$$

$$\begin{array}{r|l} 1.21550625 & 17.240740650000 \\ 1 & \text{Sub.}\ \ 24065000 \text{ remainder rejected in 1st. case.} \\ \hline .21550625\,) & 17.24050000(80\$ = u.\ \text{Ans.} \end{array}$$

2. I demand what annuity may be bought for 5 years, with $1053. 10cts.+ ready money, allowing the purchaser $6 per cent. per annum for prompt payment? Ans. $250.

3. If the present worth of a lease, to continue 6 years, be $1218. 16cts. 8m.+ what is the yearly rent, allowing $5 per cent. per annum for prompt payment? Ans. $240.

4. If the present worth of a pension, to continue 7 years, be $2777.46cts., what is the yearly income, allowing $5 per cent. per annum discount? Ans. $480.

Case 3.

When U, P, and R are given, to find T.

RULE.—$\frac{u}{p+u-pr}=r^t$ { then involve r, till it is $=r^t$ and the index of the power will be $=t$.

Examples.

1. How long may a lease of $80 a year, be purchased for $283. 68cts. ready money, allowing the purchaser $5 per cent. for prompt payment?

$283.68cts.=p.
80.=u.

r.	363.68=p+u.	$=u. 80.00000000000
283.68×1.05=	297.864=pr.	24065000 rem. subtracted
	65.816	) 79.99975935000(1.21550625=r^t,

which is equal to 4 involutions of r, consequently the time is 4 years.

2. How long may an estate of $250 a year be rented for $1053. 10cts.+ in ready money, allowing $6 per cent. for prompt payment? Ans. 5 years.

3. How long may a pension of $240 a year be enjoyed for $1218. 16cts. 8m.+ allowing the purchaser $5 per cent. for prompt payment? Ans. 6 years.

4. How long may a salary of $480 a year be bought for $2777. 46cts.+ allowing 5 per cent. discount? Ans. 7 years.

OF ANNUITIES, &c. IN REVERSION AT COMPOUND INTEREST.

Case 1.

To find the present worth of an annuity, pension, &c. taken in reversion at Compound Interest.

Rule.

1. *Find the* present worth of the yearly income for the time of

its continuance, at the given rate per cent. To do which, there are given U, T, and R to find P, by the following theorem:

$$\text{Viz. } \overline{u-\frac{u}{r^t}} \div r-1=P.$$

2. Find what sum will amount to the present worth of the annuity, &c. at the same rate and for the time before it commences, and that will be the present worth of the annuity in reversion; to do which, change P into A, and proceed by the following theorem:

$$\text{Viz. } \frac{a}{r^t}=P.$$

EXAMPLES.

1. What is the present worth of the reversion of a lease of $80 a year, to continue 4 years, but not to commence till the end of 2 years, allowing $5 per cent. to the purchaser?

$$\overline{1.05}|^4=1,21550625)\overset{\$=u.}{80.00000000000}(65.816=u\div r^t$$

24065000 rem.

$r=1.05$	$\$80=u.$
$1.$	$65.816=u\div r^t$
$r-1=.05$	$14.184=u-u\div r^t$

$283.68cts.=the present worth for the time of continuance.

Now I change the present worth for the time of continuance into A, and proceed by the second theorem.

$$\overline{1.05}|^2=1.1025)\overset{\$\ \text{cts.}=a.}{283.6800000}(\overset{\$\ \text{cts.m.}}{257.30,6}=P,$$ or the present worth in reversion. Ans.

1350 rem.

2. I have rented a plantation for 5 years at $250 a year, but am not to get possession till the end of 3 years; what is the reversion worth in ready money, allowing discount at $5 per cent. per annum? Ans. $884.20cts.+36528 rem.

3. There is a lease on a certain tract of land worth $240 a year, which is to continue 4 years, and the lessee is desirous to take a lease in reversion for 6 years, to begin when the old lease terminates; I demand the present worth of the said lease in reversion, allowing the purchaser $5 per cent. per annum discount?

Ans. $1002.18cts. 9m.+100681875, rem.

4. There is a tenement now building which will be worth $480 a year, and I have a mind to lease it for 7 years, but I cannot get

possession till the expiration of 5 years; what is the reversion worth in ready money, allowing discount at \$5 per cent. per annum?

Ans. \$2176.21cts.+3300871875 rem.

Case 2.

When the present worth, time, and ratio are given, to find the annuity, &c.

Rule.

1. Find the amount of the present worth of the yearly sum, at the given rate per cent., and for the time before the annuity commences, to do which there are given P, T, and R to find A, by the subsequent theorem:

$$\text{Viz. } p\times r^{t}=a.$$

2. Find what yearly sum will produce the present worth of the annuity, &c. at the same rate and for the time of continuance, and that will be the annuity, &c. required, to do which, change A into P, and proceed by the subsequent theorem:

$$\text{Viz. } \frac{p\times r^{t}\times r-1}{r^{t}-1}=U.$$

Examples.

1. What annuity, to be entered on 2 years hence, and then to continue 4 years, may be purchased for \$257.30cts. 6m.+1350rem. ready money, allowing discount at \$5 per cent. per annum?

$$\begin{array}{ll} & \overset{\$\ \ cts.\ m.}{257.306}=p. \\ \overline{1.05}|^{2}= & 1.1025=r, \text{ raised to t before the reversion.} \\ \hline & 2836798650 \\ & \quad 1350=\text{rem. added.} \\ \hline \end{array}$$

\$283.6800000=a before the commencement, which I change into p, and proceed by the second theorem.

$$\overline{1.05}|^{4}=1.21550625=r^{t} \text{ for the time of continuance.}$$

$$\begin{array}{l} 28368=p, \text{ or the amount before the commencement.} \\ \hline 344.8148130000 \\ \quad .05=r-1. \\ \hline \end{array}$$

$$\begin{array}{r|l} r=\overline{1.05}|^{4}=1.21550625 & 17.240740650000 \\ 1. & \quad 240650000 \text{ rem. subtracted.} \\ \hline r^{t}-1=.21550625 & 17.240500000000(80\$=u. \text{ Ans.} \end{array}$$

2. I have paid \$884.20cts.+36528 rem. for the rent of a plantation (in reversion) 5 years, but am not to get possession till the end of 3 years; what is the yearly rent worth, allowing \$6 per cent. discount for prompt payment? Ans. \$250.

3. A certain lessee in reversion has paid his lessor \$1002.18cts. 9m.+100681875 rem. in hand, for a lease of 6 years, but he is not to have possession till the termination of 4 years; what is the yearly rent worth, allowing discount at \$5 per cent. per annum?
Ans. \$240.

4. Several lessees in reversion have paid the lessor \$2176.21cts. +3300871875 rem. in hand for the lease of a certain tenement 7 years, but they are not to have possession till the end of 5 years; I demand the yearly rent, allowing discount at \$5 per cent. per annum? Ans. \$480.

Case 3.

When the annuity, present worth, and ratio, are given, to find the time of its continuance.

Rule.

1. Find the amount of the present worth, at the given rate per cent., and for the time before the annuity &c. commences, by the subsequent theorem:

$$\text{Viz. } p \times r^t = a.$$

2. Find the time that will be necessary, for the annuity, &c. to produce the amount for the present worth, at the same rate per cent. and that will be the time required; therefore, change A into P, and then proceed by the subsequent theorem:

$$\text{Viz.} \quad \frac{u^*}{p+u-pr} = r^t \quad \left\{ \begin{array}{l} \text{then involve } r \text{ till it is} = r^t\text{, and the} \\ \text{index of the power will be} = t. \end{array} \right.$$

*The remainder in the first operation of case the 1st, must always be substracted from u, before the division is performed.

Examples.

1. The present worth of a certain lease in reversion, to commence 2 years hence, is \$257.30cts.6m.+1350 remainder, and the yearly rent is \$80—what is the time of its continuance, allowing the lessee \$5 per cent. per annum discount for prompt payment?

$$\begin{array}{rl}
 & \overset{\$\ \ c.\ m.}{257.30,6} = p \\
\overline{1.05}|^2 = & 1.1025 = r^t \\
\hline
 & 283.6798\,65\,0 = p \times r^t \\
 & 1\,35\,0 = \text{last rem. added.} \\
\hline
 & \$283.6800000 = a, \text{ or the amount of the pre-}
\end{array}$$

sent worth for the time before the annuity commences, which I change into P, and proceed by the second theorem.

$$\text{Add}\left\{\begin{array}{l}\$283.68\text{cts.}=p.\\ \quad 80.00\text{cts.}=u.\end{array}\right. \qquad \left.\begin{array}{r}\$283.68\text{cts.}=p\\ 1.05=\quad r\end{array}\right\}\text{multiply.}$$

$$363.68=p+u. \qquad \$297.8640=pr.$$

$$\text{Subtract}\quad 297.864=pr.$$

$$\$=u$$

$$65.816\ )\ 80.000000000000$$

$$24065000=\text{the 1st rem. subtracted.}$$

$$t$$

$$79.999759935000\ (1.21550625=r,$$ which is equal to the fourth power of r, therefore the time of continuance is 4 years, the answer required.

2. I have paid \$384.20 cts.+36523 rem. ready money, for the rent of a plantation, but I am not to have possession till the expiration of 3 years—the yearly rent is \$250, what is the time of continuance, allowing discount at \$6 per annum? Ans. 5 years.

3. The present worth of a certain annuity in reversion, to commence 4 years hence, is \$1002.18cts. 9m.+100681875 rem.; the yearly income is \$240; I demand the time of its continuance, allowing discount at \$5 per cent. per annum? Ans. 6 years.

4. Several lessees in reversion have paid their landlord \$2176.21 cts.+3300871875 rem. in ready money, for the lease of a certain tenement, but they are not to have possession till the end of 5 years; the yearly rent is worth \$180; what is the time of continuance, allowing discount at \$5 per cent. per annum for prompt payment?

Ans. 7 years.

OF PERPETUITIES.

Perpetuities are perpetual annuities, or such estates as are bought to continue forever.

NOTE.—U represents the annuity, or yearly rent; R, the ratio, or amount of one dollar, for one year, and P the present worth.

CASE 1.

When U and R are given, to find P.

$$\text{RULE.}\ \frac{u}{r-1}=P.$$

EXAMPLES.

1. Suppose a freehold estate of \$150 a year, is to be sold; what is it worth in ready money, allowing the buyer \$5 per cent. for prompt payment?

$$r=1.05.$$

$$1.\qquad \$=u$$

$$r-1=.05\ )\ 150.00$$

Ans. \$3000=p

2. What is an estate of $285 a year, to continue forever, worth in present money, allowing the purchaser $6 per cent. for prompt payment? Ans. $4750

Case 2.

When P and R are given, to find U.

Rule.—$P \times R - 1 = U$.

Examples.

1. If a freehold estate be bought for $3000, and the allowance of $5 per cent. is made to the buyer, what is the yearly rent? Ans $150.

$$\begin{array}{r} \$3000 = p \\ .05 = r - 1 \\ \hline \text{Ans. } \$150.00 = u. \end{array}$$

2. If an estate be sold for $4750, present money, and $6 per cent. is allowed to the purchaser for prompt payment, what is the yearly rent? Ans. $285.

Case 3.

When P and U are given, to find R.

Rule.—$\dfrac{p+u}{p} = R$.

Examples.

1. If a real estate of $150 a year be sold for $3000 in cash, what rate per cent. was allowed to the buyer for prompt payment?

$$\begin{array}{r} \$3000 = p. \\ 150 = u. \\ \hline \end{array}$$

p=3000)3150.00(1.05=r=$5 per cent.

2. If a freehold estate of $285 a year be sold for $4750 in cash, what rate per cent. was allowed to the purchaser for his ready money? Ans. $6 per cent.

OF PERPETUITIES IN REVERSION.

Note.—T represents the time before the reversion, U and R as before.

Case 1.

When U, R, and T are given, to find P.

Rule.—$u \div r^{t} \times r - 1 = P$.

Examples.

1. If a freehold estate of $150 a year, to commence 3 years hence, be put up to sale, what is the present worth in cash, allowing the buyer $5 per cent. for prompt payment?

$$\overline{1.05}|^{3}=1.157625=r^{t}$$
$$r-1=\quad .05$$

$$\quad \$=u. \qquad \$ \text{ cts.}$$
$$.05788125)150.0000000000(2591.51=p. \text{ Ans.}$$
$$1618125 \text{ rem.}$$

2. What is the real estate of \$285 a year, to commence 4 years hence, worth in present money, allowing the purchaser \$6 per cent. for prompt payment? Ans. \$3762.44cts.+371197056 rem.

Case 2.

When P, T, and R are given, to find U.

Rule.—$p\times r^{t}\times\overline{r-1}=U.$

Examples.

1. A freehold estate has been bought for \$2591.51cts.+1618125 rem., which does not commence till the end of 3 years; what is the yearly income, allowing the buyer \$5 per cent. for his ready money? Ans. \$150.

$$\overline{1.05}|^{3}=1.157625=r^{t}$$
$$.05=r-1.$$
$$.05788125=r^{t}\times r-1.$$
$$2591.51=p.$$
$$149.9998381875=p\times r^{t}\times r-1.$$
$$1618125=\text{rem.}$$
$$\$150.0000000000=u.$$

2. A real estate has been sold for \$3762.44cts.+371197056 rem. which does not commence till the end of 4 years; what is the yearly income, allowing the purchaser \$6 per cent. for his ready money? Ans. \$285.

ALLIGATION.

Alligation is a rule by which two or more simple quantities, of different qualities, may be mixed together, so that the composition may be of a mean or middle quality: it consists of two kinds, namely, Alligation Medial and Alligation Alternate.

OF ALLIGATION MEDIAL.

Alligation Medial is used to find the mean price of any part of *a composition,* when the several ingredients and their respective *prices are given.*

Rule.

As the whole composition is to its whole value, so is any part thereof to its respective value.

Examples.

1. A farmer mixed 20 bushels of wheat, at 83⅓cts. per bushel, and 36 bushels of rye, at 50cts. per bushel, with 40 bushels of barley, at 33⅓cts. per bushel; please to inform me how much one bushel of the composition is worth?

bushels.	*cts.*	*\$ cts.*
20 of wheat,	at 83⅓ per b.	=16.66⅔
36 of rye,	at 50 per b.	=18.00
40 of barley,	at 33⅓ per b.	=13.33⅓
96=the whole composition.		48.00=value of the comp'n.

$$\begin{array}{c} B. \quad \$ \quad B. \\ \text{As } 96 : 48 :: 1 \\ 1 \\ \hline 96)48.00 \\ \hline \end{array}$$

Ans. 50cts. per bush.

Abbreviated.

bush.	*cts.*	*\$ cts.*
20×	83⅓	=16.66⅔
36×	50	=18.00
40×	33⅓	=13.33⅓
96	)	48.00

Ans. 50cents per bushel.

2. A farmer would mix 20 bushels of wheat at \$1.00 per bushel, 18 bushels of rye at 66⅔cts. per bushel, 12 bushels of barley at 50cts. per bushel, and 10 bushels of oats at 40cts. per bushel, together, what will one bushel of the composition cost? Ans. 70cts.

3. A vinter mingled 5 gallons of Canary, at \$1.34cts. per gallon, 6 gallons of Malaga, at \$1.16cts. per gal., and 7 gallons of white wine, at \$1.25cts per gallon; what will one gallon of the mixture cost? Ans. \$1.24½cts.

4. A refiner had 5lb. of silver bullion of 8oz. fine, 10lb. of 7oz. fine, and 15lb. of 6oz. fine, which he melted together; I demand the fineness of 1lb. of the mass? Ans. 6oz. 13pwts. 8grs. fine.

5. If 4oz. of silver, worth 60cts. per oz. be melted with 8oz. at 48cts per oz., what is one ounce of the mixture worth? Ans. 52cts.

6. A grocer mixed 2cwt. of sugar, at 56s. per cwt., 8cwt. at 43s. per cwt. and 2cwt. at 50s. per cwt. together; what is the value of 3 cwt. of the said mixture? Ans. 6£ 19s.

7. A wine merchant mixed 12 gallons of wine, at 4s. 10d. a gal. with 24 gals. at 5s. 6d., and 16 gals. at 6s. 3¼d., what is a gallon of the said mixture worth? Ans. 5s. 7d.

OF ALLIGATION ALTERNATE.

Alligation Alternate is the method of finding what quantity of each of the ingredients whose rates are given, will be necessary to compose a mixture that may bear a given rate; so, that it is the reverse of *Alligation* Medial, and may be proved by it.

Case 1.

When the respective prices of the several ingredients are given, to find how much of each ingredient will be required to make a composition that will bear a certain price propounded.

Rule.

1. Place the prices of the several ingredients in a perpendicular order, with the mean rate of the intended composition on the left hand.

2. Link or connect each price which is less than the mean rate of the composition, with one or more that is greater.

3. Take the difference between the price of each ingredient and the mean rate, and place them alternately, each against its yoke fellow.

4. If only one difference stand opposite to any given price, it will be the quantity required at that price—but if there be several, their sum will be the quantity required.

5. If all the given prices are greater or less than the mean rate of the composition, they must be linked to a cipher.

6. Different modes of linking will produce different answers; all of which may be proved by Alligation Medial. It makes no difference which of the answers is obtained first.

Examples.

1. How much rye at 4s. per bushel, barley at 3s. per bushel, and oats at 2s. per bushel, will be required to make a composition worth 2s. 6d. per bushel?

$$\text{Mean rate}=30\ d.\begin{cases}48\\36\\24\end{cases}\qquad \begin{matrix}D. & & B.\\ & & 6 \text{ of rye}\\ & & 6 \text{ of barley}\\ & 18+6= & 24 \text{ of oats}\end{matrix}\Bigg\}\ \text{Ans.}$$

Explanation.—The difference between 24 and 30 is 6, which is the quantity of rye and barley, because 48 and 36 are both linked to 24; the difference between 30 and 48 is 18, and the difference between 30 and 36 is 6, the sum of these two differences is 24, which is the quantity of oats required.

Proved by Alligation Medial.

$$\begin{array}{rl}
6\times4= & 24\\
6\times3= & 18\\
24\times2= & 48\\
\hline
36 &)96(2s.\ 6d.\ \text{proof.}\\
 & 72\\
\hline
 & 18\\
 & 12\\
\hline
\end{array}$$

36)216(6d.

Rule.

1. Multiply the number of terms, less 1, by the common difference, and to that product add the first term—the sum will be the last term or greater extreme.

2. Multiply the sum of the two extremes by the number of terms, and half the product will be the sum of all the terms; or, multiply the sum of the two extremes by half the number of terms, and the product will be the sum of all the terms.

Examples.

1. A merchant sold 19 yards of shalloon at 3cts. for the first yd. 5cts. for the second, 7cts. for the third, &c. increasing 2cts. every yard; what did the shalloon amount to?

The number of terms=	19	3cts.=the less extreme.
Subtract	1	39cts.=the greater extreme.
	18	42cts.=sum of the extremes.
The common difference=	2	19=the number of terms.
	36	2)798=the whole product.
The first term	= 3	
		399cts.=$3.99cts. Ans.
The last term	=39	

Exemplification.—If another series of the same kind with the given one be placed under it in a reversed order, then the sum of every two corresponding terms will be equal to the sum of the extremes. Consequently, any one of those terms being multiplied by the number of terms, must give the whole sum of the two series; therefore, the half of the whole sum of any two series will be the sum of all the series.

Let 1, 2, 3, 4, 5, 6, 7, 8, be the given series,
And 8, 7, 6, 5, 4, 3, 2, 1, of the same kind reversed.
9, 9, 9, 9, 9, 9, 9, 9,

$$\left.\begin{array}{ll}\text{Now} & 9\times 8\div 2=36. \\ \text{And} & 1+2+3+4+5+6+7+8=36.\end{array}\right\}\text{Proof.}$$

2. Suppose 100 stones were laid a yard distant from each other, and the first stone a yard from a basket; what distance will a man travel who gathers them up singly, and returning to the basket with one stone at a time? Ans. 10100 yards.

3. Sixteen persons bestowed charity on a poor man; the first gave him 5cts., the second 9cts., and so on in Arithmetical Progression; how much did the last person give, and what sum did the indigent person receive? Ans. The last person gave 65cts., and the poor man received $5.60 cents.

4. How many strokes will the hammer of a regular clock strike in 12 hours? Ans. 78.

5. *In a certain* triangular field, there are 41 rows of corn, the

Rule.

Find the differences between the price of each ingredient and the mean rate of the whole mixture, and place them alternately, as in case the first. Then say—

As the difference opposite the price of the given quantity,
Is to the given quantity;
So is each of the other differences,
To its respective quantity.

Prove the questions by Alligation Medial.

Examples.

1. A grocer would mix 30lbs. of sugar, at 14cts. per pound, with some at 9cts., 10cts., and 13cts. per lb.; how much of each sort must he mix with the 30 pounds, that the mixture may be sold at 12 cents per pound?

Mean Rate.	cts.	
12 cts.	9	1
	10	2
	13	3
	14	2=the

difference opposite to the price of the given quantity; therefore I say:

	dif.		lbs.		dif.		lbs.	cts.	
1. As	2	:	30	::	1	..	15	at 9 per lb.	1. Ans.
2. As	2	:	30	::	2	..	30	at 10 per lb.	
3. As	2	:	30	::	3	..	45	at 14 per lb.	

Proved.

	lbs.		cts.		cts.
Given quantity=	30	×	14	=	420
	15	×	9	=	135
	30	×	10	=	300
	45	×	13	=	585
	120	)			1440(12cts. proof.

Mean Rate.	cts.	
12 cts.	9	2
	10	1
	13	2
	14	3=the

difference opposite to the price of the given quantity; therefore, I say:

	d.		lbs.		d.		lbs.	cts.	
1. As	3	:	30	::	2	..	20	at 9 per lb.	2. Ans.
2. As	3	:	30	::	1	..	10	at 10 per lb.	
3. As	3	:	30	::	2	..	20	at 13 per lb.	

A man would mix 10 bushels of wheat at 75cts. per bushel, with rye at 50cts., barley at 34cts., and oats at 18cts. per bushel; I demand how much rye, barley, and oats must be mixed with the 10 bushels of wheat, that the whole compound may be sold at 39cts. per bushel?

Mean Rate.	cts.	
39	75	5+21=26
	50	21 =21
	34	36 =36
	18	11+36=47

	B.		cts.		cts.
	10	×	75	=	750
	$8\frac{1}{13}$	×	50	=	$403\frac{11}{13}$
	$13\frac{11}{13}$	×	34	=	$470\frac{8}{13}$
	$18\frac{1}{13}$	×	18	=	$325\frac{5}{13}$
	50				)1950 (39cts. proof.
					0000

	D.		B.		D.		B.		
1. As	26	:	10	::	21	..	$8\frac{1}{13}$	of rye	1. Ans.
2. As	26	:	10	::	36	..	$13\frac{11}{13}$	barley	
3. As	26	:	10	::	47	..	$18\frac{1}{13}$	oats	

Mean Rate.

	c's.	
39	18	36
	34	11
	50	5
	75	21

B.		cts.		cts.
10	×	75	=	750
$2\frac{8}{21}$	×	50	=	$119\frac{1}{21}$
$5\frac{5}{21}$	×	34	=	$178\frac{2}{21}$
$17\frac{3}{21}$	×	18	=	$308\frac{12}{21}$
$34\frac{16}{21}$				$1355\frac{15}{21}$
730	)			28470 (39cts. proof.

D. B. D. B.

2. Ans.
1. As 21:10:: 5.. $2\frac{8}{21}$ of rye
2. As 21:10::11.. $5\frac{5}{21}$ barley
3. As 21:10::36..$17\frac{3}{21}$ oats

	cts.		
39	18	11	=11
	34	36+11	=47
	50	21+ 5	=26
	75	5	= 5

D. B. D. B.

3. Ans.
1. As 5:10::26..52 of rye
2. As 5:10::47..94 of barley
3. As 5:10::11..26 of oats

4. Ans. B. 42 rye, 72 barley, 22 oats

5. Ans. B. $12\frac{8}{21}$ rye, $5\frac{5}{21}$ barley, $22\frac{8}{21}$ oats

6. Ans. B. $1\frac{12}{13}$ rye, $18\frac{1}{13}$ barley, $13\frac{11}{13}$ oats

7. Ans. B. 10 rye, $18\frac{1}{13}$ bar., $18\frac{1}{13}$ oats

8. A man would mix 10 bushels of wheat at 4s. per bushel, with rye at 3s., barley at 2s., and oats at 1s. per bushel; how much rye, barley, and oats must be mixed with the 10 bushels of wheat, that the whole compound may be sold at 28d. per bushel?

Ans. 1. B. $2\frac{1}{2}$ rye, 5 barley, $12\frac{1}{2}$ oats

Ans. 2. B. 40 rye, 50 barley, 20 oats

Ans. 3. B. 8 rye, 10 barley, 14 oats

Ans. 4. B. 10 rye, 14 barley, 14 oats

Ans. 5. B. $12\frac{1}{2}$ rye, 5 barley, $17\frac{1}{2}$ oats

Ans. 6. B. 2 rye, 14 barley, 10 oats

Ans. 7. B. 50 rye, 70 barley, 20 oats

4. A man being determined to mix 12 bushels of oats at 1s. 6d. per bushel, with barley at 2s. 6d., rye at 3s., and wheat at 4s. per bushel; I demand how much rye, barley, and wheat must be mixed with the 12 bushels of oats, that the whole compound may be afforded at 2s. 9d. per bushel?

Ans. 1. 60 bushels of barley, 60 of rye, and 12 of wheat.
Ans. 2. 2 bushels $1\frac{3}{5}$ peck of barley, 2b. $1\frac{3}{5}$p. of rye, and 12 of wheat.
Ans. 3. 10 bushels of barley, 10 of rye, and 12 of wheat.
Ans. 4. 72 bushels of barley, 72 of rye, and 12 of wheat.
Ans. 5. 2 bushels of barley, 12 of rye, and 10 of wheat.
Ans. 6. 14 bush. $1\frac{3}{5}$ peck of barley, 2b. $1\frac{3}{5}$p. of rye, 14b. $1\frac{3}{5}$p. of wheat.
Ans. 7. 12 bushels of each sort.

Case 3.

When the prices of all the ingredients, the quantity to be compounded, and the mean rate of the whole composition are given, to find how much of each ingredient will be necessary to make up the quantity required.

Rule.

Find the differences between the price of each ingredient, and the mean rate of the whole composition, and place them alternately, as in case the 1st; then say—

As the sum of all the differences,
Is to the quantity to be compounded;
So is each respective difference,
To the quantity required at that rate.

Examples.

1. Suppose I have four sorts of currants, viz: at 8 cents, 12 cents, 18 cents, and 22 cents per pound; the worst will not sell, and the best are too dear, I therefore conclude to make a composition of 120 pounds, which I propose selling at 16 cents per pound. How much of each sort will be required?

	cts.			D.		lbs.		D.		lbs.		cts.		
Mean rate 16	8	6	1. As	20	:	120	::	6	..	36	at	8	per lb.	1st Ans.
	12	2	2. As	20	:	120	::	2	..	12	at	12	per lb.	
	18	4	3. As	20	:	120	::	4	..	24	at	18	per lb.	
	22	8	4. As	20	:	120	::	8	..	48	at	22	per lb.	

20 = the sum of all the differences.

	cts.			D.		lbs.		D.		lbs.		cts.		
16	8	2	1. As	20	:	120	::	2	..	12	at	8	per lb.	2d Ans.
	12	6	2. As	20	:	120	::	6	..	36	at	12	per lb.	
	18	8	3. As	20	:	120	::	8	..	48	at	18	per lb.	
	22	4	4. As	20	:	120	::	4	..	24	at	22	per lb.	
		20												

	lbs.	cts. per lb.
3d Ans.	$9\frac{3}{13}$	at 8
	$36\frac{12}{13}$	at 12
	$55\frac{5}{13}$	at 18
	$18\frac{6}{13}$	at 22

	lbs.	cts. per lb.
4th Ans.	32	at 8
	8	at 12
	48	at 18
	32	at 22

	lbs.	cts. per lb.
5th Ans.	24	at 8
	32	at 12
	16	at 18
	48	at 22

	lbs.	cts.
6th Ans.	$28\frac{4}{17}$	at 8 per lb.
	$21\frac{3}{17}$	at 12 per lb.
	$28\frac{4}{17}$	at 18 per lb.
	$42\frac{6}{17}$	at 22 per lb.

	lbs.	cts.
7th Ans.	24	at 8 per lb.
	24	at 12 per lb.
	36	at 18 per lb.
	36	at 22 per lb.

2. How many gallons of water must be mixed with wine at 48cts. per gallon, so as to fill a vessel of 80 gallons, that may be sold at 33cts. per gallon? Ans. 25 of water.

3. How much sugar at 10cts., 12cts., and 15cts. per lb., will be required to compose a mixture of 240lbs. that may be sold at 13cts. per lb.? Ans. 60lbs. at 10cts; 60lbs. at 12cts., and 120lbs. at 15cts. per lb.?

4. How much gold of 15, of 17, of 20, and of 22 carats fine, must be melted together, to form a composition of 40oz. of 18 carats fine? Ans. 1. 16oz. of 15 carats, 8oz. of 17 carats, 4oz. of 20 carats, and 12oz. of 22 carats fine.—Ans. 2. 8oz of 15 carats, 16oz. of 17 carats, 12oz. of 20 carats, and 4oz. of 22 carats fine.—Ans. 3. 12oz. of 15 carats, 12oz. of 17 carats, 8oz. of 20 carats, and 8oz. of 22 carats fine.

The student will please to find the four remaining answers, and prove them all by Alligation Medial.

POSITION.

Position is a rule which teaches us how to find an unknown number, by using one or more supposed numbers. It is divided into two parts, namely, Single and Double.

I. OF SINGLE POSITION.

Single Position teaches us how to resolve those questions whose answers are proportional to their suppositions: that is, such as require the multiplication or division of the number sought by any proposed number; or, when it is to be increased or diminished by itself, or any part or parts of itself, a given number of times, &c.

Rule.

Choose any convenient number at pleasure, and work with it agreeably to the tenor of the question; then say,

As the result of the operation,
Is to the supposed number;
So is the given number,
To the number required.

Proof.

Work with the answer according to the tenor of the question, and the result will be equal to the given number.

N. B.—Many questions which are commonly wrought by Position, may be solved very concisely by Vulgar Fractions without a supposition.

Case 3.

When the prices of all the ingredients, the quantity to be compounded, and the mean rate of the whole composition are given, to find how much of each ingredient will be necessary to make up the quantity required.

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	cts.			D.	lbs.	D.	lbs.	cts.	
Mean rate 16	8	6	1.	As 20 :	120 ::	6 ..	36 at	8 per lb.	1st Ans.
	12	2	2.	As 20 :	120 ::	2 ..	12 at	12 per lb.	
	18	4	3.	As 20 :	120 ::	4 ..	24 at	18 per lb.	
	22	8	4.	As 20 :	120 ::	8 ..	48 at	22 per lb.	

20 = the sum of all the differences.

	cts.			D.	lbs.	D.	lbs.	cts.	
16	8	2	1.	As 20 :	120 ::	2 ..	12 at	8 per lb.	2d Ans.
	12	6	2.	As 20 :	120 ::	6 ..	36 at	12 per lb.	
	18	8	3.	As 20 :	120 ::	8 ..	48 at	18 per lb.	
	22	4	4.	As 20 :	120 ::	4 ..	24 at	22 per lb.	
		20							

	lbs.	cts. per lb.		lbs.	cts. per lb.		lbs.	cts. per lb.
3d Ans.	$9\frac{3}{13}$ at	8	4th Ans.	32 at	8	5th Ans.	24 at	8
	$36\frac{12}{13}$ at	12		8 at	12		32 at	12
	$55\frac{5}{13}$ at	18		48 at	18		16 at	18
	$18\frac{6}{13}$ at	22		32 at	22		48 at	22

	lbs.	cts.		lbs.	cts.
6th Ans.	$28\frac{4}{17}$ at	8 per lb.	7th Ans.	24 at	8 per lb.
	$21\frac{3}{17}$ at	12 per lb.		24 at	12 per lb.
	$28\frac{4}{17}$ at	18 per lb.		36 at	18 per lb.
	$42\frac{6}{17}$ at	22 per lb.		36 at	22 per lb.

2. How many gallons of water must be mixed with wine at 48cts. per gallon, so as to fill a vessel of 80 gallons, that may be sold at 33cts. per gallon? Ans. 25 of water.

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Rule.

Choose any convenient number at pleasure, and work with it agreeably to the tenor of the question; then say,

As the result of the operation,
Is to the supposed number;
So is the given number,
To the number required.

Proof.

Work with the answer according to the tenor of the question, and the result will be equal to the given number.

N. B.—Many questions which are commonly wrought by Position, may be solved very concisely by Vulgar Fractions without a *supposition.*

Examples.

1. What number is that of which the $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ will make 104?

Suppose the number to be 60,
Then $\frac{1}{2}$ of $60=30$
And $\frac{1}{3}$ of $60=20$
And $\frac{1}{4}$ of $60=15$

The result $=65$ which is too little; therefore, I say, as 65:60::104

$$\begin{array}{r} 60 \\ \hline 65)6240 \\ \hline \end{array}$$

The true number $=96$ Ans.
Now $\frac{1}{2}$ of $96=48$
And $\frac{1}{3}$ of $96=32$
And $\frac{1}{4}$ of $96=24$

104 proof.

Examine the same question worked by Vulgar Fractions.

$\frac{1}{2}+\frac{1}{3}+\frac{1}{4}=\frac{26}{24}$ or, $\frac{13}{12}$ numerator. denom.

$$\begin{array}{cccc} N. & D. & & given \\ \text{As } 13 : & 12 & :: & 104 \text{ number.} \\ & & & 12 \\ \hline & & & 13)1248 \\ \hline \end{array}$$

Ans. 96 as before.

2. A person after expending $\frac{1}{3}$ and $\frac{1}{4}$ of his money, had \$60 left; how many had he at first? Now $\frac{1}{3}+\frac{1}{4}=\frac{7}{12}$ expended, and $\frac{12}{12}$, or $1-\frac{7}{12}=\frac{5}{12}$ left; therefore As 5 : 12 :: 60 .. \$144 Ans.

3. A, B, and C bought a quantity of goods amounting to \$612, of which sum A paid three times more than B and B four times more than C; how much did each man pay?

A paid \$432, B \$144, and C \$36.

4. A man overtaking a maid driving a flock of geese, said to her, how do you do, sweetheart? where are you going with your 40 geese? Indeed, sir, said she, I have not 40; but if I had as many more, half as many more, and 10 geese besides, I should have 40. How many geese had she? Ans. 12 geese.

5. Suppose I have a cistern full of water, with three unequal pipes: now the greatest pipe will empty the cistern in one hour, the second in two, and the third in three. In what time will the cistern be emptied, if all three of the pipes are left open at once?

Ans. 32 minutes $43\frac{7}{11}$ seconds.

6. What is the age of a person who says, that, if $\frac{3}{5}$ of the years I have lived be multiplied by 7, and $\frac{2}{3}$ of them be added to the product, the sum will be 292? Ans. 60 years.

7. A schoolmaster being asked how many scholars he had, answered, if to double the number I now have you add $\frac{3}{4}$, $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{8}$ of them, I shall have 435; how many scholars had he? Ans. 120.

8. What sum will amount to \$860 in 12 years, at \$6 per cent. per annum? Ans. \$500.

9. A certain sum of money is to be divided among A, B, and E, in such a manner that A may have $\frac{1}{4}$, B $\frac{1}{5}$, C $\frac{1}{10}$, D E the remainder, which is \$40; what is the sum and ea *share of it?* Ans. The sum is \$100, of which A gets \$ *C 10, D 5,* and E the rest, which is \$40.

10. A man after expending $\frac{1}{2}$ and $\frac{1}{8}$ of his yearly income, had \$26$\frac{2}{3}$ left; what did his yearly salary amount to? Ans. \$160.

11. When A, B, and C were talking of their ages, B said his age was equal to 1$\frac{1}{2}$ times the age of A, to which C replied, I am twice and $\frac{1}{10}$ the age of you both, and the sum of our ages is equal to 93; what was the age of each person?

Ans. A was 12, B 18, and C 63 years of age.

12. A man having found a bag of dollars, said that the $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{1}{6}$ of them made up the sum of 57; please to tell me how many dollars were in the bag? Ans. \$60.

13. The yearly interest of Miss Charlotte's money, at \$6 per cent., exceeds one-twentieth of the principal by \$100, and she does not intend to marry any man who is not scholar enough to tell the amount of her fortune. Pray, sir, can you obtain her consent?

Yes, sir, she is worth exactly \$10000.

II. OF DOUBLE POSITION.

Double Position teaches us how to resolve questions by making use of two supposed numbers. Those questions in which the results are not proportional to their suppositions, belong to this rule: that is, such as those in which the number sought is increased or diminished by some given number which is not any known part of the number required.

Rule.

1. Suppose any two convenient numbers, and work with each of them agreeably to the tenor of the question, and if the error be too great in either of the operations mark it with +, but if it be too small mark it with —.

2. Multiply the first position by the second error, and the second position by the first error.

3. If the errors be alike, that is, both greater or both less than the given number, take their difference for a divisor and the difference of the products for a dividend.

4. If the errors be unlike, that is, one too much and the other too little, then take their sum for a divisor and the sum of the products for a dividend; the quotient in either case will be the number required.

Proof.

Work with the answer according to the conditions of the question, and the result will be equal to the given number.

Examples.

1. A man was hired 50 days on these conditions, that is, for every day he worked he should receive 75 cents, and for every day he was idle he should forfeit 25 cents; at the expiration of the time,

he received $27.50cts. How many days did he work, and how many was he idle?

1st. Suppose he worked 30 days, then he was idle 20, of course:

	D.	cts.	$ cts.	
Now	30×75	=	22.50	he earned.
And	20×25	=	5.00	forfeited.
The sum rec'd			17.50	
The given sum			27.50	
The first error			10.00—	

2d. Suppose he worked 36 days, then he was idle 14, of course:

	D.	cts.	$ cts.	
Now	36×75	=	27.00	he earned.
And	14×25	=	3.50	forfeited.
The sum rec'd			23.50	
The given sum			27.50	
The 2d error			4.00—	

The first position $=30\times 4.00=120.00$
The second position $=36\times 10.00=360.00$

6.00) 240.00

He worked 40 days. }
And he was idle 10 days. } Ans.

2. The head of a certain fish is 9 inches long, and its tail is as long as its head and half of its body, and the length of its body is equal to the length of its head and tail; what is the whole length of the fish?

		inches.
1. Suppose the body to be		20
Then one half of the body	=	10
The length of the head	=	9
The length of the tail	=	19
The length of the head	=	9
The head and tail added	=	28
The length of the body	=	20
The first error	=	8

		inches.
2. Suppose the body to be		30
Then one half of the body	=	15
The length of the head	=	9
The length of the tail	=	24
The length of the head	=	9
The head and tail added	=	33
The length of the body	=	30
The second error	=	3

The first position $=20\times 3=60$
The second position $=30\times 8=240$

5) 180

The true length of the body = 36 inches.

Half the body = 18 inches.
The head = 9 inches.

The true length of the tail = 27 inches;
therefore, $36+27+9=72$ inches. Answer.

3. Two men, namely, A and B, have both the same income; A

saves one fifth of his every year, but B, by spending $150 per annum more than A, at the end of 8 years finds himself $400 in debt. What is their yearly income, and what does each one expend per annum? Ans. Their true income is $500 per annum; A expends $400 and B $550.

4. A sheep fold was robbed three nights in succession; the first night half the sheep were stolen and half a single sheep more; the second night half the remainder were taken and half a single sheep more; the third night they took half of what yet remained and half a single sheep more: all of which was done without killing any sheep, and there remained at least 20 sheep in the flock. How many were there at first? Ans. 167 sheep.

5. By his will a father has directed that his eldest son is to have in the first instance $1000 out of the whole estate, and then to receive $\frac{1}{6}$ part of the remainder; the second son is to have first $2000 out of the residue left by his elder brother, and then $\frac{1}{6}$ part of the remainder; the third son is to have $3000 out of the balance left by his two elder brothers, and then $\frac{1}{6}$ part of the remainder; and so on to the last son, whose portion will be what his elder brothers have left, and all their portions must be equal. Required: 1st. The father's estate? 2d. The number of his sons? 3d. The portion of each son? Ans. The father's estate was $25000, he had 5 sons, and the portion of each son is $5000.

6. A man having sold his cattle, received for them all the sum of $900, being paid for every steer $20, for every cow $12, and for every calf $2; there were as many steers as cows, and for every cow two calves; how many were there of each sort?

Ans. 25 steers, 25 cows, and 50 calves.

7. One being asked what time of the day it was, answered, the day at this time is 14 hours long; therefore, if you add $\frac{1}{2}$ of the time past since sun rising to $\frac{1}{4}$ of the remaining time till sun setting the sum will be the time required; what o'clock was it when the question was propounded? Ans. 8 hours 24 minutes from sun rising, consequently the remaining time is 5 hours 36 minutes.

8. A son asked his father how old he was; his father replied, your age is now $\frac{1}{5}$ of mine, but 4 years ago your age was only $\frac{1}{7}$ of what mine is now; what were their ages? Ans. The father's age was 70, the son's age was 14.

9. A gentleman has two horses of considerable value and a carriage worth 100£; now, if the first horse be harnessed to it, he and the carriage together will be three times the value of the second, but if the second be harnessed to it they will be 7 times the value of the first; what is the value of each horse? The value of the first horse is £20, the value of the second £40.

10. A gentleman went into a school where the scholars were very studious; he, therefore, gave each of them 10 cents, and had 20

cents left; but, if he had given them $12\frac{1}{2}$ cents apiece, he would have wanted 25 cents. How many scholars were in the said school? Ans. 18 scholars.

11. A, B, and C would divide $80 among them in such a manner that B may have $5 more than A and C $10 more than B; what is each man's share? Ans. A must have $20, B $25, and C $35.

12. D, E, and F would divide $100 among them so that E may have $3 more than D and F $4 more than E; what is the share of each man? Ans. D's share is $30, E's $33, and F's $37.

ARITHMETICAL PROGRESSION.*

Any rank or series of numbers more than two, increasing or decreasing by a common difference, is said to be in Arithmetical Progression.

When the succeeding terms of a progression increase by a common difference, they form an ascending series, but when they decrease by a common difference, they form a descending series.

Thus { 2 4 6 8 10 12 &c. is an ascending series.
{ 12 10 8 6 4 2 &c. is a descending series.

The numbers which form the series are called the terms of the progression. The first and last terms are extremes, and the other terms are called the means.

There are five terms in Arithmetical Progression, any three of which being given, the other two may be easily found.

1. The first term
2. The last term } Extremes.
3. The number of terms
4. The common difference
5. The sum of all the terms } Means.

N. B.—In any series of numbers in Arithmetical Progression, the sum of the two extremes is equal to the sum of any two means, which are equally distant from the said extremes, as in the above series, where, 12+2=14, 10+4=14, and 8+6=14. When the number of terms is odd, the middle one supplies the place of two terms, as in the following series, 7, 6, 5, 4, 3, 2, 1, in which, 7+1=8, 6+2=8, 5+3=8, and 4+4=8. See the exemplification following the first example.

Case 1.

When the first term, the common difference, and number of terms are given, to find the last term and sum of all the terms.

* Arithmetical Progression will admit of 20 problems. See Mr. Nicholas Pike's large Arithmetic, second edition, 1797.

RULE.

1. Multiply the number of terms, less 1, by the common difference, and to that product add the first term—the sum will be the last term or greater extreme.

2. Multiply the sum of the two extremes by the number of terms, and half the product will be the sum of all the terms; or, multiply the sum of the two extremes by half the number of terms, and the product will be the sum of all the terms.

EXAMPLES.

1. A merchant sold 19 yards of shalloon at 3cts. for the first yd. 5cts. for the second, 7cts. for the third, &c. increasing 2cts. every yard; what did the shalloon amount to?

The number of terms=	19	3cts.=the less extreme.
Subtract	1	39cts.=the greater extreme.
	18	42cts.=sum of the extremes.
The common difference=	2	19=the number of terms.
	36	2)798=the whole product.
The first term	= 3	
		399cts.=$3.99cts. Ans.
The last term	=39	

EXEMPLIFICATION.—If another series of the same kind with the given one be placed under it in a reversed order, then the sum of every two corresponding terms will be equal to the sum of the extremes. Consequently, any one of those terms being multiplied by the number of terms, must give the whole sum of the two series; therefore, the half of the whole sum of any two series will be the sum of all the series.

Let 1, 2, 3, 4, 5, 6, 7, 8, be the given series,
And 8, 7, 6, 5, 4, 3, 2, 1, of the same kind reversed.
9, 9, 9, 9, 9, 9, 9, 9,

$$\left.\begin{array}{ll}\text{Now} & 9\times8\div2=36.\\ \text{And} & 1+2+3+4+5+6+7+8=36.\end{array}\right\}\text{Proof.}$$

2. Suppose 100 stones were laid a yard distant from each other, and the first stone a yard from a basket; what distance will a man travel who gathers them up singly, and returning to the basket with one stone at a time? Ans. 10100 yards.

3. Sixteen persons bestowed charity on a poor man; the first gave him 5cts., the second 9cts., and so on in Arithmetical Progression; how much did the last person give, and what sum did the indigent person receive? Ans. The last person gave 65cts., and the poor man received $5.60 cents.

4. How many strokes will the hammer of a regular clock strike in 12 hours? Ans. 78.

5. In a certain triangular field, there are 41 rows of corn, the

first row being in one corner, contains but a single hill, and the last row on the opposite side contains 81 hills; how many hills of corn are in the said field? Ans. 1681 hills.

Case 2.

When the two extremes and the number of terms are given, to find the common difference of all the terms.

Rule.

Divide the difference of the extremes by the number of terms less 1, and the quotient will be the common difference of all the terms.

Examples.

1. A certain debt is to be discharged in a year by weekly payments; the first payment to be $8\frac{1}{3}$ cents and the last \$8.58$\frac{1}{3}$ cents; what is the common difference of each payment, and whole debt?

w. | \$ cts.
52 | 8.58$\frac{1}{3}$
1 | .08$\frac{1}{3}$
—— | ——
51) 8.50 .(16$\frac{2}{3}$cts.=the common difference of each payment
5 1
——
3 40
3 06
——
$17)\frac{34}{51}=\frac{2}{3}$ of a cent.

\$. cts.
8.58$\frac{1}{3}$
.08$\frac{1}{3}$
8.66$\frac{2}{3}$=2600 thirds.
52
5200
13000
2)135200
3)67600 thirds.
The debt=\$225.33$\frac{1}{3}$cts. Ans.

2. A man is to travel from the city of Washington to a certain place in 19 days, and go but 6 miles the first day, increasing every day by an equal excess, so that the last day's journey may be 60 miles; what is the common difference and distance of the journey?

Ans. Common difference=3 miles; distance, 627 miles.

3. A man had 10 sons whose several ages differed alike; the youngest was 3 years old, and the eldest 48; what was the common difference of their ages? Ans. 5 years.

4. There are 21 persons whose ages are equally distant from each other; the youngest is 20 years old, and the eldest 60; what is the common difference of their ages, and the age of each person?

Ans. The common difference is 2 years.

years.
20=the age of the first person.
20+2=22= of the 2d person.
22+2=24= of the 3d person.
24+2=26= of the 4th person. &c.

5. A certain debt is to be discharged at 11 separate payments, the first payment is to be \$5, and the last \$75; what is the common difference of each payment? Ans. \$7.

GEOMETRICAL PROGRESSION.*

Any rank or series of numbers continually increasing by a common multiplier, or decreasing by a common divisor, is said to be in Geometrical Progression.

Thus $\begin{cases} 1, 2, 4, 8, 16, \&c. \text{ is an increasing geometrical series.} \\ 16, 8, 4, 2, 1, \&c. \text{ is a decreasing geometrical series.} \end{cases}$

There are five terms in Geometrical Progression, any three of which being given, the other two may be easily found.

1. The first term. } Extremes.
2. The last term. }
3. The number of terms. }
4. The ratio. } Means.
5. The sum of all the terms. }

The common multiplier or divisor, by which the series is increased or decreased, is called the ratio.

N. B.—In any rank or series of numbers, which increase or decrease by a common ratio, the product of the two extremes is equal to the product of any two means, equally distant from the said extremes, as in the series, 2, 4, 8, 16; the product of 16×2 is equal to the product of 8×4, each product being 32. When the number of terms is odd, the middle one supplies the place of two terms, as in the series 27, 9, 3, the product of 27×3 is equal to the product of 9×9, each product being 81. Also, the product of any two means divided by either of the extremes will produce the other extreme, &c. &c.

When the first term, the ratio, and number of terms are given, to find the last term, and the sum of all the series.

Rule.

1. Raise the ratio to the power whose index is one less than the number of terms given in the question; which, being multiplied by the first term of the series, will give the last term or greater extreme.

2. Multiply the last term by the ratio, and from the product subtract the first term of the series; then divide the remainder by the ratio, less one, and the quotient will be the sum of all the series.

Examples.

1. If the first term of a geometrical series be 5, the ratio 3, and the number of terms 7, what is the last term, and the sum of the series.

* Geometrical Progression will admit of 18 problems. See Mr. Nicholas Pike's large Arithmetic, second edition, 1797.

=1st power. =2d power. =3d power. =4th power. =5th power.

The ratio=3 . 9 27 81 . 243
3=the ratio.

729=the 6th power of the ratio.
Multiply by 5=the first term of the series.

Answer 3645=the last term of the series.
Multiply by 3=the ratio.

10935
Subtract 5=The 1st term of the series.

The ratio=3−1=2)10930

Answer 5465=the sum of the series.

2. If the first term of a geometrical series be 4 and the ratio 4, what is the 9th term only?

The ratio=4, and the 8th power of 4=65536
Multiply by the first term 4
The 9th term of the series=262144

3. A gentleman by his will left his estate to his five sons, in the following manner, that is, to his youngest son \$1000, to his second \$1500, and ordered that each son should exceed the next younger by the equal ratio of $1\frac{1}{2}$; what was the whole amount of the gentleman's estate?

The ratio= $\overline{1.5|}^{4}$ =5.0625
Multiply by 1st term= 1000

The last term= 5062.5000
Multiply by the ratio= 1.5

253125000
50625000

7593.75000
Subtract 1000=1st ter.

1.5−1.=.5)6593.75000

Answer \$13187.50,00

4. A man bought a horse, and by agreement was to give a farthing for the first nail, two for the second, four for the third, &c.; there were 4 shoes and 8 nails in each shoe; I demand what the horse is worth at that rate?

Ans. 44739242£ 5s. 3¾d.

5. A thresher wrought 20 days and received for the first day's labor 4 grains of wheat; for the second 12; for the third 36, &c. How much did his wages amount to, allowing 7680 grains to make a pint, and the whole to be disposed of at \$1 per bushel?

Ans. \$14187.75cts.

6. A gentleman, whose daughter was married on a new year's day, gave her a guinea, promising to triple it on the first day of each month in the year; what did her portion amount to?

Ans. 265720 guineas.

7. An ignorant fop wanting to purchase an elegant house, a facetious gentleman told him he had one which he would sell upon these moderate terms, namely: that he should give him one cent for the first door, two for the second, four for the third, and so on, doubling at every door, which were 36 in all. It is a bargain, cried the simpleton, and here is a dollar to bind it. How much did the house cost him? Ans. \$687194767.35cts.

8. A crafty servant agreed with a farmer (ignorant in numbers) to serve him 12 years, and to have nothing for his service but the produce of a wheat corn for the first year, and that produce to be sowed for the second year, and so on from year to year till the end of the said time. I demand the worth of the whole produce, allowing the increase to be but in a tenfold proportion, and sold out at 50 cents per bushel? Ans. \$113028.

9. A man threshed wheat 9 days for a farmer, and agreed to receive but 8 wheat corns for the first day's work, 64 for the second, and so on, in an eightfold proportion. What did his 9 days' labor amount to, rating the wheat at 83⅓cts. per bushel?

Ans. \$260, rejecting remainders.

10. A merchant sold 30 yards of fine silk velvet, trimmed with gold very curiously, at 2 pins for the first yard, 6 pins for the second, 18 pins for the third, &c., in triple proportion. I demand how much the velvet produced when the pins were afterwards sold at 100 for a farthing; also, whether the merchant gained or lost by the transaction, and how much, supposing the velvet to have cost him 100£ by the yard? Ans. The velvet produced 2144699292£ 13s. 0½d., and the merchant gained 2144696292£ 13s. 0½d.

PERMUTATION.

The permutation of quantities is the showing how many different ways any given number of things may be changed, so as to assume different positions: thus—abc, acb, bac, bca, cab, cba, are six different positions of three letters.

Case 1.

To find the number of permutations or changes that may be made of any number of things, all different from each other.

Rule.

Multiply all the terms of the natural series of numbers, from 1 up to the given number, continually together, and the last product will be the number of permutations required.

Examples.

1. How many changes of position can a company of 6 men assume?

$1\times2\times3\times4\times5\times6=720$ Ans.

2. How many different numbers can be made with 123456789?

1×2×3×4×5×6×7×8×9=362880. Ans.

3. How many changes may be rung upon 12 bells, and how long will they be ringing but once over, supposing 24 changes to be rung in one minute, and the year to contain 365 days and 6 hours?

```
1×2×3×4×5×6×7×8×9×10×11=39916800
                              12
                        ---------
                     24)479001600=changes.
                        ---------
                      60)19958400=minutes.
                        ---------
365 days 6 hours=8766 hours)332640hrs.(37 years.
                        ---------
                       24)8298 hours rem.
                          ----
                         7)345 days 18 hours.
                          ----
                            49 weeks 2 days;
```

consequently, the number of changes is 479001600, and the time is 37 years, 49 weeks, 2 days, and 18 hours.

4. Seven gentlemen that were travelling met together by chance at a certain inn upon the road, where they were so well pleased with their host and each other's company, that, in a frolic, they offered him $100 to let them stay at that place so long as they, together with him, could sit every day at dinner in a different order. The host, thinking that they could not sit in many different positions, because there were but few of them, and that himself could make no considerable alteration, he being but one, imagined that he should make a good bargain, and readily (for the sake of a good dinner and better company) entered into an agreement with them, and so made himself the eighth person. I demand how many different positions they sat in, and how long they staid at the said inn, allowing the year to be 365 days 6 hours? Ans. They sat in 40320 positions, and staid at the said inn 110 years, 142 days, and 12 hours.

Case 2.

Any number of different things being given, to find how many changes can be made out of them, by taking any given number of quantities at a time.

Rule.

Take a series of numbers, beginning at the number of things given, and decreasing by 1, to the number of quantities to be taken at a time, and the product of all the terms, taken as above directed, will be the answer required.

EXAMPLES.

1. How many changes may be rung with 4 bells out of 8?

$$\begin{array}{r} 8 \text{ bells.} \\ 7 \\ \hline 56 \\ 6 \\ \hline 336 \\ 5 \\ \hline 1680 \end{array}$$

Ans. 1680 changes.

2. How many words can be made with 6 letters out of 24, admitting a number of consonants may make a word?

$$24\times23\times22\times21\times20=5100480$$
$$\begin{array}{r} 19 \\ \hline \text{Ans. } 96909120 \end{array}$$

CASE 3.

Any number of things being given, whereof there are several things of one sort, several of another sort, &c., to find how many changes may be made out of them all.

RULE.

1. Take the series 1, 2, 3, 4, &c. up to the whole number of things given, and find the product of all the terms.
2. Take the series 1, 2, 3, 4, &c. up to the number of things given of the first sort, and do the same by the second, third, &c. sorts.
3. Divide the product of all the terms by the joint product of the different terms, and the quotient will be the answer.

EXAMPLES.

1. How many changes or variations can be made out of the letters in the word Zaphnathpaaneah?

There are fifteen letters in the given word; therefore, the number of things given is 15—consequently

$1\times2\times3\times4\times5\times6\times7\times8\times9\times10\times11\times12\times13\times14\times15=$ $1307674368000=$the product of the whole number of terms—and

$$\begin{array}{rl} 1\times2\times3\times4\times5 & (=\text{the number of ays})=120 \\ 1\times2 & (=\text{the number of pees})=2 \\ 1\times2\times3 & (=\text{the number of aitches})=6 \\ 1\times2 & (=\text{the number of ens})=2 \end{array}$$

Now, $120\times2\times6\times2=2880=$the divisor.

2880)1307674368000(454053600 Ans.

. . . .

2. How many different numbers can be made of the following figures 1223334444? Ans. 12600.

COMBINATION.

The combination of quantities is the showing how often a less number of things may be taken out of a greater, and combined together, without considering the order they stand in: thus—out of the letters abc there are three combinations of two, that is, ab, ac, and bc.

Rule.

1. Take the series 1, 2, 3, 4, &c. up to the number to be taken at a time, and find the product of all the terms.

2. Take a series of as many terms, decreasing by 1, from the given number out of which the combination is to be made and find the product of all the terms.

3. Divide the last product by the first, and the quotient will be the answer required.

Examples.

1. How many combinations can be made of 7 letters out of 12?

1	12
2	11
2	132
3	10
6	1320
4	9
24	11880
5	8
120	95040
6	7
720	665280
7	6

5040)3991680(792 Ans.

2. How many combinations can be made of 6 letters out of the 24 letters of the alphabet?

Ans. 134596.

3. A butcher bargained with a farmer (well skilled in numbers) for a dozen sheep, (at 2dolls. per head) which were to be picked out of 2 dozen; but, being long in choosing them, the farmer told him that if he would give him a a cent for every different dozen which might be chosen out of the 2 dozen, he should have the whole, to which the butcher readily agreed; what did they cost him? Ans. $27041.56cts.

4. A general was asked by his king what reward he should confer on him for his services; the general required a penny for every file of 10 men in a file, which he could make out of a company of 90 men; what sum did it amount to? Ans. 23836022841£ 5s. 3d.

OF MENSURATION.

Mensuration teaches us how to find the area or superficial content of any plain surface or superficies. Also, the solidity or cubical content of any solid body.

N. B.—1. A superficies or surface is an extension of two dimensions, namely, length and breadth, but is not considered as having thickness.

2. A solid body of any thing has three dimensions, viz., length, breadth, and thickness.

TABLES OF DIFFERENT MEASURES.

I. Of Lineal Measures.

12 inches	make	1 foot.
3 feet	make	1 yard.
6 feet	make	1 fathom.
$16\frac{1}{2}$ feet or $5\frac{1}{2}$ yards	make	1 rod, pole or perch.
40 poles	make	1 furlong.
8 furlongs	make	1 mile.

II. Of Square Measures.

144 inches	make	1 foot.
9 feet	make	1 yard.
36 feet	make	1 fathom.
$272\frac{1}{4}$ feet or $31\frac{1}{4}$ yards	make	1 pole or rod.
1600 poles	make	1 furlong.
64 furlongs	make	1 mile.

N. B.—The chain made use of in measuring land, commonly called Gunter's chain, is 4 poles, or 22 yards in length, and contains 100 equal links, each link being $\frac{22}{100}$ of a yard, equal to .66 decimals of a foot, or 7 inches and .92 decimals of an inch long. An acre of land is equal to 10 square chains; that is, 10 chains in length and 1 in breadth; 4840 square yards, 160 square poles, or 100,000 square links (each being the same in quantity) make 1 acre.

III. Of Cubic Measures.

1728 cubic inches	make	1 cubic foot.
27 cubic feet	make	1 cubic yard.
$166\frac{3}{8}$ cubic yards	make	1 cubic rod.
128 cubic feet, or 8 ft. in length, 4 in breadth, and 4 in height	make	1 cord of wood.
$24\frac{3}{4}$ cubic feet, or $16\frac{1}{2}$ ft. long, $1\frac{1}{2}$ thick and 1 in height	make	1 perch of stone.
231 cubic inches	make	1 gallon wine measure.
282 cubic inches	make	1 gallon ale measure.
$268\frac{4}{5}$ cubic inches	make	1 gallon dry measure.

MENSURATION OF SUPERFICIES.

The area of any plain surface is estimated by the number of squares contained within the bounds thereof, without any regard to thickness. The side of a square may be an inch, a foot, a yard, a link, a chain, or a pole, whichever may suit best in the opinion of the calculator; and hence the area is said to be so many square inches, square feet, square yards, square links, &c. according to the dimension of the measuring unit.

Problem 1.

To find the area of a square.

Rule.

Multiply the side of the given square into itself, and the product will be the area or superficial content, and of like name with the denomination of the measuring unit, let it be inches, feet, or yards respectively.

Examples.

1. Required the area of a square whose side is 15 feet 9 inches.

ft.	*in.*		
15	9		
15	9		
225	0		
11	3		
11	3	″	
00	6	9	
Ans. 248	0	9	

by duodecimals, cross multiplication.

in.	*ft.*	*in.*	
6=½	15	9	
		3	
	47	3	
		5	
	236	3	″
3=½ of 6	7	10	6
	3	11	3
Ans.	248	0	9

by practice.

By whole numbers.

ft. in. inches.
15 9=189
15 9=189

12)35721=product in seconds.

12)2976 9″

Ans. 248 0 9″

2. Required the area or superficial content of a square floor, each side of which is 16 feet.

16 feet long.
16 feet wide.
96
16

Ans. 256 square feet, because the measuring unit is one foot.

3. What is the area of a square field whose side is 37 chains and 25 links? Ans. 138a. 3ro. 1po.

4. What is the area of a square field whose side is 250 yards? Ans. 12a. 3r. 26p.+560.

5. What is the area of a square floor that measures 24 feet 6 in. 9 sec. every way?

Ans. 603 sq. feet, 3in. 9sec. 6 thirds and 9 fourths.—The student is requested to prove the answer to be right by Practice and Duodecimals.

Problem 2.

To find the area or superficial content of a parallelogram or long square.

Rule.

Multiply the length by the breadth, and the product will be the area or superficial content required.

Examples.

1. What is the superficial content of a plank that is 14 feet 6 inches long, and 4 feet 9 inches wide? Ans. 68ft. 10in. 6 sec.

2. What is the area of a field, in acres, whose length is 14 chains 50 links, and breadth 9 chains 75 links? Ans. 14a. 0roo. 22per.

Problem 3.

When the breadth of a plank is given, to find how much in length *will make a square foot.*

or yard, &c. square; but enriched mouldings, and some other articles, are often estimated by lineal or running measure, and some things are rated by the piece.

1. Of Flooring.

Joists are measured by multiplying their breadth by their depth, and that product by their length. They receive various names according to the position in which they are laid to form a floor, such as trimming joists, common joists, girders, binding joists, bridging joists, and ceiling joists. See Mr. Hawney's Mensuration, page 221.

Examples.

1. If a floor be 57 feet 3 inches long and 28 feet 6 inches wide, how many squares of flooring will it contain?

ft. in. in.	*ft. in. in.*	*ft. in.*
57 3=687	57 3=57.25	6in.=½)57 3
28 6=342	28 6= 28.5	28ft. 6in.
12)234954	1,00)16,31.625	1603 0
12)19579 6″	Ans. 16sq. 31ft. 625	28 7 6″
1,00)16,31 7in. 6″	by decimals.	1,00)16,31 7 6″
Ans. 16sq. 31ft. 7in. 6″ by whole numbers.		Ans. 16sq. 31ft. 7in. 6″ by Practice.

2. If a floor be 53ft. 6in. long and 47ft. 9in. broad, how many squares of flooring are contained therein? Ans. 25sqs. 54ft.

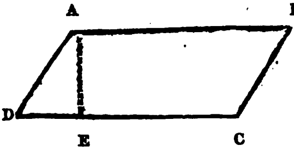
3. In a naked floor the girder is 14 inches deep, 1 foot wide, and 20 feet long; there are 8 bridging joists 6½ inches deep, 3 inches wide, and 20 feet long; 8 binding joists 8½ inches deep, 4 inches wide, and 9 feet long; 24 ceiling joists 4 inches deep, 2½ inches wide, and 6 feet long. Required the solidity of the whole?

	ft.	*in.*
The girder contains - - -	23	4 solid
8 bridging joists contains - -	21	8 ——
8 binding —— —— - -	17	0 ——
24 ceiling —— —— - -	10	0 ——
Ans.	72	0 in all.

4. Suppose a house of three stories, besides the ground floor, is to be floored at 6£ 10s. per square, (the workmen to find materials,) the house measures 20 feet 8 inches by 16 feet 9 inches; there are 7 fire places—2 of them measure, each, 6 feet by 4 feet 6 inches—2 others, each, 6 feet by 5 feet 4 inches—2 more, each, 5 feet 8 inches by 4 feet 8 inches, and the 7th 5 feet 2 inches by 4 feet; the well-hole for the stairs is 10 feet 6 inches by 8 feet 9 inches. What will the whole amount to?

AN EXAMPLE.

Suppose ABCD to represent a rhomboid, the two longest sides, namely, AB and CD, each being 10 chains 50 links, and the perpendicular height AE 7 chains 60 links; required, the area in acres, &c.



ch. li. *ch. li.* *acres.* *a.* *roo.* *po.* *tenths.*

$10.50 \times 7.60 = \frac{79.8000}{10} = 7.98000 = 7 \quad 3 \quad 36 \quad 8$ Ans.

PROBLEM 6.

To find the area of a triangle.

RULE.

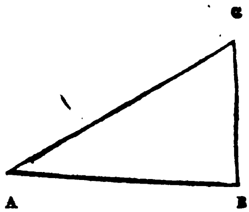
1. If it be a right angled triangle, multiply the base by half the perpendicular, or half the base by the whole perpendicular, and the product in either case will be the area or superficial content required.

2. If it be an oblique angled triangle, (whether obtuse or acute,) multiply the base by half the perpendicular, let fall from the opposite angle, or half the base by the whole perpendicular, and the product in either case will be the area required; or, multiply the whole base by the whole perpendicular, and half the product will be the area.

NOTE.—An obtuse angle contains more than 90 degrees, and an acute angle less.

EXAMPLES.

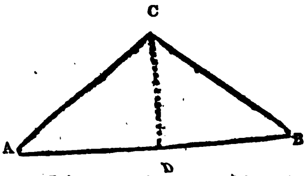
1. Suppose the base AB of the right angled triangle ABC to measure 10 feet 9 inches, and the perpendicular BC 7 feet 3 inches, what is the area?



in.	*ft.*	*in.*	
3=¼)	10	9=base AB.	
	7	3in.=per. BC.	
	75	3	
	2	8	3″
2)	77	11	3
	38	11	7 6‴ Ans.

Prove theanswer to be right by decimals and duodecimals.

2. The oblique angled triangle ABC being given, draw a perpendicular from the obtuse angle at C to the base AB, and that perpendicular is the height of the triangle. The base AB is supposed to be 84 poles *and the perpendicular* CD 56; what *is the area in* acres, &c.?



Ans. 14 acres 2 roods 32 poles.

Problem 7.

To find the area of a triangle when the three sides are given.

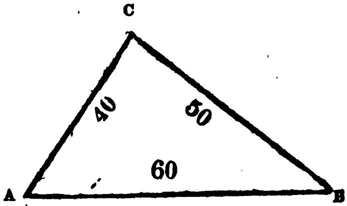
Rule.

From half the sum of the three sides subtract each side severally, then multiply the half sum and the three remainders continually together, and the square root of the product will be the area required.

An Example.

In the triangle ABC let the side AC be 40 poles, CB 50, and AB 60; required, the area in acres, &c.?

Ans. 6a. 0ro. 32po.



75—40=35
75—50=25
75—60=15
2)150
Half 75=sum.
35=1 dif.
2625
25=2 dif.
65625
15=3 dif.
984375=product of the half sum and 3 dif.

po.
984375(992
81
189)1743
1701
1982)4275
3964
311 rem.
4,0)99,2 poles.
4)24 32po.
6a. 0r 32po.

Problem 8.

To find the area of a trapezium.

Note.—A trapezium is an irregular figure of four unequal sides and unequal angles.

Rule.

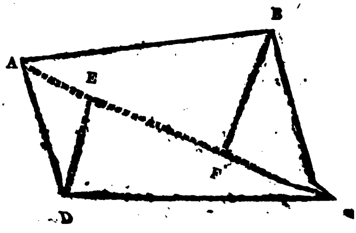
Draw a diagonal line from one of the angles to the opposite angle, as AC, then the trapazium will be divided into two triangles, of which the diagonal AC is the common base; then draw perpendicular lines to the diagonal from the opposite angles and add them together, and multiply half that sum into the diagonal, or half the diagonal into the whole sum of the perpendiculars, and the product either way will be the area of the trapezium.

An Example.

In the trapezium ABCD, the diagonal AC is 48 poles, the perpendicular DE 16, and the perpendicular BF 28, the sum of the perpendiculars is 44, whose half is 22, which, being multiplied into 48, will give the area required.

Ans. 6a. 2ro. 16po.

Examine the work carefully every way.



PROBLEM 9.

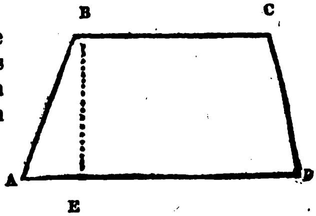
To find the area of a trapezoid.

NOTE.—A trapezoid is the segment of a triangle, cut off by a line parallel to the base.

RULE.

Add the parallel sides together, and multiply half that sum by the perpendicular breadth, and the product will be the area required.

AN EXAMPLE.

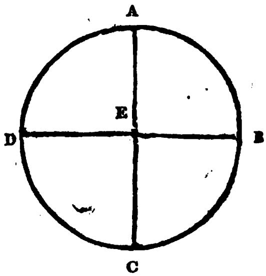


In the trapezoid ABCD, the side AD is 24 feet, and the side BC is 16ft., and the perpendicular breadth BE is 12 feet; what is the area in square feet?

$24+16\div 2\times 12=240$ square feet, the area required.

PROBLEM 10.

To measure a circle and its parts.



In the annexed circle ABCD, the arch line ABCD is called the periphery or circumference; any line, as DB or AC, passing the center E, cuts the circle into two equal parts, called semicircles, and such lines are called diameters of the circle. If two diameters be drawn through a circle at right angles to each other, they divide the circle into four equal parts, called quadrants; half the diameter of any circle is called the radius or semidiameter, and is the distance taken in the compasses to describe the circle.

PROBLEM 11.

The diameter of a circle being given, to find the circumference.

RULE.

This may be done by several proportions.

1. As 7 is to 22, so is the diameter to the circumference of the circle.

Or, more exactly—

2. As 113 is to 355, so is the diameter to the circumference required.
3. As 1 is to 3.1416, so is the diameter to the circumference required.

EXAMPLES.

1. If the diameter of a circle be 1, what is the circumference? As 7 : 22 :: 1 to 3.142857+1 rem. By the 2d rule—As 113 : 355 :: 1 to 3.1415929+23 rem. or rather 3.1416 which is the circumference of a circle, whose diameter is 1, and is considered and used by mathematicians as being the most accurate.

2. If the diameter of a circle be 2, what is the circumference?
Ans. 6.2857+ by the 1st rule, and 6.28318+ by the 2d rule.

3. If the diameter of a mill wheel be 16ft. what is the circumference? Ans. 50.2857ft. by 1st rule, and 50.26548ft.+by 2d rule.

Problem 12.

The circumference of a circle being given, to find the diameter.

Rule.

1. As 22 is to 7, so is the circumference to the diameter of the circle.
2. As 355 is to 113, so is the circumference to the diameter required.
3. As 3.1416 is to 1, so is the circumference to the diameter.

Examples.

1. If the circumference of a circle be 3.142857+1, (by the first rule,) what is the diameter?

$$22 : 7 :: 3.142857+1$$
$$7$$

$$22)22.000000$$

Ans. 1=diameter.

2. If the circumference of a circle be 3.1415929+23, (by the 2d rule,) what is the diameter? Ans. 1.

3. If the circumference of a circle be 3.1416, what is the diameter by the 3d rule? Ans 1.

4. If the circumference of a circle be 6.28318584+8, what is the diameter by the 2d rule. Ans. 2.

Problem 13.

To find the area of a circle.

Rule.

1. Multiply half the circumference by half the diameter, or multiply the whole circumference by the whole diameter, and divide the product by 4, and the result either way will be the area required.

2. Multiply the square of the diameter by .7854, and the product will be the area required. For the area of a semicircle take one half the area of the whole circle, and for a quadrant take one fourth.

Examples.

1. What is the area of a circle whose diameter is 1, and circumference 3.1416, by both rules?

2)3.1416

1.5708=half circum.
.5=half diameter.

Ans. .78540=the area of a circle whose diameter is 1.

3.1416=whole circum.
1=whole diameter.

4)3.1416

Ans. .7854=the same as before.

2. What is the area of a circle whose diameter is 2, and circumference 6.2832, by all the rules? Ans. 3.1416.

3. Required the area of a circle whose diameter is 22.6, and circumference 71? Ans. 401.15.

4. There is a round table whose diameter is 5 feet; what is the superficial content? Ans. 19ft. 91in. 44 decimals.

PROBLEM 14.

The diameter of a circle being given, to find the side of a square that is equal in area to the given circle.

RULE.

1. Multiply the diameter of the circle by .88623, and the product will be the side of a square equal in area to the given circle.

2. As 19 : 17 :: the diameter to the side of an equal square.

NOTE.—.88623 is the side of a square which is equal in area, nearly, to the area of a circle whose diameter is unity.

EXAMPLES.

1. What is the side of a square that is equal in area to a circle whose diameter is 22.6?

$.88623 \times 22.6 = 20.028798 =$ the side required, and $20.0287 \times 20.0287 = 401.1488+$. See 3d example under problem 13th.

2. If the diameter of a circle be 12 inches, what is the side of a square equal in area to the circle? Ans. 10in. .63476 decimals.

PROBLEM 15.

The side of a square being given, to find the diameter of a circle that is equal in area to the square whose side is given.

RULE.

Divide the side of the given square by .88623, and the quotient will be the diameter required.

NOTE.—.886 will be sufficient in common business.

EXAMPLES.

1. If the side of a square be 20ft. .028798, what will the diameter of a circle be that is equal to the square?

$20.028798 \div .88623 = 22\text{in}. 6 =$ the diameter required.

2. What is the diameter of a circle that is equal in area to a square whose side is 10in. .63476? Ans. 12in.

PROBLEM 16.

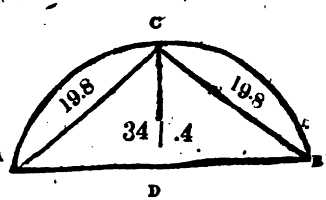
To find the length of any arch of a circle.

RULE.

From 8 times the chord of half the arch, subtract the chord *of the whole arch*, and one-third of the remainder will be the length of *the arch line* required.

EXAMPLES.

1. In the annexed segment of a circle ABCD, let the whole chord ADB be 34.4, and the chord of half the arch AC or BC 19.8; what is the length of the whole arch line ACB?



19.8=the chord AC or BC.
8
―――
158.4
34.4=whole chord of ADB.
―――
3)124.0
―――
Ans. 41⅓=the length of the whole arch line ACB.

N. B.—This problem is useful in finding the concave surface of a vaulted or arched roof. See problem 19th.

2. If the chord of half the arch AC be 126 feet, and the chord of the whole arch ADB 216 feet; what is the length of the arch line ACB? Ans. 264 feet.

PROBLEM 17.

To find the area of the segment of a circle that is less than a semicircle.

RULE.

To two thirds of the product of the chord and height of the segment, add the cube of the height divided by twice the chord, and the sum will be the area.

AN EXAMPLE.

In the preceding segment ABCD, the chord ADB is 34 4ft., and the height CD is 12ft.; required the area?

34.4=the chord ADB.
12=the height CD.
―――
3)412.8
―――
137.6
2
―――
275.2=⅔ of 344×12
25.=cube of 12÷twice 34.4
―――
Ans. 300.2 area.

	12
	12
34.4	144
2	12
68.8	1728.0 =the cube of CD.
	25+80rem.

PROBLEM 18.

To find the convex surface of a sphere or globe, or any part of the same.

RULE.

Multiply the circumference of the given globe or sphere by its diameter, or by the height of the part whose surface is required, and the product will be the answer. The diameter is the height of the whole *sphere or globe*.

EXAMPLES.

1. What is the convex surface of a globe whose diameter is 7 inches? As 113 : 355 :: 7 .. 21.99115+=circumference.

7=diameter.

The surface required=153.93805 inches. Ans.

2. If a globe be 12 inches in diameter, what is the convex surface of that part whose diameter is only 5 inches? Ans. 188.4960.

PROBLEM 19.

To find the concave surface of a circular or elliptic vaulted roof.

RULE.

Multiply the length of the arch line by the length of the roof or vault, and the product will be the concave surface required.

EXAMPLES.

1. What is the concave surface of a semicircular roof whose span is 40 feet and length 120?

3.1416
40
2)125.6640
*62.832=length of arch line.
120=length of roof.
7539.840=concave surface.

2. What is the concave surface of a vaulted roof in a church, the chord of half the arch being 30 feet, the whole chord 54, and the length 60 feet?

$30 \times 8 - 54 \div 3 = 62$=arch line.
60
Ans. 3720 square ft.

* N. B.—When the height of the vault is less than the radius of the arch line, it must be found by the 16th problem.

PROBLEM 20.

To find the area of an ellipsis or oval.

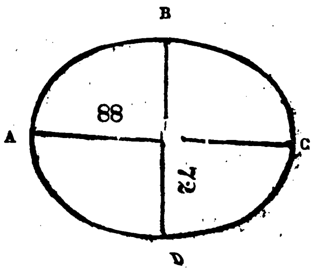
NOTE.—An ellipsis or oval is a curve which returns into itself like a circle, but has two diameters, one longer than the other; the longest is called the transverse and the shortest the conjugate diameter.

RULE.

Multiply the two diameters together, then multiply the product by .7854, and the last product will be the area of the ellipsis.

AN EXAMPLE.

In the annexed ellipsis, ABCD, the transverse diameter AC is 88ft. and the conjugate diameter BD is 72ft.; what is the area or superficial content in feet?



$88 \times 72 \times .7854 = 4976.2944$ feet= area required.

MENSURATION OF SOLIDS.

The measure of any solid body is the whole capacity of that body, when considered under the triple dimensions of length, breadth, and thickness. A cube whose side is one inch, one foot, or one yard, &c., is called the measuring unit, and the content or solidity of any figure is computed by the number of those cubes contained in that figure. A cube is a solid contained by six equal square sides.

PROBLEM 1.—To find the solidity or solid content of a cube.

RULE.

Multiply the side of the given cube into itself, and that product again by the same side, and the last product will be the solidity required.

EXAMPLES.

1. The side of a given cube is 1 foot 6 inches, what is the solidity in feet, &c.? Ans. 3ft. .375=3ft. 4in. 6 sec. Perform all the work carefully the several ways.

1ft. 6in.=1.5ft. and $1.5 \times 1.5 \times 1.5 = 3.375$ft. by decimals.

1ft. 6in.$=1\frac{1}{2}$ft.$=\frac{3}{2}$ft. and $\frac{3}{2}\times\frac{3}{2}\times\frac{3}{2}=\frac{27}{8}$ft.$=3\frac{3}{8}$ft. by vulgar fractions.

Again: 1ft. 6in.=18in. and $18 \times 18 \times 18 = 5832$ and $5832 \div 1728$ gives 3ft. 4in. 6sec. by whole numbers.

2. How many cubical blocks of 6 inches in diameter are equal to one cubical block of 12 inches in diameter? Ans. 8.

3. A cellar is to be dug, whose length, breadth, and depth are each 10ft. 4in. respectively; how many solid feet and yards does it contain, and what will the digging cost at 1s. 6d. per solid yard?

Ans. 1103 sol. ft. 4in. 5″ 4‴, and 40 sol. yds. 23ft. 4in. 5″ 4‴. The digging will cost 3£ 1s. $3\frac{1}{2}$d. Examine the calculation by Practice, and perform the work by whole numbers and Vulgar Fractions.

PROBLEM 2.—To find the solidity of a sphere or globe.

RULE.

Multiply the circumference of the given sphere or globe by its diameter, and that product by $\frac{1}{6}$ part of the same, and the last product will be the solidity required; or, multiply the cube of the diameter by .5236, and the product will be the solidity, as before.

EXAMPLES.

1. Required the solidity of a globe whose diameter is 9 inches:

As 113 : 355 :: 9 .. 28.274336+32 rem.=cir.

```
                    9
            ———————————
            254.469024=superficies.
                   1.5=1/6 of the diameter.
            ———————————
            1272345120
            254469024
            ———————————
Ans. 381.7035360=solidity required in inches
```

The solidity of a globe whose diameter is one $\}=.5236$

$9\times9\times9=729=$cube of 9.

```
   47124
  10472
 36652
```

Ans. 381.7044 as before nearly. The difference arises by taking the number a little too much.

2. Required the solidity of a globe whose diameter is 3 inches? Ans. 15in. .1844 by the 2d rule.

Problem 3.

To find the solidity of hewn timber.

If a piece of timber be of an equal bigness through its whole length, though there should be a difference between the breadth and thickness, if the breadth and thickness are multiplied together, and that product being multiplied by the length will produce the solid content required.

Examples.

1. If a piece of timber be 18 inches wide, 9 inches thick, and 9 feet 6 inches long, what is the solid content?

$1.5\times.75\times9.5=10.6875$ft. Ans. by decimals.

$18\times9\times114=18468$, and $18468\div1728=10.6875$, as before.

N. B.—1. If the piece of timber tapers regularly from one end to the other, the breadth and thickness taken in the middle will be the mean breadth and thickness.

2. If the timber does not taper regularly, take several dimensions and divide their sum by the number, and the quotient will be the true dimension, nearly.

Example 2.—If a piece of timber be 20½ft. long, breadth at the greater end 21 inches, at the less 18 inches, and thickness at the greater end 15 inches, and at less end 1 foot, what is the solidity?

21in.=1.75 greater breadth.
18in.=1.5 less breadth.

2)3.25

1.625=mean breadth.

15in.=1.25 greater thickness.
12in.=1.00 less thickness.

2)2.25

1.125 mean thickness.

Now, 1.625 ft. $\times$ 1.125 ft. $\times$ 20.5 ft. $=$ 37.4765625 ft. = the solid content required.

Example 3.—If a piece of timber be 25 inches square at the greater end, and 9 inches square at the less end, and 20 feet long, what is the solid content? Ans. 40ft. 1in. 8″.

$25+9\div2=17$ mean square, and $17\times17\times20\div144=40$ft. 1in. 8″.

Problem 4.

To find the solidity of round timber.

Rule.

1. Multiply the square of the quarter girt (or one fourth part of the circumference) by the length, and the product will be the solidity, according to the common practice, though very erroneous.

2. Multiply the square of one fifth of the whole girt by twice the length, and the product will be the solidity, extremely near the truth. See Dr. Hutton's Mensuration.

3. Multiply the square of the circumference by the length, divide the product by 11, and subtract one-eighth part of the quotient from itself, the remainder will be the true solidity required.

Note.—1. Mr. Bonnycastle says, the 3d rule differs from the truth by only one foot in 2300.

Examples.

1. If a piece of timber be $9\frac{3}{4}$ feet long, and the quarter girt be 39 inches, what is the solidity?

Here 39=3.25ft. quarter girt.
3.25
10.5625 squared.
9.75=the length.
Ans. 102.984375 feet by the 1st rule, which is 28 feet too little.

3.25ft.=$\frac{1}{4}$ girt.
4
5)13.00=circumference.
2.6=$\frac{1}{5}$ circum.
2.6
6.76 squared.
19.5=the length×2.
Ans. 131.820 feet by the 2d rule.

13ft.=circum.
13=circum.
169 squared.
9.75=length.
11)1647.75=product.
8)149.7954+6=quotient.
18.7244+2=subtract.
Ans. 131.0710ft. by the 3d rule

Note.—2. By a careful inspec- of the above operations, the inquisitive student will be enabled to decide for himself, and say which of them ought to be used in business.

3. When trees have their bark on, an allowance is generally made, by deducting so much from the girt as is judged sufficient to reduce it to such a circumference as the tree would have without the bark. In oak or pine this allowance is about $\frac{1}{10}$ or $\frac{1}{12}$ part of the girt; but for elm, beach, ash, &c. whose bark is not so thick, the deduction ought to be less. See Mr. Bonnycastle's note, page 211, and Mr. Hawney's on page 210.

Example 2.—The girts of a tree in 4 different places are as follows: in the first place 5 feet 9 inches, in the second 4 feet 6 inches, in the third 4 feet 9 inches, and in the fourth 3 feet 9inches. The length of the whole tree is 15feet; what is the solidity?

Ans. 26ft. 4in. 4' 10'' 6''' by the 2d rule.

f. in. f. in. f. in. f. in. f. in. ″
5 9+4 6+4 9+3 9÷4=4 8 3=the whole girt or circumference.
consequently, 1ft. 2in. 0″ 9‴= the quarter girt.
Now, 4ft. 8in. 3″=4.6875, and 4.6875÷5=.9375ft.=$\frac{1}{5}$ of the whole circumference.

The student will please to perform the rest of the operation.

Problem 5.

To find how much round timber will square to.

Rule.

Multiply the diameter by .7071, or by .71 in common business, and the product will be the answer.

2. Multiply the circumference by .225 and the product will be the answer.

Examples.

1. Suppose a round piece of timber to be 24 inches in diameter, and 76 in circumference; how much will it square.

Ans. 17 inches, nearly, both ways.

circum. *inches.*
76×.225=17.100 Ans. by the 2d rule.
24×.71 =17.04 Ans. by the 1st rule.

2. If a log of round timber be 13 feet in circumference, how much will it square? Ans. 2ft. 11in. .1

3. If a round stock of timber be 12 inches in diameter, how much will it square to, when hewn? Ans 8in. 4 tenths.

OF ARTIFICERS' WORK.

Artificers estimate or compute the content and value of their works by different measures; that is—

1. Glazing and mason's flat work, &c. by the foot.
2. Painting, plastering, paving, &c. by the square yard.
3. Flooring, roofing, partitioning, ceiling, wainscoting, tiling, &c. by the square of 100 feet.
4. Brick work, &c. by the rod of 16½ feet, whose square is 272¼ feet. In America, bricklayers' work is generally calculated by the 1000.
5. Stone walls are commonly measured by the solid perch of 24¾ feet. The measures generally used by workmen are contained in the preceding tables.

Of Carpenters' and Joiners' Work.

Carpenters' and Joiners' work is that of flooring, roofing, partitioning, weather-boarding, &c. The dimensions are usually taken in feet and inches, and the content required in squares of 100 feet each.

N. B.—Large and plain articles are usually measured by the foot

or yard, &c. square; but enriched mouldings, and some other articles, are often estimated by lineal or running measure, and some things are rated by the piece.

1. Of Flooring.

Joists are measured by multiplying their breadth by their depth, and that product by their length. They receive various names according to the position in which they are laid to form a floor, such as trimming joists, common joists, girders, binding joists, bridging joists, and ceiling joists. See Mr. Hawney's Mensuration, page 221.

Examples.

1. If a floor be 57 feet 3 inches long and 28 feet 6 inches wide, how many squares of flooring will it contain?

ft. in. in.	ft. in. in.	ft. in.
57 3=687	57 3=57.25	6in.=½)57 3
28 6=342	28 6= 28.5	28ft. 6in.
12)234954	1,00)16,31.625	1603 0
12)19579 6''		28 7 6''
1,00)16,31 7in. 6''	Ans. 16sq. 31ft. 625 by decimals.	1,00)16,31 7 6''
Ans. 16sq. 31ft. 7in. 6'' by whole numbers.		Ans. 16sq. 31ft. 7in. 6'' by Practice.

2. If a floor be 53ft. 6in. long and 47ft. 9in. broad, how many squares of flooring are contained therein? Ans. 25sqs. 54ft.

3. In a naked floor the girder is 14 inches deep, 1 foot wide, and 20 feet long; there are 8 bridging joists 6½ inches deep, 3 inches wide, and 20 feet long; 8 binding joists 8½ inches deep, 4 inches wide, and 9 feet long; 24 ceiling joists 4 inches deep, 2½ inches wide, and 6 feet long. Required the solidity of the whole?

	ft.	in.
The girder contains - - -	23	4 solid
8 bridging joists contains - -	21	8 ——
8 binding —— —— - -	17	0 ——
24 ceiling —— —— - -	10	0 ——
Ans.	72	0 in all.

4. Suppose a house of three stories, besides the ground floor, is to be floored at 6£ 10s. per square, (the workmen to find materials,) the house measures 20 feet 8 inches by 16 feet 9 inches; there are 7 fire places—2 of them measure, each, 6 feet by 4 feet 6 inches—2 others, each, 6 feet by 5 feet 4 inches—2 more, each, 5 feet 8 inches by 4 feet 8 inches, and the 7th 5 feet 2 inches by 4 feet; the well-hole for the stairs is 10 feet 6 inches by 8 feet 9 inches. What will the whole amount to?

ft. in.	ft. in.	no.	feet	in.	″	
20 8×	16 9×	4=	1384	8	0=	the area of all the floors.
4 6×	6 0×	2=	54	0	0=	the area of the first 2 fire places.
5 4×	6 0×	2=	64	0	0=	the area of the second 2 fire places.
5 8×	4 8×	2=	52	10	8=	the area of the third 2 fire places.
5 2×	4 0×	1=	20	8	0=	the area of the seventh fire place.
10 6×	8 9×	4=	367	6	0=	the area of the 4 well-holes.
Deduct			559	0	8	from the whole area.
			825	7	4=	8sqs. 25ft. 7in. 4 seconds.

ft. sq.	£ s.		ft. sq.	$ cts.
$25=\frac{1}{4}$	6 10		$25=\frac{1}{4}$	$21.66\frac{2}{3}$
	8			8
in.	52 00 d.		in.	$173.33\frac{1}{3}$
$6=\frac{1}{50}$	1 12 6		$6=\frac{1}{50}$	$5.41\frac{2}{3}$
$1=\frac{1}{6}$	0 00 $7\frac{3}{4}$+		$1=\frac{1}{6}$	$10\frac{5}{6}$+
$4=\frac{1}{3}$	0 00 $1\frac{1}{4}$+		$4''=\frac{1}{3}$	$1\frac{5}{8}$+
	0 00 $0\frac{1}{4}$+			$\frac{1}{3}$+
	53 13 $3\frac{1}{4}$= Hawney's answer.			$178.87\frac{5}{8}$=answer in federal money.

Of Roofing.

Rule.

When the roof is of a true pitch, (commonly called a square roof,) that is, when the length of the rafters is equal to $\frac{3}{4}$ of the breadth of the building, multiply the area of the floor by $1\frac{1}{2}$, and the product will be the area of the roof; but when the roof is higher or lower than the true pitch, measure over it from eave to eave with a string and multiply that distance by the length of the roof; the product will be the area or superficial content required. To find the area of the gable ends, multiply the breadth of the house by the height of the roof, and the product will be equal to the area of both gable-ends.

Examples.

1. If the length of a roof be 45 feet 9 inches, and the girt over the top from eave to eave 34 feet 3 inches, what is the superficial content in squares, &c.? 45ft. 9in.×34ft. 3in.=1566ft. 11in. 3″=15sqs. 66ft. 11in. 3″ the answer.

2. If a house within the walls be 44 feet 6 inches long, and 18 feet 3 inches broad, how many squares of roofing will cover that house, supposing the roof to be of the true pitch? 44ft. 6in.×18ft. 3in.×$1\frac{1}{2}$=1218ft. 2in. 3″=12sqs. 18ft. 2in. 3″ answer.

3. What will the shingling of a house cost at $1.75cts. per

square, the length within the walls being 52 feet 8 inches, and the breadth 30 feet 6 inches, the roof being of a true pitch?

Ans. \$42.16½cts.

4. The roof of a house is of a true pitch, the house measures 40 feet 6 inches in length and 20 feet 6 inches in breadth; how many squares of roofing are contained thereon, and what will it cost at \$2.25cts. per square? Ans. 12sqrs. 45ft. 4in. 6″ and \$28.03cts.+

3. Of Slaters' and Tilers' Work.

In slating, an allowance must be made for the double row at the bottom. In taking the girt, the line must be made to ply over the lowest row of slates, and return up the under side till it meet with the wall or eaves-board, but in tiling the line is stretched down only to the lowest part without returning it up again. Double measure is generally allowed for hips, valleys, gutters, &c., but no deductions are made for chimneys. Slating and tinning are measured by the square yard of 9 feet.

Examples.

1. If the length of a slated roof be 36 feet 8 inches, and the girt 34 feet 6 inches, how many square yards of roofing will it contain, and what will it cost at 80 cents per yard?

Ans. 140yds. 5ft., cost \$112.36cts.

2. If a roof that is covered with tin be 24 feet 6 inches long, and 27 feet 6 inches girt, what will the tinning cost at 90 cents per square yard? Ans. \$67.37½cts.

3. What will the tiling of a barn cost at \$3.40cts. per square, the length being 44 feet, and the breadth 29 feet 6 inches, the roof being of the true pitch? Ans. \$66 19cts.8m.

4. Of Partitioning.

Boarded or planked partitions are measured by the square in the same manner as flooring—deductions are usually made for doors and windows, except they are included by agreement.

Examples.

1. If a partition between rooms be 82 feet 6 inches long, and 12 feet 3 inches high, how many squares are contained therein, and what will the work cost at \$6.50cts. per square? Ans. 10 squares, 10 feet, and the work will cost \$65.65cts.

2. There are 8 partition walls between rooms, each being 14 feet 8 inches long, and 10 feet 6 inches high; how many squares do they contain, and what will the work cost at \$7.50cts. per square?

Ans. 12sqs. 32ft., the cost will be \$92.40cts.

5. Of Wainscoting.

In wainscotting, take the compass of the room for the length, and multiply it by the height of the room, from the floor to the ceiling,

and the product will be the area in square feet, which may be changed into square yards of 9 feet each, or squares of 100 feet each, as may be required by the workman. In taking the dimensions, the string must be made to ply close into all the mouldings.

EXAMPLES.

1. The compass of a certain room is 188 feet 8 inches, and the height from the floor to the ceiling 9 feet 9 inches, what will the wainscotting amount to at 50cts. per square yard? Ans. \$102.19⅓cts.

2. If a room be 52 feet 6 inches long, and 44 feet 8 inches wide, and 12 feet 9 inches high from the floor to the ceiling, how many squares of wainscotting will there be, and what will the work cost at \$5.60cts. per square? Ans. 24sqs. 77ft. 9in. and the work will cost \$138.75cts. 4m.

6. OF PLASTERERS' WORK.

Plasterers' work is of two kinds, that is, plastering upon laths, called ceiling, and plastering upon walls called rendering. The dimensions are taken in feet and inches, and content required in square yards. Deductions must be made for doors and windows.

EXAMPLES.

1. If a ceiling be 59 feet 9 inches long, and 24 feet 6 inches wide, how many yards of plastering does it contain, and what will it cost at 12½ cents per yard? Ans. 162yds. 5ft. 10in. 6 seconds—the cost will be \$20.33cts. 1m + .593 rem.

2. If a room measure 141 feet 6 inches round within the walls, and 11 feet 3 inches high from the floor to the ceiling, what will a double coat of plastering cost at 24 cents per yard? Ans. 176 yds. 7ft. 10in. 6sec., and the work will cost \$42.44cts.9m. + .98rem.

7. OF PAINTERS' AND GLAZIERS' WORK.

Painting is measured by the square yard; glazing by the square foot, or at a stipulated price by the pane according to the size of the glass; the dimensions are taken in feet and inches, &c. or in inches and parts—the line must be made to ply close into all the corners and mouldings. Suitable deductions must be made for doors, windows, &c.

EXAMPLES.

1. If the compass of a room be 81ft. 8in., and the height 9ft. 6 inches, how many square yards does it contain, and what will the painting come to at 18cts. per square yard?

Ans. 86yds. 1ft. 10in.—cost \$15.51,6m. 66 rem.

2. If a house be 42ft. long within the walls, and 28 feet wide, with a square roof, how many square yards are contained on the said roof, and what will the painting cost at 12½cts. per yard?

Ans. 226 square yards—cost of painting \$28.25cts.

Decimally.	in. ft. in.
12.75ft. wide.	6=½)52 6 long.
52.5 long.	12 9 wide.
6375	in. 630 0
2550	3=½)26 3 ″
6375	13 1 6
669.375=superficial content	669 4 6=superficial con.
5=no. of half bricks.	5=no. of half bricks.
3)3346.875	3)3346 10 6
1115.625=standard thickness	1115 7 6=standard thick.
18	6
8925.000	6693 9 0
11156 25	3
20.081.25=the number of bricks required to build the wall.	20081 3 0=number of bricks required as before.

Superficial content in standard thickness.

272)1115ft. 7in. 6″(4 rods 27ft. 7in. 6″
1088 Ans. in rods, &c.

27ft. rem.

2. How many square rods are contained in a wall that is 62ft. 6 inches long, 14ft. 8in. high, and 2½ bricks thick; and how many bricks will be required to complete the said wall? Ans. 5 rods 167 feet 9 inches 4″, and 27,500 bricks of the common size will complete the wall.

3. If each side wall of a building 45ft. long on the outside, each end wall 15ft. broad on the inside, the height of the building 20ft., and the gable at each end of the wall 6ft. high, the whole being two bricks thick; what number of bricks will be required to complete the said building? Ans. 59,760.

10. Of Stone Masons' Work.

Stone work is sometimes measured by the solid perch of 24¾ feet, but more frequently by the superficial or solid foot. Solid measure is principally used for materials, and the superficial for workmanship. Masons usually take the whole girt round the outside of the building. The thickness of the wall is not reckoned to the mason at less than 18 inches; but if it is more than that thickness it must be reduced to it by multiplying the solid content of the wall by its thickness in inches, and dividing the product by 18, which will reduce it to the standard thickness.

Examples.

1. Required the solid content of a wall whose length is 48ft. 6in.,

its height 10ft. 9in., and thickness 2 feet; also, what is the solid content in standard thickness?

ft. in.
48 6=length.
10 9=height.

521 4 6″=superficies.
2ft.=thickness.

Ans. 1042 9 0=solidity 2ft. thick
24 inches=thickness.

18)25026 0 0

Ans. 1390 4 0=solidity in standard thickness.

By Decimals.
$48.5 \times 10.75 \times 2 = 1042.75$ = the solid content at 2 feet thick.
$1042.75 \times 24\text{in.} \div 18 = 1390.333$ + =the solid content in standard thickness.

2. The dimensions of a certain building are 58ft. long by 26ft. broad on the outside; the height is 22ft., and the gable at each end is 12ft. high, and the wall is 15 inches thick; there are two doors 4½ by 8 feet each, 28 windows each 3½ by 6 feet; what will the mason's work amount to at 56cts. a perch, and what must be paid for the stone at 44cts. a perch? Ans. For the Mason's work $136.02cts. 9m.+.04, and for the stone $74.36cts. The student is requested to perform the work at large very carefully.

3. In a chimney piece, suppose the length of the

		ft. in.
mantle and slab, each	=	4 6
The breadth of both them together	=	3 2
The length of each jamb	=	4 4
The breadth of both together	=	1 9

What will be the content of the chimney piece?

$$\left.\begin{array}{l} \overset{ft.\,in.}{4\ 6} \times \overset{ft.\,in.}{3\ 2} = \overset{ft.\ \ in.}{14\ \ 3} \\ 4\ 4 \times 1\ 9 = \ \ 7\ \ 7 \end{array}\right\} = 21\text{ft. } 10\text{in. Ans.}$$

4. What is a marble slab worth at 80cts. per superficial foot, the length of which is 5ft. 7in. and the breadth 1 foot 10 inches.

Ans. $8.75⅓cts.+.

OF GAUGING.

Gauging is the art of measuring and finding the cubical inches contained in any corn-house, garner, tun, back, cooler, wagon body, &c., either square or oblong; also, in all sorts of hogsheads, pipes, barrels, tubs, casks, and other vessels in common use; likewise, the contents of the same, in gallons, bushels, &c., as may be required.

Problem 1.

To find the several multipliers, divisors, and gauge-points belonging to the several measures now in use.

1. *For square figures, &c.*

N. B. 282 solid inches make 1 gallon of ale or beer measure.
231 solid inches make 1 gallon of wine or rum measure.
268⅘ solid inches make 1 gallon of dry measure.
2150 solid inches make 1 bushel of dry measure.

Now, if one gallon of any measure be divided by the solid inches contained in a gallon of that measure, the quotient will be a fixed multiplier in the said measure.

gallon. multiplier.

282)1.0000(.003546+ for ale gallons.
231)1.0000(.004329+ for wine gallons.
268.8)1.0000(.00372 + for dry gallons.
2150)1.0000(.000465+ for bushels.

So, if the solid inches in any square thing be multiplied by any one of the above multipliers, or divided by any one of the above divisors, the product or quotient will be the answer in gallons, or in bushels, according to the multiplier or divisor used.

2. *For circular areas, &c.*

Now, as the area of a circle whose diameter is 1 inch, is .7854 decimal parts of an inch, if the solid inches in a gallon or bushel be divided by .7854, the quotient will be a fixt divisor for the square of any diameter, to reduce the cubic inches into gallons or bushels. The fixt multipliers, which answer the same purpose, are found by dividing .7854 by the solid inches in a gallon or bushel, or by dividing 1 gallon by the divisor belonging to the measure for which you want to find a fixt multiplier for circular areas.

solid inches.

.7854)282.0000(359=the divisor for ale gallons.
.7854)231.0000(294=the divisor for wine gallons.
.7854)268.0000(342.24=the divisor for dry gallons.
.7854)2150.000(2738=the divisor for bushels.

Now let us find the fixt multipliers.

282).785400(.002785=the multiplier for ale gallons.
231).785400(.0034=the multiplier for wine gallons.
268.8).785400(.00292=the multiplier for dry gallons.
2150).785400(.000365=the multiplier for bushels.

N. B.—The square root of the divisor is the gauge-point.

For square figures.	*Divisors.*	*Gauge-point.*
	$\sqrt{282}$	=16.79 for ale gal.
	231	=15.19 for wine gal.
	268.8	=16.39 for dry gal.
	2150	=46.37 for bushels.

For circular figures.	*Divisors.*	*Gauge-point.*
	$\sqrt{359}$	=18.95 for ale gal.
	294	=17.15 for wine gal.
	342	=17.9 for dry gal.
	2738	=52.32 for bushels.

PROBLEM 2.

Of gauging Corn-houses, &c.

RULE.

1. Multiply the length, breadth, and depth (reduced to inches) continually together, and the last product will be the cubical inches, which must be divided by 268.8, and the quotient will be gallons, *or by 2150.4,* and the quotient will be bushels of dry measure.

Problem 3.

Of the gauging of Casks, &c.

1. To find the content of a cask by the mean diameter.

Rule.

1. When the difference between the bung and head diameter exceeds 6 inches, multiply that difference by .7 tenths of an inch; if it be 6, or between 6 and 5 inches, multiply by .65; if it be 5, or between 5 and 4 inches, multiply by .6; if it be 4, or between 4 and 3 inches, multiply by .55; if it be 3 inches, or less, multiply by .5 tenths; then add the product in each case to the head diameter, and that sum will be the mean diameter required.

2. Multiply the square of the mean diameter by the length of the cask, then divide the product by 359 for ale, and by 294 for wine gallons.

Examples.

1. Required the content of a hogshead in ale and wine gallons, whose bung diameter is 35 inches, head diameter 27 inches, and length 45 inches? Ans. 133gal. 1pi. 2gi.+.88 of ale, and 162gal. 5pi. 1gi.+.344 of wine.

inches.
35=the bung diameter.
27=the head diameter.

8=the difference.
.7 tenths.

Add { 5.6=the product 8×.7 tenths.
27. =the head diameter.

32.6=the mean diameter.
32.6=— —— ———

1062.76=the square of the mean diameter.
45=the length.

359)47824.200(133gal. 215dec. of ale or beer.

15 rem.

294)47824.200(162gal. 667dec. of wine.

102 rem.

2. Suppose the bung diameter of a cask to be 32 inches, the head diameter 24 inches, and the length 40 inches; how many ale and wine gallons will it hold? Ans. 97gal. 5pi.+ of ale, and 119 *gal. 1pi. 2 gills .4* of wine.

2. To find the content of a cask by two dimensions.

RULE.

To the square of the head diameter add twice the square of the bung diameter, and multiply the sum by the length of the cask; then divide the product by 1077 for ale and 882 for wine gallons. The answers may be found sufficiently accurate and more concisely by multiplying the solid inches by $.0009\frac{1}{3}$ for ale, and by $.0011\frac{1}{3}$ for wine gallons.

THE SAME EXAMPLES.

35in. × 35in. = 1225 = the square of the bung diameter.
2

2450 = twice the square of the same.
27in. × 27in. = 729 = the square of the head diameter.

3179
45 = the length of the cask.

1077)143055(132 gals. 6pi. 2 gills of ale. Ans.

510 rem.

882)143055(162gals. 1pi. 2 gills of wine. Ans.

180 rem.

More concisely by Multiplication—

$143055 \times .0009\frac{1}{3} = 132$gals. 4pi.+ of ale. Ans.
$143055 \times .0011\frac{1}{3} = 162$gals. 1pi.+ of wine. Ans.

N. B.—The divisors 1077 and 882 are found by multiplying 359 and 294 by 3, because there are 3 squares used in this method and one in the first. The multipliers are found by dividing 1 gallon by the said divisors.

DIRECTION 1.—In taking the dimensions of a cask, &c., be very careful to measure the length of the stave exactly; then take the depth of the chimes with the thickness of the heads from the length of the stave, and the remainder will be the length of the cask within.

2. You must take the head diameter close to the outside, and for small casks of about 30, 40, or 50 gallons, add .4 tenths; for large casks of about 60 or 70 gallons, add .5 tenths, and for larger casks add .6 or .7 tenths, and the sum will be the head diameter within the cask very near to the truth.

3. In taking the bung diameter be very careful to observe, by moving the rod backward and forward, whether the stave opposite to the bung stave be thicker or thinner than the rest, and if it be so, make allowance accordingly; the bung-hole should be exactly in the middle of the cask.

3. To gauge a Cask by the diagonal.

Rule.

1. With your gauging rod, or some other rod that is perfectly straight, take the distances from the middle of the bung-hole within the cask both ways, to the meeting of the head of the cask with the stave exactly opposite to the bung stave. If these distances differ, half their sum will be the true diagonal required.

2. Divide the cube of the diagonal by 448.56 for ale gallons, and by 366.7 for wine gallons. The answers may be found sufficiently accurate and more concisely by multiplying the cube of the diagonal by $.0022\frac{1}{3}$ for ale gallons, and by $.0027\frac{1}{3}$ for wine gallons.

Note.—1. The common divisor used in gauging by the diagonal in all cases is 370, which will give nearly one gallon too little in every 110 gallons of wine, and a little more than 19 gallons too much in the same quantity of ale or beer, if Mr. Bonnycastle be correct in the answers to his first question, which are found by the gauging rod.

2. In taking the diagonal distance be very careful to observe whether the bung-stave and the one opposite be thinner than the rest, and if they are so, make allowance accordingly by reducing the diagonal.

An Example.

The diagonal distance from the middle of the bung-hole to the lower chime opposite to the bung-stave in a certain cask is 34 inches 4 tenths; what is the content thereof in ale and wine gallons?

Ans. 90gals. 3qts. of ale, and 111gals. of wine, according to Mr. Bonnycastle's gauging rod. See his 1st question, p. 242.

```
         34.4
         34.4
       ------
      1183.36
         34.4
      -------
448.56)40707.584(90.75gals.ale.
       -------
       7640 rem.

366.7)40707.584(111.01 gallons
      ---------        of wine.
      217 rem.

   The common way.
370)40707.584(110.020gals. of
    ---------          all sorts.
    184 rem.
```

Now, by multiplication.
$40707.584 \times .0022\frac{1}{3} = 90.9136042$ + which is $=$ 90gals. 3qts. 1pt. 1 gill of ale. $40707.584 \times .0027\frac{1}{3} = 111.2673962+$ which is $=$ 111 gallons 1 quart + of wine.

N. B. The operation may be abbreviated by omitting the decimal .584, and the answer will be more accurate.

$40707 \times .0022\frac{1}{3} = 90.9123$, which is $=$ 90gals. 3qts. 1pt. 1g. the same as before.

$40707 \times .0027\frac{1}{3} = 111.2658$, which is $=$ 111gals. 1qt. of wine as before.

Problem 4.

To gauge a round mash-tub, or any other similar vessel.

RULE.

To the product of the top and bottom diameters, add one-third of the square of their difference, then multiply that sum by the height, and divide the product by 359 for ale or beer gallons, and by 342 for gallons of dry measure.

AN EXAMPLE.

A round mash-tub is 42 inches diameter at the top, 36 at the bottom, and 48 inches high; what is the content thereof in ale and dry gallons?

Ans. 203gals. 3qts.+ of ale, and 214gals. nearly of dry measure.

```
42 × 36=1512
 6×6÷3=   12
        ----
        1524
 Height=  48
        ----
    359)73152(203gal.3qt. ale.
        718..
        ---
         1352
         1077
         ----
          275
            4
         ----
     359)1100(3 quarts.
         1077
         ----
           23 rem.
```

```
342)73152(214gals. dry measure.
    684..
    ---
     475
     342
     ---
     1332
     1368
     ----
     —36 wanting.
```

N. B.—The above method of forcing the last figure in the quotient is sometimes admitted in the higher branches of Arithmetic, because remainders are often rejected in finding the divisors.

PROBLEM 5.

The difference of the diameters of any mash-tub, or other similar vessel, the height or length thereof, and the content of the same in gallons being given, to find the diameters.

RULE.

Multiply the content for ale measure by 359, by 294 for wine, &c. and by 342 for dry measure, then divide the product by the height or length; from the quotient subtract $\frac{1}{3}$ of the square of the difference of the diameters, and extract the square root of the remainder; from the square root subtract $\frac{1}{2}$ of the difference of the diameters, and this last remainder will be the less diameter to great exactness; to which add the difference of the diameters, and the sum will be the greater diameter required.

EXAMPLES.

1. The difference of the diameters in a mash-tub is 6 inches, the height is 48 inches, the content is 203 gallons of ale+275 rem. and 214 *gallons of dry* measure—36; what are the diameters?

Ans. 36 and 42.

Ale measure.
203gals.+275 rem.
359
———
2102
1015
609
———
48)73152
———
1524
6×6÷3=12
———
√1512 (39=square root.
9.. | 3=½ difference.
——— | ——
69)612 | 36=less diam.
621 | 6=difference.
—— | ——
—9 | 42=greater diam.

Dry measure.
214gals.—36.
342
———
428
856
642
———
73188
—36
———
48)73152
———
1524
6×6÷3=12
√1512 (39=square root.
9.. | 3=½ difference.
69)612 | 36=less diam.
621 | 6=difference.
——— | ——
—9 | 42=greater diam.

2. Required the diameters of a measuring-tub, whose difference is 3.75 inches, height 36 inches, and content 5 bushels?

Ans. The top diameter must be 21¼in., and the bottom 17½in.

3. Required the diameters of a beer-stand, whose difference is 4½ inches, height 40 inches, and the content to be 50 gals. of beer?

Ans. The top diameter must be 23¼in. and the bottom 18¾in.

Problem 6.

The difference between the bung and head diameters, the length of the cask, and the content in wine gallons being given, to find the bung and head diameters.

Rule.

Multiply the given content by 294, then divide the product by the length of the cask, and the square root of the quotient will be the mean diameter, from which subtract the proper allowance for the curvature of the staves, (found by the first rule under the gauging of casks,) and the remainder will be one of the head diameters, to which add the difference between the bung and head diameters, and that sum will be the bung diameter required.

Examples.

1. The difference between the bung and head diameters of a wine hogshead is 8 inches, the length is 45 inches, and the content 162 gallons 667 dec. +102 rem.; what are the bung and head diameters?

Ans. The bung diameter is 35 inches, and each head diameter 27in.

```
gals. dec.   rem.
162.667+102
     294
  -------
   650770
  1464003
  325334
  -------
45)47824.200
   ---------
  ³√1062.76 (32.6in.=the mean diameter.
     9          5.6=8in×.7 tenths.
   ----         ---
  62)162       27=the head diameter.
     124        8in.=the difference.
   ----         --
  646)3876     35=the bung diameter.
      3876
      ----
      ....
```

2. You are required to find the bung and head diameters of a n hogshead, whose difference is 7in., the length being 42in. and t content 123.48 gallons? Ans. The bung diameter must be 3 inches, and each head diameter 24½ inches.

PROMISCUOUS QUESTIONS.

1. You are required to write down eleven thousand eleven hu dred and eleven, with five figures only. Ans. 121

2. Add fifteen thousand fifteen hundred and fifteen, and ei teen thousand, eighteen hundred and eighteen together, and tell amount. Ans. 363

3. What is the difference between 9 feet square and 9 squ feet? Ans. 72 square fe

4. What is the difference between twice forty and four, and tw four and forty? Ans.

5. What is the difference between 6 dozen dozen and hal dozen dozen? Ans. 7

6. Jacob by contract was to serve Laban 14 years for his t daughters, and when he had accomplished 11 years, 11 months, weeks, and 11 days, the remaining time is required?

Ans. 1 year, 11 lunar months, 3 weeks, and 3 da

7. The sum of two numbers is 750, the less number is 248; w is their difference, product, and the square of their difference?

Ans. Their difference is 254, their product is 124496, and square of their difference is 64516.

8. What is the product of .50 cents multiplied by .50 cents? Ans. .25 cents.

9. What is the difference between three times thirty-three, and three times three and thirty? Ans. 60.

10. There is a certain number which, being divided by 7, the quotient resulting multiplied by 3, that product divided by 5, from this quotient 20 being subtracted, and 30 added to the remainder, the half of this sum will be 35; what is the number? Ans. 700.

11. What part of 3 pence is equal to $\frac{4}{7}$ of 2 pence? Ans. $\frac{4}{7}$.

12. What part of $6\frac{1}{4}$ cents is equal to $\frac{2}{3}$ of $12\frac{1}{2}$ cents? Ans. $\frac{4}{3}$.

13. The hour and minute hands of a clock or watch are exactly together at 12 o'clock; when will they be together between 1 and 2, between 2 and 3, and between 3 and 4 o'clock, &c.

N. B. the velocities of the two hands of a clock or watch are to each other as 12 is to 1; therefore, the difference of their velocities is $12-1=11$.

$$\text{Consequently, As}\left\{\begin{array}{l} 11 : 12 :: 1 \;..\; 1 \quad 5 \quad 27\frac{3}{11} \\ 11 : 12 :: 2 \;..\; 2 \quad 10 \quad 54\frac{6}{11} \\ 11 : 12 :: 3 \;..\; 3 \quad 16 \quad 21\frac{9}{11} \\ 11 : 12 :: 4 \;..\; 4 \quad 21 \quad 49\frac{1}{11} \\ 11 : 12 :: 5 \;..\; 5 \quad 27 \quad 16\frac{4}{11} \end{array}\right\}\text{Answer.} \qquad (h.\ m.\ sec.)$$

Now observe, they will be together at 5m. $27\frac{3}{11}$sec. after 1 o'clock, and at 10m. $54\frac{6}{11}$sec. after 2 o'clock, &c.

14. A cubical foot of brass is to be drawn into a wire of $\frac{1}{40}$ of an inch in diameter; what will be the length of the wire, allowing no loss in the metal? Ans. 55 miles 984yds. 1ft. 8in. +.46.

15. A young gentleman making his addresses in a lady's family who had five daughters, she told him that their father had made a very curious will, which imported that the first four of the girls' fortunes were together to make 50,000£; the last four 66,000£; the three last with the first 60,000£; the three first with the last 56,000£, and the two first with the two last 64,000£, which, if he would unravel and make it appear what each one was to have as he appeared to have a partiality for Harriet, her third daughter, he should be welcome to her; pray what was Miss Harriet's fortune? Ans. 10,000£.

16. In a certain school $\frac{1}{16}$ of the scholars learn geometry; $\frac{3}{8}$ learn grammar, $\frac{3}{10}$ learn arithmetic, $\frac{3}{20}$ learn to write, and 9 learn to read; how many scholars were in the said school, and how many were in each class? Ans. 5 geometricians, 30 grammarians, 24 arithmeticians, 12 writers, and 9 readers—whole number of scholars 80.

17. If A can do a piece of work alone in 7 days, and B in 12 days, set them both about it together, in what time will they finish it? Ans. $4\frac{8}{19}$.

18. A and B together can build a boat in 20 days, with the assistance of C they can do it in 12 days; in what time can C do it by himself? Ans. 30 days.